A1 Doubles Tracking Test 4 Part A

(36 marks: 44 minutes)

1. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \text{f}(x)$ is shown in Figure 1.

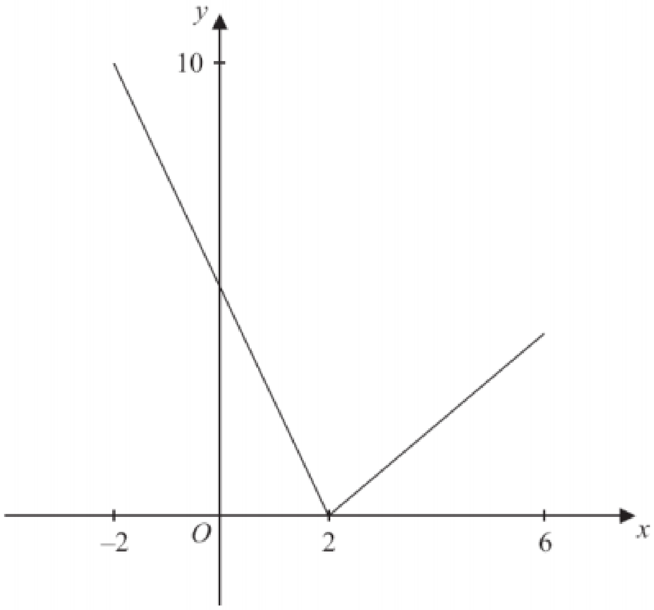


Figure 1

(*a*) Write down the range of f.

(1)

(*b*) Find ff(0).

(2)

The function g is defined by

$$\text{g} : x \to \frac{4+3x}{5-x}, \qquad x \in \mathbb{R}, \qquad x \neq 5.$$

(*c*) Find $\text{g}^{-1}(x)$.

(3)

(*d*) Solve the equation $\text{gf}(x) = 16$.

(5)

2.

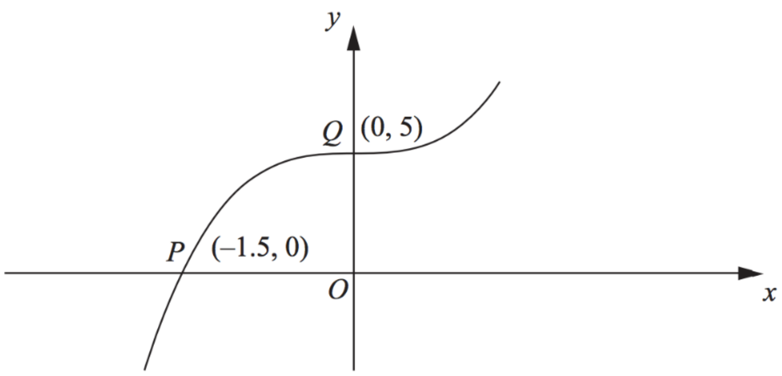


Figure 2

Figure 2 shows part of the curve with equation $y = f(x)$.

The curve passes through the points P(-1.5, 0) and Q(0, 5) as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$

(2)

(b) $y = f(|x|)$

(2)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

3. Find the integral

$$\int cosx sin^2 x \, dx$$

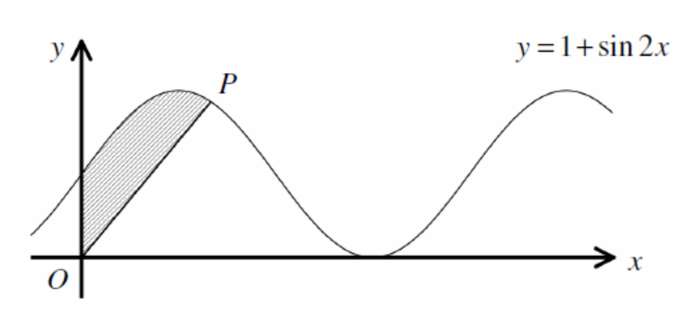
(3)

4. Prove, from first principles, that the derivative of $\cos 3x$ is $-3sin3x$.

You may assume that as $h \to 0, \frac{\sin 3h}{h} \to 3$ and $\frac{cos3h-1}{h} \to 0$

(5)

5.



The figure above shows the graph of the curve with equation

$$y = 1 + \sin 2x, \quad x \in \mathbb{R}.$$

The point P lies on the curve where $x = \frac{\pi}{3}$.

Show that the area of the finite region bounded by the curve, the y axis and the straight line segment OP is exactly

(8)

$$\frac{1}{12}\left(2\pi + 9 - \pi\sqrt{3}\right).$$

6. The curve C has equation

$$y = \frac{kx^2 - a}{kx^2 + a},$$

where k and a are non zero constants.

a) Find a simplified expression for $\frac{dy}{dx}$ in terms of a and k.

b) Hence show that C has a single turning point for all values of a and k, and state its coordinates.

(5)

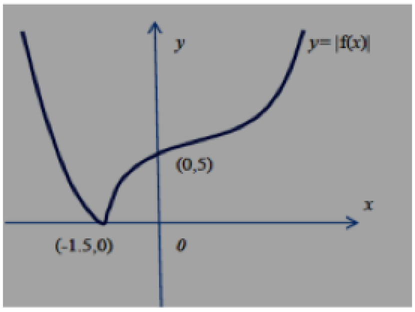
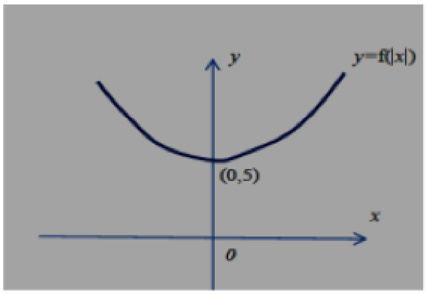
END OF TEST

Mark Scheme TT4 Part A

1.

(a)	$0 \leqslant f(x) \leqslant 10$	B1 (1)
(b)	$ff(0) = f(5),\ = 3$	B1, B1 (2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y - 4 = xy + 3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1 (3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1 A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4$ oe	M1 A1 (5)
		[11]

2.

Question Number	Scheme	Marks
(a)	 Shape including cusp	B1
	(-1.5, 0) **and** (0, 5)	B1
		(2)
(b)	 Shape	B1
	(0,5)	B1
		(2)

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the *x*-axis at (-1.5, 0) **and** crossing the *y*-axis at (0,5)

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the *y*- axis

B1 (0,5) lies on the curve

3.

$\int \cos x\,(\sin x)^2\ dx$ $y = (\sin x)^3$ $\frac{dy}{dx} = 3\cos x\ (\sin x)^2$	M1 A1
$= \frac{1}{3} sin^3 x + c$	A1

4.

Let $f(x) = \cos(3x)$ $f'(x) = \lim_{h\to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$ $= \lim_{h\to 0}\left(\frac{\cos(3x+3h) - \cos(3x)}{h}\right)$ $= \lim_{h\to 0}\left(\frac{\cos 3x \cos 3h - \sin 3x \sin 3h - \cos 3x}{h}\right)$	M1 A1
$= \lim_{h\to 0}(\left(\frac{cos3h - 1}{h}\right) cos3x - \left(\frac{sin3h}{h}\right) sin3x)$	M1
As $h \to 0, \left(\frac{sin3h}{h}\right) \to 3$ and $\left(\frac{cos3h-1}{h}\right) \to 0$, So the expression in side the limit tends to 0x$cos3x$ − 3x$sin3x = -3sin3x$	M1
Hence the derivative of $\cos(3x)\ is - 3\sin(3x)$	A1

5.

$\int 1 + sin2x\ dx = \left[x - \frac{1}{2}cos2x\right]_0^{\frac{\pi}{3}}$	M1 A1
$= \left(\frac{\pi}{3} + \frac{1}{4}\right) - (-\frac{1}{2})$ $= \frac{\pi}{3} + \frac{3}{4}$	M1 A1
When $x = \frac{\pi}{3}, y = \frac{2+\sqrt{3}}{2}$	B1
Area of triangle = $\frac{1}{2}$ x $\frac{\pi}{3}$ x $\frac{2+\sqrt{3}}{2}$ = $\frac{2\pi+\sqrt{3}\pi}{12}$	M1
Shaded region = $\frac{\pi}{3} + \frac{3}{4} - \frac{2\pi+\sqrt{3}\pi}{12}$ = $\frac{4\pi+9-2\pi+\sqrt{3}\pi}{12}$	M1
$= \frac{1}{12}(2\pi + 9 - \pi\sqrt{3})$ *	A1

6.

$$y = \frac{kx^2 - a}{kx^2 + a}$$	
$$\frac{dy}{dx} = \frac{(kx^2 + a)(2kx) - (kx^2 - a)(2kx)}{(kx^2 + a)^2}$$	M1 A1
$$\frac{dy}{dx} = \frac{4akx}{(kx^2 + a)^2}$$	A1
Turning point where $\frac{dy}{dx} = 0$, therefore $4akx = 0$ So when $x = 0$, $y = \frac{-a}{a} = -1$ Single turning point occurs at $(0, -1)$	M1 A1