Tracking Test 3 Part A

(36 marks: 43 minutes)

1 $f(x) = x^3 - 3x + 2.$

(a) Use the factor theorem to show that (x+2) is a factor of $f(x)$. **(2)**

(b) Given that $f(x) = (x+2)(x-1)^2$,

express $\frac{3x^3+x^2-18x+20}{x^3-3x+2.}$ in partial fractions.

(6)

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2 (a) Prove that

$$tan\theta + cot\theta = 2cosec2\theta, \qquad \theta \neq \frac{n\pi}{2},\ n \in \mathbb{Z}$$

(4)

(b) Given the equation

$$tan\theta + cot\theta = k$$

has real solutions, find all possible values of k.

Write your answer in set notation.

(1)

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3. Given that

$$2\cos(x + 50)° = \sin(x + 40)°$$

(a) Show, without using a calculator, that

$$\tan x° = \frac{1}{3}\tan 40°$$

(4)

(b) Hence solve, for $0 \leqslant \theta < 360$,

$$2\cos(2\theta + 50)° = \sin(2\theta + 40)°$$

giving your answers to 1 decimal place.

(4)

4. The curve C has the equation

$$f(x) = e^{3x} sin5x$$

Show that the turning points of C occur when $tan5x = -\frac{5}{3}$

(5)

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5. The circle C has equation $x^2 + y^2 - 4x + 8y = 33$.

(a) Express C in the form $(x-a)^2 + (y-b)^2 = r^2$ **(2)**

The points P(-5,-2) and Q(9,-6) both lie on C.

(b) Show that PQ is a diameter of C. **(2)**

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6. The curve C has equation $y = x^3 + 6x^2 - 12x + 6$

a) Show that C is concave on the interval [-5,-3].

(3)

b) Find the coordinates of the point of inflection.

(3)

END OF PART A

Mark Scheme

1.

1a	$(f(-2)) = (-2^3) - 3 \times (-2) + 2$	Attempts $f(-2)$. Some sight of (-2) embedded or calculation is required.	M1
	$f(-2) = 0$ so $(x + 2)$ is a factor.	Requires correct statement and conclusion. Both "$f(-2) = 0$" and "$(x + 2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $f(-2) = 0$, but in these cases there must be a minimal statement ie QED, "proved" etc.	A1
1b	$\frac{3x^3 + x^2 - 18x + 20}{x^3 - 3x + 2} = 3 + \frac{x^2 - 9x + 14}{x^3 - 3x + 2}$ $\frac{x^2 - 9x + 14}{x^3 - 3x + 2} = \frac{4}{x + 2} + \frac{2}{(x - 1)^2} - \frac{3}{x - 1}$ $\frac{3x^3 + x^2 - 18x + 20}{x^3 - 3x + 2} = 3 + \frac{4}{x + 2} + \frac{2}{(x - 1)^2} - \frac{3}{x - 1}$	Division attempted, by any method Denominators and multiplying Eliminating to find constants Correct form with 2 constants correct Correct form with 3 constants correct	M1 A1 M1 M1 A1 A1

2.

Question	Scheme	Marks
(a)	$\tan\theta + \cot\theta \equiv \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$	M1
	$\equiv \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$	A1
	$\equiv \frac{1}{\frac{1}{2}\sin 2\theta}$	M1
	$\equiv 2\text{cosec}2\theta$ *	A1*
		(4)
(b)	Real solutions when $\{k: k \le -2\} \cup \{k: k \ge 2\}$	B1
		(1)
		(5 marks)

Notes:

(a)

M1: Writes $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

A1: Achieves a correct intermediate answer of $\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

M1: Uses the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$

A1*: Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

$\tan\theta + \cot\theta \equiv \tan\theta + \frac{1}{\tan\theta} \equiv \frac{\tan^2\theta + 1}{\tan\theta} \equiv \frac{\sec^2\theta}{\tan\theta} \equiv \frac{1}{\cos^2\theta \times \frac{\sin\theta}{\cos\theta}} \equiv \frac{1}{\cos\theta \times \sin\theta}$ then as the main scheme.

(b)

B1: Scored for sight of $\sin 2\theta = 2$ and a reason as to why this equation has no real solutions. Possible reasons could be $-1 \leqslant \sin 2\theta \leqslant 1$........and therefore $\sin 2\theta \neq 2$ or $\sin 2\theta = 2 \Rightarrow 2\theta = \arcsin 2$ which has no answers as $-1 \leqslant \sin 2\theta \leqslant 1$

3.

Question Number	Scheme	Marks
3(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$	
	$\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$	M1
	$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28)	A1
	States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x^\circ = \frac{1}{3}\tan 40^\circ$ * cao	A1 *
		(4)
(b)	Deduces $\tan 2\theta = \frac{1}{3}\tan 40$	M1
	$2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer	A1
	Also $2\theta =$ 195.6, 375.6 , 555.6 $\Rightarrow \theta = ..$	M1
	$\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers	A1
		(4)
		[8 marks]

Alt 1 **3(a)**	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$	
	$\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$	M1
	$\tan x = \dfrac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$	A1,A1
Alt 2 **3(a)**	$2\cos(x+50) = \sin(x+40) \Rightarrow 2\sin(40-x) = \sin(x+40)$	
	$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$	M1
	$\tan x = \dfrac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$	A1,A1

Notes for Question 3

(a)

M1 Expand both expressions using $\cos(x+50)=\cos x\cos 50-\sin x\sin 50$ and $\sin(x+40)=\sin x\cos 40+\cos x\sin 40$. Condone a missing bracket on the lhs.

The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions.

Allow if written separately and not in a connected equation.

M1 Divide by $\cos x$ to reach an equation in $\tan x$.

Below is an example of M1M1 with incorrect sign on left hand side

$2\cos x\cos 50+2\sin x\sin 50=\sin x\cos 40+\cos x\sin 40$

$\Rightarrow 2\cos 50+2\tan x\sin 50=\tan x\cos 40+\sin 40$

This is independent of the first mark.

A1 $\tan x=\dfrac{2\cos 50-\sin 40}{\cos 40+2\sin 50}$

Accept for this mark $\tan x=$ awrt 0.28... as long as M1M1 has been achieved.

A1* States or uses cos50=sin40 and cos40=sin50 leading to showing

$\tan x=\dfrac{2\cos 50-\sin 40}{\cos 40+2\sin 50}=\dfrac{\sin 40}{3\cos 40}=\dfrac{1}{3}\tan 40$

This is a given answer and all steps above must be shown. The line above is acceptable.

Do not allow from $\tan x=$ awrt 0.28...

(b)

M1 For linking part (a) with (b). Award for writing $\tan 2\theta=\frac{1}{3}\tan 40$

A1 Solves to find one solution of θ which is usually (awrt) 7.8

M1 Uses the correct method to find at least another value of θ. It must be a full method but can be implied by any correct answer.

Accept $\theta=\dfrac{180+their\alpha}{2}, (or)\dfrac{360+their\alpha}{2}, (or)\dfrac{540+their\alpha}{2}$

A1 Obtains all four answers awrt 1dp. $\theta=7.8, 97.8, 187.8, 277.8$.

Ignore any extra solutions outside the range.

Withhold this mark for extras inside the range.

Condone a different variable. Accept x= 7.8, 97.8, 187.8, 277.8

Answers fully given in radians, loses the first A mark.

Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85

Mixed units can only score the first M 1

4.

	$f'(x) = 3e^{3x}sin5x + 5e^{3x}cos5x$	Applying product rule $\frac{du}{dx}$ correct $\frac{dv}{dx}$ correct	M1 A1 A1
	$f'(x) = e^{3x}(3sin5x + 5cos5x) = 0$ $\boldsymbol{tan5x = \frac{-5}{3}}$*	Solving bracket =0	M1 A1

5.

a	$(x-2)^2 + (y+4)^2 = \sqrt{53}^2$		M1 A1
b	Midpoint =$(\frac{-5+9}{2}, \frac{-2-6}{2})$ $= (2, -4)$ which is centre of circle $\therefore PQ \text{ is a diameter}$	Many other ways this could be shown.	M1 A1

6.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7a	Finds $\frac{dy}{dx} = 3x^2 + 12x - 12$	M1	1.1b	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $\frac{d^2y}{dx^2} = 6x + 12$	M1	1.1b	
	States that $\frac{d^2y}{dx^2} = 6x + 12 \leq 0$ for all $-5 \leqslant x \leqslant -3$ and concludes this implies C is concave over the given interval.	B1	3.2a	
		(3)		
7b	States or implies that a point of inflection occurs when $\frac{d^2y}{dx^2} = 0$	M1	3.1a	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $x = -2$	A1	1.1b	
	Substitutes $x = -2$ into $y = x^3 + 6x^2 - 12x + 6$, obtaining y = 46	A1	1.1b	
		(3)		(6 marks)