TRACKING TEST 2 FP2 PRACTICE PAPER 3 63 marks
Time: one hour and 15 minutes

1. **Figure 1**

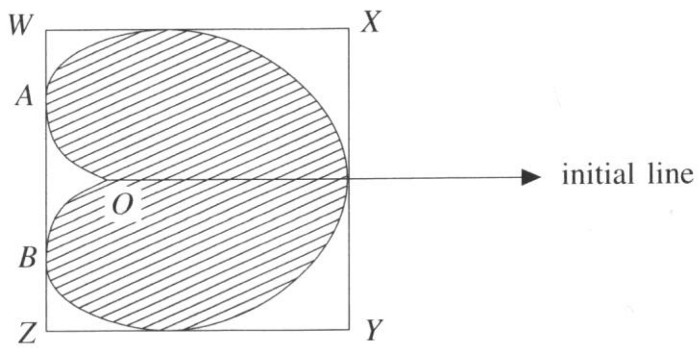

Figure 1 shows a sketch of the cardioid C with equation $r = a(1 + \cos\theta)$, $-\pi < \theta \leq \pi$. Also shown are the tangents to C that are parallel and perpendicular to the initial line. These tangents form a rectangle $WXYZ$.

(a) Find the area of the finite region, shaded in Fig. 1, bounded by the curve C.

(6)

(b) Find the polar coordinates of the points A and B where WZ touches the curve C.

(5)

(c) Hence find the length of WX.

(2)

Given that the length of WZ is $\dfrac{3\sqrt{3}a}{2}$,

(d) find the area of the rectangle $WXYZ$.

(1)

A heart-shape is modelled by the cardioid C, where $a = 10$ cm. The heart shape is cut from the rectangular card $WXYZ$, shown in Fig. 1.

(e) Find a numerical value for the area of card wasted in making this heart shape.

(2)

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2. (*a*) Express as a simplified fraction $\frac{1}{(r-1)^2} - \frac{1}{r^2}$.

(2)

(*b*) Prove, by the method of differences, that

$$\sum_{r=2}^{n} \frac{2r-1}{r^2(r-1)^2} = 1 - \frac{1}{n^2}.$$

(3)

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3. Solve the inequality $\frac{1}{2x+1} > \frac{x}{3x-2}$.

(6)

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4. (*a*) Using the substitution $t = x^2$, or otherwise, find

$$\int x^3 e^{-x^2} \, dx.$$

(6)

(*b*) Find the general solution of the differential equation

$$x\frac{dy}{dx} + 3y = xe^{-x^2}, \quad x > 0.$$

(4)

5.

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 4e^{3t}, \quad t \geq 0.$$

(*a*) Show that Kt^2e^{3t} is a particular integral of the differential equation, where K is a constant to be found.

(4)

(*b*) Find the general solution of the differential equation.

(3)

Given that a particular solution satisfies $y = 3$ and $\frac{dy}{dt} = 1$ when $t = 0$,

(*c*) find this solution.

(4)

Another particular solution which satisfies $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$ when $t = 0$, has equation

$$y = (1 - 3t + 2t^2)\mathrm{e}^{3t}.$$

(*d*) For this particular solution draw a sketch graph of y against t, showing where the graph crosses the t-axis. Determine also the coordinates of the minimum of the point on the sketch graph.

(5)

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6. (*a*) Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(6)

(*b*) Hence find 3 distinct solutions of the equation $16x^5 - 20x^3 + 5x + 1 = 0$, giving your answers to 3 decimal places where appropriate.

(4)

TOTAL MARKS: 63 marks
