Test yourself Tracking Test 5

1) (*a*) Express 3 cos θ + 4 sin θ in the form R cos ($\theta - \alpha$), where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(*b*) Hence find the maximum value of 3 cos ϑ + 4 sin ϑ and the smallest positive value of ϑ for which this maximum occurs.
The temperature, f(t), of a warehouse is modelled using the equation
f (t) = 10 + 3 cos (15t)° + 4 sin (15t)°, where t is the time in hours from midday and $0 \le t < 24$.
(*c*) Calculate the minimum temperature of the warehouse as given by this model.
(*d*) Find the value of t when this minimum temperature occurs.

2) $I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,dx$.

(*a*) Given that $y = \frac{1}{4+\sqrt{(x-1)}}$, copy and complete the table below with values of y corresponding to x = 3 and x = 5 . Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(*b*) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.
(*c*) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I.

3) The points A and B have position vectors 2**i** + 6**j** – **k** and 3**i** + 4**j** + **k** respectively.
The line l_1 passes through the points A and B.
(*a*) Find the vector $\overrightarrow{AB}$.
(*b*) Find a vector equation for the line l_1.
A second line l_2 passes through the origin and is parallel to the vector **i** + **k**. The line l_1 meets the line l_2 at the point C.
(*c*) Find the acute angle between l_1 and l_2.
(*d*) Find the position vector of the point C.

4) Find the particular solution of the differential equation $\frac{dy}{dx} = 2\cos^2 y \cos^2 x; \quad y = \frac{\pi}{4}, x = 0$

Answers

1) (*a*) 5 cos(θ-53.1) (*b*) max value = 5 where 53.13 (*c*) 5° (*d*) t = 15.5

2) (*a*) 0.1847, 0.1667 (*b*) 0.543 (*c*) $2 + 8 \ln \frac{5}{6}$

3) a) **i** – 2**j** + 2**k** b) 2**i** +6**j** – **k** + λ(**i** – 2**j** + 2**k**) c) 45° d) 5**i** + 5**k**

4) tan y = ½ sin 2x + x + 1