BHASVIC Maths

PS5

Section A: Q1 - 6	90 mins
Section B: Q1 - 6	90 mins
Section C: Q1 - 6	90 mins

Notices:

- Remember teacher subject extension drop ins are every lunch time in room 24
- A2 Doubles students are available for drop in help almost every period, every day, in room 7

1. Sketch the following graphs, writing down any equations of asymptotes or any intersections with the coordinate axis

a) $y = \frac{1}{(x-2)^2} + 3$ b) $y = -5\ln(x+3)$ c) $y = -2cosec\left(x - \frac{\pi}{3}\right)$ between $-2\pi \leq x \leq 2\pi$

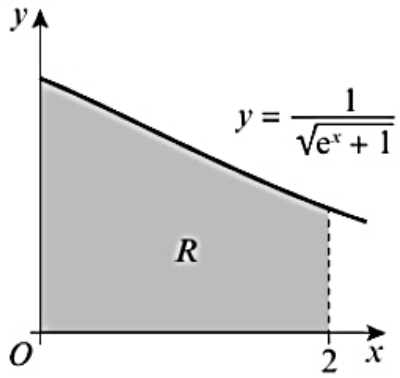
2. The diagram shows a sketch of the curve with equation $y = \frac{1}{\sqrt{e^x + 1}}$

The shaded region R is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

a Complete the table giving values of y to 3 decimal places. **(2 marks)**

x	0	0.5	1	1.5	2
y	0.707	0.614	0.519		0.345

b Use the trapezium rule, with all the values from your table, to estimate the area of the region R, giving your answer to 2 decimal places. **(4 marks)**



Answers

3. Summarised below are the distances, to the nearest mile, travelled to work by a random sample of 120 commuters.

Distance, x miles	Number of commuters
$0 < x \leqslant 10$	10
$10 < x \leqslant 20$	19
$20 < x \leqslant 30$	43
$30 < x \leqslant 40$	25
$40 < x \leqslant 50$	8
$50 < x \leqslant 60$	6
$60 < x \leqslant 70$	5
$70 < x \leqslant 80$	3
$80 < x \leqslant 90$	1

a For this distribution, use linear interpolation to estimate its median.

The midpoint of each class was represented by x and its corresponding frequency f giving

$\Sigma fx = 3610$ and $\Sigma fx^2 = 141\,600$

b Estimate the mean and standard deviation of this distribution.

Answers

4

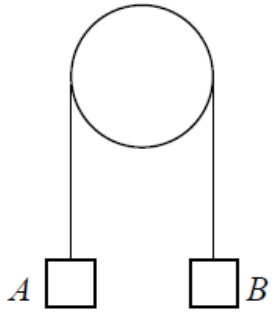


Fig. 3

Figure 3 shows two particles A and B of masses m and km respectively, connected by a light inextensible string which passes over a smooth fixed pulley.

When the system is released from rest with both particles 0.5 m above the ground, particle A moves vertically upwards with acceleration $\frac{1}{4}g\ \text{m s}^{-2}$.

(a) Write down, with a brief justification, the magnitude and direction of the acceleration of B.

(2 marks)

(b) Find the value of k. **(6 marks)**

Given that A does not hit the pulley,

(c) calculate, correct to 3 significant figures, the speed with which B hits the ground.

(3 marks)

Answers

5.

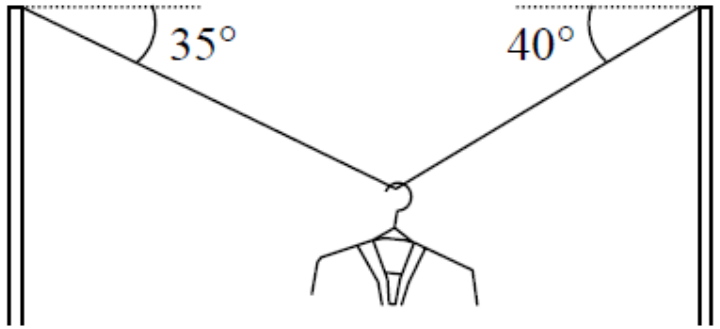


Fig. 2

Figure 2 shows a washing line suspended at either end by vertical rigid poles. A jacket of mass 0.7 kg is suspended in equilibrium part of the way along the line. The sections of the washing line on either side of the jacket make angles of 35° and 40° with the horizontal.

(a) Find the tension in the washing line on each side of the jacket. **(7 marks)**

(b) Explain why, in practice, the angles are likely to be very similar in value. **(1 mark)**

Answers

6. **a** When θ is small, show that the equation

$$32\cos 5\theta + 203\tan 10\theta = 182$$

can be written as

$$40\theta^2 - 203\theta + 15 = 0$$

b Hence, find the solutions of the equation

$$32\cos 5\theta + 203\tan 10\theta = 182$$

c Comment on the validity of your solutions.

Answers

1. Put the following in terms of partial fractions:

a) $\frac{2x-13}{(2x+1)(x-3)}$

b) $\frac{3x^2+x+1}{x^2(x+1)}$

2. Find the following integrals.

a $\int (x+1)(x^2+2x+3)^4\,dx$

c $\int \sin^5 3x \cos 3x\,dx$

e $\int \frac{e^{2x}}{e^{2x}+3}\,dx$

g $\int (2x+1)\sqrt{x^2+x+5}\,dx$

3. Applied book 1 page 202 Q3

Answers

3. A biased dice has a probability distribution as shown in the table below:

x	1	2	3	4	5	6
$\mathbf{P}(X = x)$	0.1	0.2	0.15	p	0.1	0.25

a Find the value of p.

b Find $P(2 \leqslant X \leqslant 5)$.

c The dice is rolled 10 times Find the probability that it lands on an odd number:

i exactly twice

ii more than 6 times.

Answers

4. At 6 a.m. a cargo ship has position vector $(7\mathbf{i} + 56\mathbf{j})$ km relative to a fixed origin O on the coast and moves with constant velocity $(9\mathbf{i} - 6\mathbf{j})$ km h^{-1}.

A ferry sails from O at 6 a.m. and moves with constant velocity $(12\mathbf{i} + 18\mathbf{j})$ km h^{-1}. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed due east and due north respectively.

(a) Show that the position vector of the cargo ship t hours after 6 a.m. is given by

$$[(7 + 9t)\mathbf{i} + (56 - 6t)\mathbf{j}] \text{ km},$$

and find the position vector of the ferry in terms of t. **(3 marks)**

(b) Show that if both vessels maintain their course and speed, they will collide and find the time and position vector at which this occurs.

(6 marks)

At 8 a.m. the captain of the ferry realises that a collision is imminent and changes course so that the ferry now has velocity $(21\mathbf{i} + 6\mathbf{j})$ km h^{-1}.

(c) Find the distance between the two ships at the time when they would have collided.

(5 marks)

Answers

5. A factory makes plates using a production line process. On average, 3 out of every 10 plates have flaws. A new production process is introduced designed to make the average number of flaws less. A new sample of 20 plates is taken.

   a Describe the test statistic and state suitable null and alternative hypotheses.

   b Using a 5% level of significance, find the critical region for a test to check the belief that the process has improved.

   c State the actual significance level.

   In the new sample, only 1 plate has flaws.

   d Conclude whether there is evidence that the process has improved.

6. **Finite binomial Expansion Question**

In the binomial expansion of $(2 + px)^5$ where p is a constant, the coefficient of x^3 is 135.

i) Calculate the value of p

ii) Calculate the value of the coefficient x^4 in the expansion

Answers

1. Infinite binomial expansion question

Find the binomial expansions up to and including the x^3 term and state the range of values of x for which the expansions are valid:

a) $\sqrt{4+2x}$ b) $\frac{1+x}{2+x}$

2. The point A lies on the circle with equation $x^2 + y^2 = 9$. Given that $\overrightarrow{OA} = 2k\mathbf{i} + k\mathbf{j}$, find the exact value of k.

3. A vertical mast is 32 m high. Two balls P and Q are projected simultaneously. Ball P is projected horizontally from the top of the mast with speed 18 m s^{-1}. Ball Q is projected from the bottom of the mast with speed 30 m s^{-1} at an angle α above the horizontal. The balls move freely under gravity in the same vertical plane and collide in mid-air. By considering the horizontal motion of each ball,

a prove that $\cos\alpha = \frac{3}{5}$ **(4 marks)**

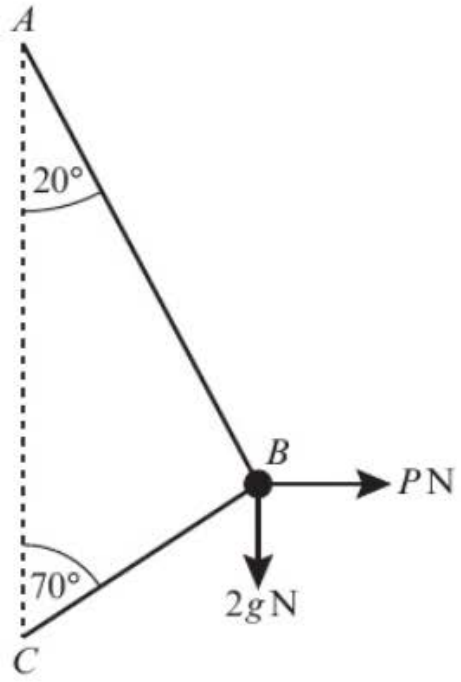
b Find the time which elapses between the instant when the balls are projected and the instant when they collide. **(4 marks)**

Answers

4. A smooth bead B of mass 2 kg is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C where A is vertically above C. The bead is held in equilibrium by a horizontal force of magnitude P N.
The sections AB and BC make angles of 20° and 70° with the vertical as shown.

a Show that the tension in the string is 33 N (2 s.f.). **(3 marks)**

b Calculate the value of P. **(3 marks)**



Answers

5. Two events A and B are such that $P(B) = 0.3$ and $P(A \cap B) = 0.15$. If A and B are independent, find:

a $P(A)$ **(1)**

b $P(A' \cap B')$ **(1)**

A third event C has $P(C) = 0.4$. Given that B and C are mutually exclusive and $P(A \cap C) = 0.1$,

c Draw a Venn diagram to illustrate this situation. **(2)**

d Find:

i $P(A|C)$ **(2)**

ii $P(A \cap (B \cup C'))$ **(2)**

iii $P(A|(B \cup C'))$ **(2)**

6. **a** Show that $\cos^4 x \equiv \frac{1}{8}\cos 4x + \frac{1}{2}\cos 2x + \frac{3}{8}$

b Hence find $\int \cos^4 x \, dx$.

Answers

BHASVIC Maths PS5 Answers: Section A

1. Use desmos to check

2. **a** 0.427 **b** 1.04

3. **a** Median 27.3 miles
   b Mean 30.1 miles, standard deviation 16.6 miles
   Mean 3.06 hours, standard deviation 3.32 hours

Section A

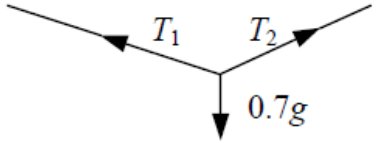
BHASVIC Maths PS5 Answers: Section A

4. *(a)* e.g. string is inextensible so B moves down same dist. A moves up — B1
$\therefore$ acceleration of B is $\frac{1}{4}g$ ms^{-2} downwards — B1

(b) eqn. of motion for A: $kmg - T = kma$ (1) — M1
eqn. of motion for B: $T - mg = ma$ (2) — M1
(1) + (2) gives $kmg - mg = kma + ma$ — M1 A1
$k(g-a) = g + a \therefore k = \frac{g+a}{g-a} = \frac{5g/4}{3g/4} \therefore k = \frac{5}{3}$ — M1 A1

(c) $u = 0, s = 0.5, a = \frac{1}{4}g$ use $v^2 = u^2 + 2as$ — M1
$v^2 = 0 + 2(0.25g)0.5 = 2.45 \therefore v = 1.57$ ms^{-1} (3sf) — M1 A1 **(11)**

5. *(a)*



resolve ↑ : $T_1\sin35° + T_2\sin40° - 0.7g = 0$ (1) — M1 A1
resolve →: $T_2\cos40° - T_1\cos35° = 0$ (2) — M1
from (2), $T_2 = 1.069\,T_1$ — M1
sub. into (1) to get $T_1 = 5.44$ N (3sf) — M1 A1
and so $T_2 = 5.82$ N — A1

(b) e.g. jacket likely to slide to a position near centre of line — B1 **(8)**

Section A

6. **a** $32 \cos 5\theta + 203 \tan 10\theta = 182$

$$32\left(1 - \frac{(5\theta)^2}{2}\right) + 203\,(10\theta) = 182$$

$$32 - 16(25\theta^2) + 2030\theta = 182$$

$$0 = 400\theta^2 - 2030\theta + 150$$

$$0 = 40\theta^2 - 203\theta + 15$$

b $5, \frac{3}{40}$

c 5 is not valid as it is not "small". $\frac{3}{40}$ is "small" so is valid.

BHASVIC Maths PS5 Answers: Section B

1. a) $\frac{4}{2x+1} - \frac{1}{x-3}$ b) $\frac{1}{x^2} + \frac{3}{x+1}$

2. **a** $\frac{1}{10}(x^2 + 2x + 3)^5 + c$
   c $\frac{1}{18}\sin^6 3x + c$
   e $\frac{1}{2}\ln|e^{2x} + 3| + c$
   g $\frac{2}{3}(x^2 + x + 5)^{\frac{3}{2}}$

3. **a** 0.2
   b 0.65
   c **i** 0.1757 **ii** 0.0260

Section B

BHASVIC Maths PS5 Answers: Section B

4. *(a)* cargo ship travels $(9t\mathbf{i} - 6t\mathbf{j})$ km in t hours

pos^n vector after t hours is $[(7\mathbf{i} + 56\mathbf{j}) + (9t\mathbf{i} - 6t\mathbf{j})]$ km — M1

$= [(7 + 9t)\mathbf{i} + (56 - 6t)\mathbf{j}]$ km — A1

pos^n vector of ferry after t hours is $(12t\mathbf{i} + 18t\mathbf{j})$ km — A1

(b) they will collide if coeffs. of **i** and **j** in pos^n vectors are equal — B1

$7 + 9t = 12t$ and $56 - 6t = 18t$ are both satisfied when $t = \frac{7}{3}$ — M1 A1

collision after $\frac{7}{3}$ hrs or 2 hrs 20 mins i.e. at 8:20 a.m. — A1

pos^n vector $= 12(\frac{7}{3})\mathbf{i} + 18(\frac{7}{3})\mathbf{j} = (28\mathbf{i} + 42\mathbf{j})$ — M1 A1

(c) at 8 a.m. ferry at $(24\mathbf{i} + 36\mathbf{j})$

$\frac{1}{3}$ hr at $21\mathbf{i} + 6\mathbf{j} = 7\mathbf{i} + 2\mathbf{j}$ so at 8:20 a.m. ferry is at $31\mathbf{i} + 38\mathbf{j}$ — M2 A1

at 8:20 a.m cargo ship is at $(28\mathbf{i} + 42\mathbf{j})$

dist. between $= \sqrt{(3^2 + 4^2)} = 5$ km — M1 A1 **(14)**

5. **a** Test statistic is the number of plates that are flawed.
$H_0: p = 0.3$, $H_1: p < 0.3$

b 0, 1, 2 **c** 3.55%

d 1 falls into the critical region therefore there is evidence to support the claim.

6. i) 1.5 ii) 50.625

Section B

BHASVIC Maths PS5 Answers: Section C

1. a) $2 + \frac{x}{2} - \frac{x^2}{16} + \frac{x^3}{64}$ $|x| < 2$ b) $\frac{1}{2} + \frac{1}{4}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$ $|x| < 2$

2. $\frac{3\sqrt{5}}{5}$

3. **a** Particle P: $x = 18t$, Particle Q: $x = 30\cos\alpha\, t$
When particles collide: $18t = 30\cos\alpha\, t \Rightarrow \cos\alpha = \frac{3}{5}$
b $\frac{4}{3}$s

Section C

4. **a** R(↑): $T\cos 20 = 2g + T\cos 70$

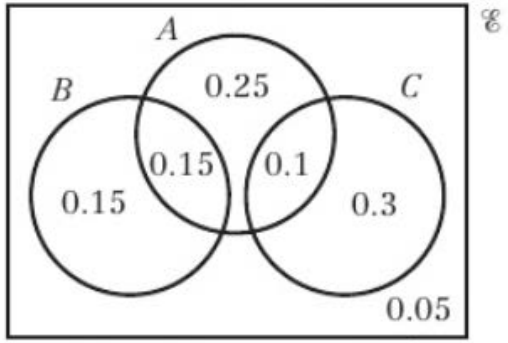
$$T = \frac{2g}{\cos 20 - \cos 70}$$

$$= 33\,\text{N (2 s.f.)}$$

b 42 N (2 s.f.)

5. **a** 0.5 **b** 0.35

c



d **i** 0.25 **ii** 0.4 **iii** $\frac{2}{3}$

6. **a** $\cos^4 x = (\cos^2 x)^2 = \left(\frac{1 + \cos 2x}{2}\right)^2 = \frac{1}{4} + \frac{1}{2}\cos 2x$

$$+ \frac{1}{4}\cos^2 2x = \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\left(\frac{1 + \cos 4x}{2}\right)$$

$$= \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

b $\frac{1}{32}\sin 4x + \frac{1}{4}\sin 2x + \frac{3}{8}x + c$