BHASVIC Maths

P.S.4

Section A: Q1 - 6	90 mins
Section B: Q1 - 6	90 mins
Section C: Q1 - 6	90 mins

Notices:

- Remember teacher subject extension drop ins are every lunch time in room 24
- A2 Doubles students are available for drop in help almost every period, every day, in room 7

1. **a** Describe the transformations of the graph of $\cos\theta$ which would give $3\cos(\theta - 30^\circ) - 1$.

   b The graph of $y = \tan\theta$ undergoes these transformations. State the equation of the resulting graph.

   i Stretch parallel to x-axis, scale factor 3, followed by a translation $\begin{pmatrix} 10^\circ \\ -2 \end{pmatrix}$

   ii A translation $\begin{pmatrix} 60^\circ \\ 4 \end{pmatrix}$, followed by a stretch parallel to the y-axis, scale factor 2

2. Find the quotient when these polynomials are divided by the expression in brackets.

   a $x^4 - 2x^3 - 4x^2 + 7x - 2$ $(x^2 + x - 2)$

   b $x^4 + 3x^3 + 3x - 1$ $(x^2 + 1)$

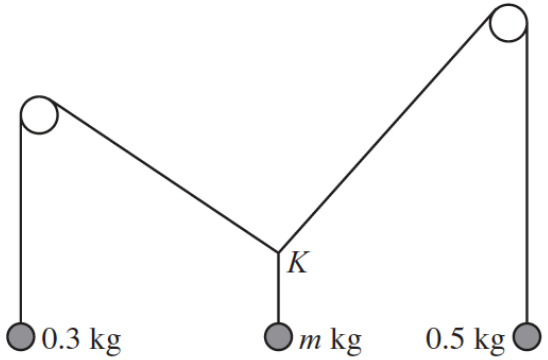
   c $3x^4 - 2x^3 + 7x^2 - 4x + 2$ $(x^2 + 2)$

   d $2a^3 - 7a^2 - a + 2$ $(a^2 - 3a - 2)$

3. The equation $kx^2 - 2kx + 3 = 0$, where k is a constant, has no real roots.
   Prove that k satisfies the inequality $0 \leqslant k < 3$.

Answers

4. The diagram shows three strings, which are tied in a knot K. Two of the strings pass over smooth pulleys and have particles of mass 0.3 kg and 0.5 kg attached to them at the ends opposite to K. The other string has a particle of mass m kg attached to it at the end opposite to K. The system is at rest.



(a) In the case $m = 0.7$, find the angle made by the sloping part of each string with the upward vertical.

(b) Give a reason why $m < 0.8$.

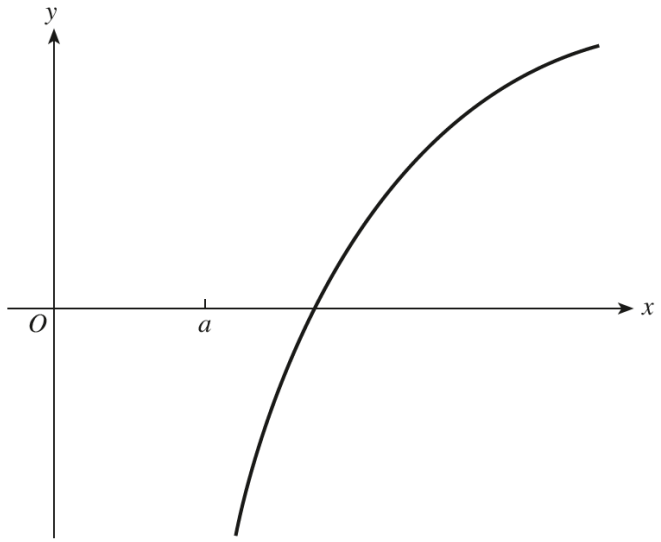
(c) In the case when $m = 0.4$, show that part of one of the strings is horizontal.

(d) If the pulleys were at the same horizontal level, give a reason why $m > 0.4$.

5. A parcel delivery service claims that at least 80% of their parcels are delivered within 48 hours of posting. A check on 200 parcels found that 152 were delivered within 48 hours of posting. Test the delivery service's claim at the 5% significance level.

Answers

6.



The function f is defined by

$$\mathrm{f}(x) = \tfrac{1}{3}\ln\left(\frac{x-a}{2}\right) \quad \text{for } x > a,$$

where the constant a is positive. The graph of $y = \mathrm{f}(x)$ is shown in the diagram.

i Explain how the graph shows that f is one–one.
ii Find an expression for $\mathrm{f}^{-1}(x)$.
iii State the range of f^{-1}.
iv Given that $\mathrm{f}(a^3) - \mathrm{f}(a^2) = \ln\frac{5}{3}$, find the exact value of a. *OCR June 2004*

Answers

1. Integrate the following with respect to x:

   (a) $(2x-1)^2$ (b) $x-\sec 2x\tan 2x$ (c) $\frac{1}{2x}$

2. Given that $f(x)=\frac{2}{x-1}-\frac{6}{(x-1)(2x+1)}$, $x>1$,

   (a) Prove that $f(x)=\frac{4}{2x+1}$ (b) Find the range of f.

   (c) Find $f^{-1}(x)$ and state its domain. (d) State the range of $f^{-1}(x)$.

3. Prove from first principles (using the small angle approximation and the compound angle formulae) that:-
   (a) the derivative of sinx is cosx
   (b) the derivative of cosx is -sinx

Answers

4. A car of mass 2150 kg is travelling down a rough road that is inclined at 10° to the horizontal. The engine of the car applies a constant driving force of magnitude 700 N, which acts in the direction of travel of the car. Any friction between the road and the tyres is initially ignored, and air resistance is modelled as a single constant force of magnitude F N that acts to oppose the motion of the car.

   a Given that the car is travelling in a straight line at a constant speed of 22 m s^{-1}, find the magnitude of F.

   The driver brakes suddenly. In the subsequent motion the car continues to travel in a straight line, and the tyres skid along the road, bringing the car to a standstill after 40 m. The driving force is removed, and the force due to air resistance is modelled as remaining constant.

   b Find the coefficient of friction between the tyres and the road.

   c Criticise this model with relation to

   i the frictional forces acting on the car

   ii the motion of the car.

5. In a tennis match, the probability that Anne wins the first set against Colin is 0.7. If Anne wins the first set, the probability that she wins the second set is 0.8. If Anne loses the first set, the probability that she wins the second set is 0.4. A match is won when one player wins two sets.

   a Find the probability that the game is over after two sets.

   b Find the probability that Anne wins given that the game is over after two sets.

   If the game is tied at one set all, a tiebreaker is played and the probability of Anne winning it is 0.55.

   c Find the probability of Anne winning the entire match.

Answers

6. Show that the curve with equation $y = \frac{\ln x}{x}$ has a maximum value of $\frac{1}{e}$ at $x = e$.

Answers

1. Find:-

(a) $\frac{d}{dx}\left(3xe^{5x}\right)$ (b) $\frac{d}{dx}\left(e^{-3x}\cot x\right)$ (c) $\frac{d}{dx}\left(x\ln(2-x)\right)$

2. Find the exact value(s) of x which satisfy the equations:

(a) $\ln(6x+1)=1$ (b) $e^{3x-1}=2$
(c) $e^{2x}=e^{x}+12$ (d) $e^{2x}e^{x+1}=28$

3. Solve the following equations on the interval $0\le\theta\le2\pi$. Give exact answers where you can, but otherwise give your answers to 3sf:

(a) $\tan^2\theta+2\sec\theta=7$ (b) $\operatorname{cosec}^2 2\theta=2$

Answers

4. As a car passes the point A on a straight road, its speed is $10\,\text{m s}^{-1}$. The car moves with constant acceleration $a\ \text{m s}^{-2}$ along the road for T seconds until it reaches the point B, where its speed is $V\ \text{m s}^{-1}$. The car travels at this speed for a further 10 seconds, when it reaches the point C. From C it travels for a further T seconds with constant acceleration $3a\ \text{m s}^{-2}$ until it reaches a speed of $20\,\text{m s}^{-1}$ at the point D. Sketch the (t, v) graph for the motion, and show that $V = 12.5$.

   Given that the distance between A and D is 675 m, find the values of a and T. (OCR)

5. Two events A and B are such that $\text{P}(A) = \frac{3}{4}$, $\text{P}(B \mid A) = \frac{1}{5}$ and $\text{P}(B' \mid A') = \frac{4}{7}$. By use of a tree diagram, or otherwise, find

   (a) $\text{P}(A \text{ and } B)$, (b) $\text{P}(B)$, (c) $\text{P}(A \mid B)$. (OCR, adapted)

6. **i** Write down the formula for $\tan 2x$ in terms of $\tan x$.

   ii By letting $\tan x = t$, show that the equation

   $$4\tan 2x + 3\cot x \sec^2 x = 0$$

   becomes

   $$3t^4 - 8t^2 - 3 = 0.$$

   iii Find all the solutions of the equation

   $$4\tan 2x + 3\cot x \sec^2 x = 0$$

   which lie in the range $0 \leqslant x \leqslant 2\pi$.

Answers

BHASVIC Maths PS4 Answers: Section A

1. **a** Translation $\begin{pmatrix} 30^\circ \\ 0 \end{pmatrix}$ followed by stretch parallel to y-axis, s.f. 3, followed by translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

   b **i** $y = \tan\left(\frac{1}{3}x - 10^\circ\right) - 2$

   ii $y = 2(\tan(x - 60^\circ) + 4)$

2. **a** $x^2 - 3x + 1$ **b** $x^2 + 3x - 1$

   c $3x^2 - 2x + 1$ **d** $2a - 1$

3. No real roots $b^2 - 4ac < 0$ $(-2k)^2 - 4 \times k \times 3 < 0$

   $4k^2 - 12k = 0$ when $k = 0$ and $k = 3$

   solution $0 \leqslant k < 3$

   note when $k = 0$ equation gives $3 = 0$

4. (a) $38.2°$, $21.8°$
(b) The length of any one side of the triangle of forces cannot exceed the sum of the lengths of the other two sides (the case $m = 0.8$ is excluded because the pulleys are not in the same vertical line).
(d) K cannot be above the level of the pulleys.

5. $H_0 : p = 0.95$, $H_1 : p < 0.95$; $X \sim B(25, 0.95)$; $P(X \leqslant 22) = 0.1271 > 0.05$, so accept H_0, there are at least 95% satisfied customers.

6. **i** The graph has no turning points
ii $f^{-1}(x) = 2e^{3x} + a$
iii $f^{-1}(x) > a$ **iv** $a = \frac{98}{27}$

BHASVIC Maths PS4 Answers: Section B

1. (a) $\frac{1}{6}(2x-1)^3+c$ (b) $\frac{1}{2}x^2-\frac{1}{2}\sec 2x+c$ (c) $\frac{1}{2}\ln x+c$

2. (a) Proof (b) $f \in \Re : f \neq 0$ (c) $f^{-1}(x)=\frac{4-x}{2x}, x \in \Re : x \neq 0$ (d) $f^{-1}(x) > 1$

3. Proof

Section B

BHASVIC Maths PS4 Answers: Section B

4. (a) F = 4400N (b) 0.59 (2 s.f.) (c) Discuss your answers in class

5. (a) 0.74 (b) 0.757 (3 s.f.) (c) 0.703 (3 s.f.)

6. Proof

BHASVIC Maths PS4 Answers: Section C

1. (a) $3e^{5x}(1+5x)$ (b) $-e^{-3x}(3\cot x+\operatorname{cosec}^2 x)$ (c) $\ln(2-x)-\dfrac{x}{2-x}$

2. (a) $\dfrac{1}{6}(e-1)$ (b) $\dfrac{1}{3}(\ln 2+1)$ (c) $2\ln 2$ (d) $\dfrac{1}{3}(\ln 28-1)$

3. (a) $\dfrac{\pi}{3}, \dfrac{5\pi}{3}, 1.82, 4.46$ (b) $\dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{9\pi}{8}, \dfrac{11\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}$

BHASVIC Maths PS4 Answers: Section C

4. $a = \frac{1}{8}, T = 20$

5. (a) $\frac{3}{20}$ (b) $\frac{9}{35}$ (c) $\frac{7}{12}$

6. i $\frac{2\tan x}{1-\tan^2 x}$ iii $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Section C