BHASVIC Maths

P S 3

Section A: Q1 - 6	90 mins
Section B: Q1 - 6	90 mins
Section C: Q1 - 6	90 mins

Notices:

- Remember teacher subject extension drop ins are every lunch time in room 24
- A2 Doubles students are available for drop in help almost every period, every day, in room 7

1. Sketch the following functions where each function is defined on domain, $x \in \mathbb{R}$..

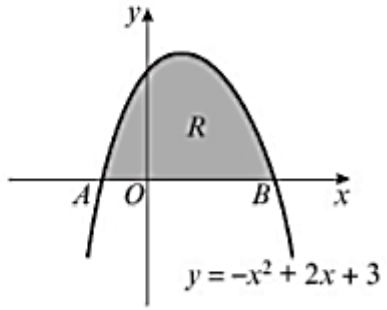
   State the range of each function.

   (a) $f(x) = (x+1)^2 + 4, \quad x \geq 0$ (b) $f(x) = 1 - e^x, \quad x \geq 0$ (c) $f(x) = 3\ln x, \quad x > 0$

2. Find the following integrals by considering what has been differentiated

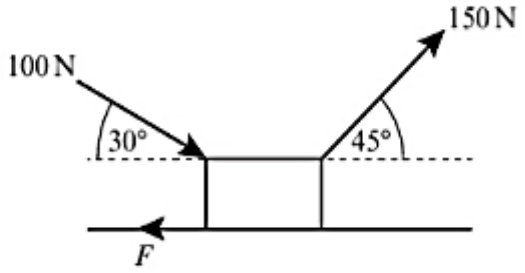
   $(a) \int \sec 3x \tan 3x dx$ (b) $\int \operatorname{cosec} x \cot x dx$ (c) $\int \sec^2 2x dx$

3. The finite region R is bounded by the x-axis and the curve with equation $y = -x^2 + 2x + 3$, $x \geqslant 0$.
   The curve meets the x-axis at points A and B.
   a Find the coordinates of point A and point B.
   b Find the area of the region R.



Answers

4. A box is being pushed and pulled across a rough surface by constant forces as shown in the diagram.
The box is moving at a constant speed. By modelling the box as a particle, show that the magnitude of the resistance due to friction F is $25(3\sqrt{2} + 2\sqrt{3})$ N.



5. For the events A and B, $P(A \cap B') = 0.34$, $P(A' \cap B) = 0.13$ and $P(A \cup B) = 0.62$.

a Draw a Venn diagram to illustrate the complete sample space for the events A and B.

b Write down the values of $P(A)$ and $P(B)$.

c Find $P(A|B')$.

d Determine whether or not A and B are independent.

6. Given that $f(x) = \dfrac{2}{x-1} - \dfrac{6}{(x-1)(2x+1)}$, $x > 1$,

(a) Prove that $f(x) = \dfrac{4}{2x+1}$

(b) Find the range of f.

(c) Find $f^{-1}(x)$ and state its domain.

(d) State the range of $f^{-1}(x)$.

Answers

1. Find the function $f'(x)$ where $f(x)$ is:

a) $\frac{\sin 3x}{e^x}$ b) $e^x \sin^2 x$ c) $\frac{\ln x}{\tan x}$

2. A diver launches herself off a springboard. The height of the diver, in metres, above the pool t seconds after launch can be modelled by the following function:

$h(t) = 5t - 10t^2 + 10, t \geqslant 0$

a How high is the springboard above the water?

b Use the model to find the time at which the diver hits the water.

c Rearrange $h(t)$ into the form $A - B(t - C)^2$ and give the values of the constants A, B and C.

d Using your answer to part **c** or otherwise, find the maximum height of the diver, and the time at which this maximum height is reached.

3. Prove that

a)

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} \equiv \cos 2\theta$$

b) Prove that $\cos\left(x+\frac{\pi}{6}\right)+\sin\left(x+\frac{\pi}{3}\right) \equiv \sqrt{3}\cos x$

c) Prove that $\sec x + \operatorname{cosec} x \cot x \equiv \sec x \operatorname{cosec}^2 x$

Answers

4. A particle P is projected from a point on a horizontal plane with speed U at an angle of elevation θ.

   a Show that the range of the projectile is $\frac{U^2 \sin 2\theta}{g}$.

   b Hence find, as θ varies, the maximum range of the projectile.

   c Given that the range of the projectile is $\frac{2U^2}{3g}$, find the two possible value of θ.

   Give your answers to the nearest 0.1°.

5. The probability that Joanna oversleeps is 0.15. If she oversleeps, the probability that she is late to college is 0.75. If she gets up on time, the probability that she is late to college is 0.1.

   a Find the probability that Joanna is late to college on any particular day.

   b Find the probability that Joanna overslept given that she is late to college.

6. Given that $\frac{6x^3 - 7x^2 + 3}{3x^2 + x - 10} \equiv Ax + B + \frac{C}{3x - 5} + \frac{D}{x + 2}$, find the values of the constants A, B, C and D.

Answers

BHASVIC Maths PS3 Section C

1. Show that $\frac{4x^3-6x^2+8x-5}{2x+1}$ can be written in the form $Ax^2+Bx+C+\frac{D}{2x+1}$ where A, B, C and D are constants to be found.

2. *(a)* Simplify

$$\frac{x^2+7x+12}{2x^2+9x+4}.$$

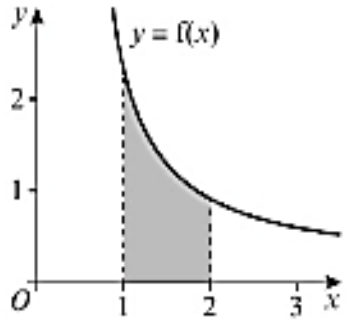
(b) Solve the equation

$$\ln(x^2+7x+12)-1=\ln(2x^2+9x+4),$$

giving your answer in terms of e.

3. The diagram shows a sketch of the curve with equation, $y=f(x)$, where $f(x)=\frac{4x+3}{(x+2)(2x-1)}$, $x>\frac{1}{2}$

Find the area of the shaded region bounded by the curve, the x-axis and the lines $x=1$ and $x=2$.



Answers

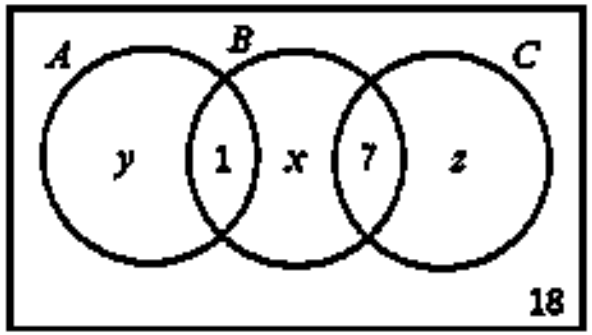
4. A packing crate of mass 10 kg rests on rough horizontal ground. It is filled with books which are evenly distributed through the crate. The coefficient of friction between the crate and the ground is 0.3.

   a Find the mass of the books if the crate is in limiting equilibrium under the effect of a horizontal force of magnitude 147 N.

   b State what modelling assumptions you have made.

5. The Venn diagram shows the number of sports club members liking three different sports.

   Given that there are 50 members in total, $P(C) = 3P(A)$ and $P(\text{not } B) = 0.76$, find the values of x, y and z.



6. The maximum point on the curve with equation $y = x\sqrt{\sin x}$ where $0 < x < \pi$ is A. Show that the x coordinate of A satisfies the equation $2\tan x + x = 0$.

Answers

BHASVIC Maths PS3 Answers: Section A

1. a) $f(x) \geq 5$ b) $f(x) \leq 0$ c) $-\infty < f(x) < \infty$ or $f(x) \in \mathbb{R}$

2. a) $\frac{1}{3}\sec 3x + c$ $(b) -\operatorname{cosec} x + c$ c) $\frac{1}{2}\tan 2x + c$

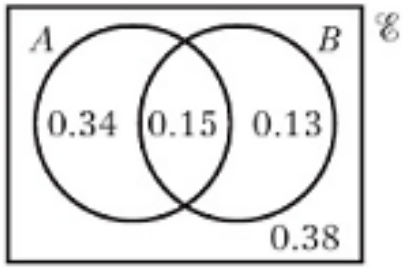
3. **a** $(-1, 0)$ and $(3, 0)$ **b** $10\frac{2}{3}$

Section A

BHASVIC Maths PS3 Answers: Section A

4. $\text{R}(\rightarrow): F = 150\cos 45 + 100\cos 30$
$= \frac{150\sqrt{2}}{2} + \frac{100\sqrt{3}}{2}$
$= 25(3\sqrt{2} + 2\sqrt{3})\text{N}$

5. **a**



b $\text{P}(A) = 0.49$, $\text{P}(B) = 0.28$
c $\frac{17}{36}$ or 0.472 (3 s.f.)
d No: $\text{P}(A) \times \text{P}(B) \neq \text{P}(A \cap B)$

6. b) $f \in \Re : f \neq 0$ c) $f^{-1}(x) = \frac{4-x}{2x}, x \in \Re : x \neq 0$ d) $f^{-1}(x) > 1$

Section A

BHASVIC Maths PS3 Answers: Section B

1. a) $\frac{3\cos 3x - \sin 3x}{e^x}$ b) $e^x \sin x(\sin x + 2\cos x)$ c) $\frac{\tan x - x\sec^2 x \ln x}{x\tan^2 x}$

2. **a** 10 m **b** 1.28 s
   c $h(t) = 10.625 - 10(t - 0.25)^2$
   $A = 10.625, B = 10, C = 0.25$
   d 10.625 m at 0.25 s

3. Proof

Section B

4. **a** R(↑): $s = U\sin\theta\, t - \frac{g}{2}t^2$

When particle strikes plane, $s = 0 = t(U\sin\theta - \frac{g}{2}t)$

So $t = 0$ or $t = \frac{2U\sin\theta}{g}$

R(→): $s = Ut = U\cos\theta\left(\frac{2U\sin\theta}{g}\right) = \frac{U^2\sin 2\theta}{g}$

b $\frac{U^2}{g}$ **c** 20.9°, 69.1° (nearest 0.1°)

5. **a** 0.198 (3 s.f.) **b** $\frac{45}{79}$ or 0.570 (3 s.f.)

6. $A = 2, B = -3, C = \frac{34}{11}, D = \frac{73}{11}$

BHASVIC Maths PS3 Answers: Section C

1. $A = 2, B = -4, C = 6, D = -11$

2. a) Check! b) $x = \frac{3-e}{2e-1}$

3. $\ln 4$

Section C

4. **a** 40 kg
   b The assumption is that the crate and books may be modelled as a particle.

5. $x = 4, y = 6, z = 14$

6. Proof