BHASVIC Maths

P S 2

Section A: Q1 - 6	90 mins
Section B: Q1 - 6	90 mins
Section C: Q1 - 6	90 mins

Notices:

- Remember teacher subject extension drop ins are every lunch time in room 24
- A2 Doubles students are available for drop in help almost every period, every day, in room 7

1. Sketch the following functions on the given domain and hence find their ranges:

(a) $f(x)=x^2+4x+3$ Domain f: $x \in \mathbb{R}$

(b) $g(t)=2t^2-4t-1$ Domain g: $t \in \mathbb{R}$

Make the following a one to one function and state its domain and range and sketch it.

(c) $h(x)=5x-x^2$

2. Simplify:

i) $\left|\frac{4a^2-4b^2}{4a-4b}\right.$ ii) $\frac{5}{x^2-4}-\frac{3}{x^2-4x+4}$ iii) $\frac{(x^2-x-2)}{(x^2-5x+6)} \div \frac{(x^2+2x+1)}{x^2-9}$

3. Solve the following equations.

(a) $3e^{2x+5}=4$ (b) $3^x=5^{1-x}$ (c) $2\ln(2x-1)=1+\ln 7$

Answers

4. A particle P is projected with velocity $(3u\mathbf{i} + 4u\mathbf{j})$ m s^{-1} from a fixed point O on horizontal ground. Given that P strikes the ground at a point 750 m from O,

   a show that $u = 17.5$

   b calculate the greatest height above the ground reached by P

   c find the angle the direction of motion of P makes with $\mathbf{i}$ when $t = 5$.

5. A factory produces a component for the motor trade and 5% of the components are defective. A quality control officer regularly inspects a random sample of 50 components. Find the probability that the next sample contains:

   a fewer than 2 defectives

   b more than 5 defectives.

   The officer will stop production if the number of defectives in the sample is greater than a certain value d. Given that the officer stops production less than 5% of the time:

   c find the smallest value of d.

6. Prove the following identities, setting out your proof clearly:

   (a) $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} \equiv \cos 2\theta$ (b) $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1-\cos\theta}{\sin\theta} \equiv \dfrac{2\sin\theta}{1+\cos\theta}$

   c) Prove that $\sin(A+B) + \sin(A-B) \equiv 2\sin A\cos B$

Answers

1. Integrate the following functions by working out what has been differentiated:

(a) $\int \sin 4x \, dx$ (b) $\int (3x+2)^5 \, dx$ (c) $\int \cos(x+2) \, dx$

2. Solve $\cot^2 x + \text{cosec} x = 11$ for $0 \leq x < 2\pi$, giving your answer(s) to 3sf.

3. $f(x) = e^{2x+3} - 1, x \in \mathbb{R}$.

a) Sketch the curve with equation $y = f(x)$, showing the coordinates of any points at which the curve meets the coordinate axes and the equation of the asymptote.

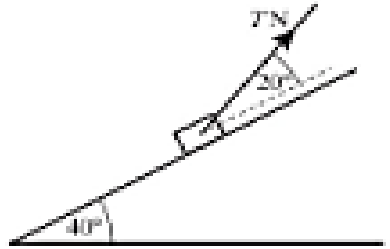
The curve with equation $y = f(x)$ has a gradient of 8 at the point P.

The x-coordinate of P is $\ln a + b$, where $a \in \mathbb{Z}$ and $b \in \mathbb{Q}$.

b) Find the value of a and the value of b.

Answers

4. A box of mass 0.5 kg is placed on a plane which is inclined at an angle of 40° to the horizontal. The coefficient of friction between the box and the plane is $\frac{1}{5}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in the diagram. The box is in limiting equilibrium and may be modelled as a particle. The tension in the string is T N.



Find the range of possible values of T.

5. The daily mean windspeed, x (kn) for Leeming is recorded in June 2015. The summary data is:

$\Sigma x = 243 \qquad \Sigma x^2 = 2317$

a Use your calculator to work out the mean and the standard deviation of the daily mean windspeed in June 2015.

The highest recorded windspeed was 17 kn and the lowest recorded windspeed was 4 kn.

b Estimate the number of days in which the windspeed was greater than one standard deviation above the mean.

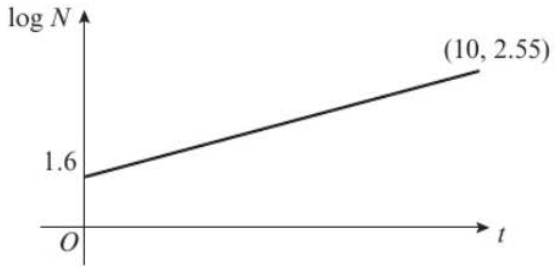
c State one assumption you have made in producing this estimate.

6. Find the stationary points of the curve C with equation $y = \frac{e^{2x+3}}{x}$, $x \neq 0$.

Answers

1. The vectors **a**, **b** and **c** are given as $\mathbf{a} = \begin{pmatrix} 8 \\ 23 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -15 \\ x \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -13 \\ 2 \end{pmatrix}$, where x is an integer. Given that $\mathbf{a} + \mathbf{b}$ is parallel to $\mathbf{b} - \mathbf{c}$, find the value of x.

2. A scientist is modelling the number of people, N, who have fallen sick with a virus after t days.



From looking at this graph, the scientist suggests that the number of sick people can be modelled by the equation $N = ab^t$, where a and b are constants to be found.

The graph passes through the points (0, 1.6) and (10, 2.55).

a Write down the equation of the line.

b Using your answer to part **a** or otherwise, find the values of a and b, giving them to 2 significant figures.

c Interpret the meaning of the constant a in this model.

d Use your model to predict the number of sick people after 30 days. Give one reason why this might be an overestimate.

Answers

3. Solve the equation $\frac{\tan 47 + \tan\theta}{1 - \tan 47 \tan\theta} = \frac{3}{2}$ for $-180° \leq x \leq 180°$ giving your answers to 2 d.p.

4. A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope. The mass of the car is 1400 kg. The mass of the trailer is 700 kg. The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively. The driving force on the car, due to its engine, is 2380 N. Find:

a the acceleration of the car

b the tension in the tow-rope.

When the car and trailer are moving at 12 m s^{-1}, the tow-rope breaks. Assuming that the driving force on the car and the resistances to motion are unchanged:

c find the distance moved by the car in the first 4 s after the tow-rope breaks.

d State how you have used the modelling assumption that the tow-rope is inextensible.

Answers

5. Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. Dhriti wants to test whether a new fertiliser increases the size of her tomatoes. She takes a sample of 40 ripe tomatoes that have been treated with the new fertiliser.

a Write down suitable hypotheses for her test. **(1)**

b Using a 5% significance level, find the critical region for her test. **(4)**

c Write down the actual significance level of the test. **(1)**

Dhriti finds that 18 out of the 40 tomatoes have a diameter greater than 4 cm.

d Comment on Dhriti's observed value in light of your answer to part **b**. **(2)**

6. A car tyre develops a puncture. The tyre pressure P, measured in suitable units called p.s.i., t minutes after the tyre got punctured, is given by the expression $P = 8 + 32\,e^{-kt}$, $t > 0$, where k is a positive constant.

a) State the tyre pressure when the tyre got punctured.

The tyre pressure halves 2 minutes after the puncture occurred.

b) Show that k = 0.4904, correct to 4 s.f.

c) Calculate the time that it takes for the tyre pressure to drop to 12 p.s.i.

d) Find the rate at which the pressure of the tyre is changing one minute after the puncture occurred.

Answers

BHASVIC Maths PS2 Answers: Section A

1. Proof

2. i) $a+b$ ii) $\dfrac{2x-16}{(x-2)^2(x+2)}$ iii) $\dfrac{x+3}{x+1}$

3. a) -2.36 b) 0.594 c) 2.68

Section A

BHASVIC Maths PS2 Answers: Section A

4. a) u = 17.5 b) 250 m c) 22°

5. a) 0.2794 b) 0.0378 c) d = 5

6. Proof

Section A

BHASVIC Maths PS2 Answers: Section B

1. a) $-\frac{1}{4}\cos 4x+c$ b) $\frac{1}{18}(3x+2)^6$ c) $\sin(x+2)+c$

2. 0.340, 2.80, 3.39, 6.03

3. a) $(0, e^3-1)$, $(-\frac{3}{2}, 0)$, $y=-1$ b) $a=2$, $b=-1.5$

4. $2.75 \leqslant T \leqslant 3.87\,\text{N}$

5. **a** Mean 8.1 kn, standard deviation 3.41 kn
   b 12 days
   c The windspeeds are equally distributed throughout the range.

6. $(0.5, 2e^4)$

BHASVIC Maths PS2 Answers: Section C

1. $x = 12$

2. **a** $\log N = 0.095t + 1.6$
   b $a = 40, b = 1.2$
   c The initial number of sick people
   d 9500 people. After 30 days people may start to recover, or the disease may stop spreading as quickly.

3. –170.69, 9.31

Section C

BHASVIC Maths PS2 Answers: Section C

4. **a** $0.7\,\text{ms}^{-2}$ **b** $770\,\text{N}$ **c** $58\,\text{m}$
   d Inextensible $\Rightarrow$ tension the same throughout, and the acceleration of the car and the trailer is the same.

5. **a** $H_0: p = 0.3$, $H_1: p > 0.3$
   b $18 \leqslant X \leqslant 40$
   c 3.2%
   d Reject the null hypothesis. Dhiriti's claim is supported.

6. b) 40 c) 4.24 d) -9.61

Section C