1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos \theta = 2 \sin \theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos \theta = 2 \sin \theta$ $\tan \theta = 2$ $\theta = 63.4°$	$\cos \theta = 2 \sin \theta$ $\cos^2 \theta = 4 \sin^2 \theta$ $1 - \sin^2 \theta = 4\sin^2 \theta$ $\sin^2 \theta = \frac{1}{5}$ $\sin \theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos \theta = 2 \sin \theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

3. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n(Ax + B)$$

where n, A and B are constants to be found.

(4)

4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 6 is 6 marks)

7. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

8.

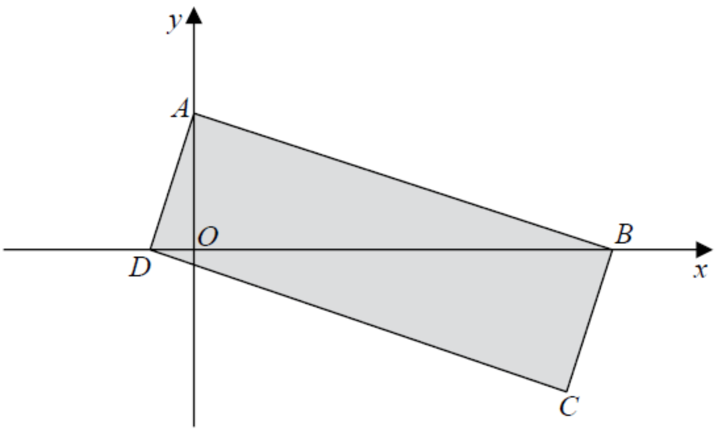

Figure 1

Figure 1 shows a rectangle $ABCD$.

The point A lies on the y-axis and the points B and D lie on the x-axis as shown in Figure 1.

Given that the straight line through the points A and B has equation $5y + 2x = 10$

(a) show that the straight line through the points A and D has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle $ABCD$.

(3)

9. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right) \mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

11.

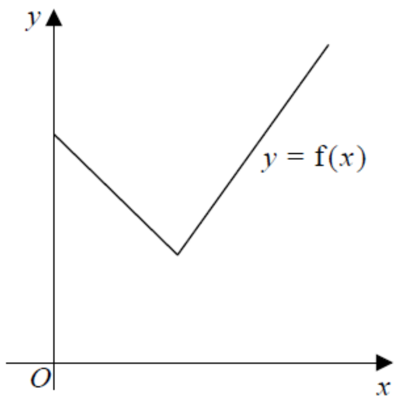

Figure 2

Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

12. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

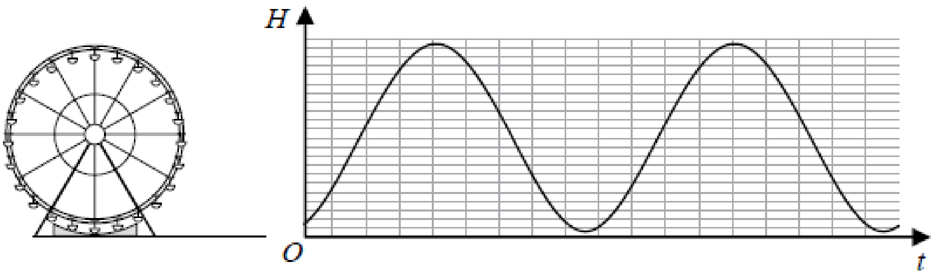

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

15.

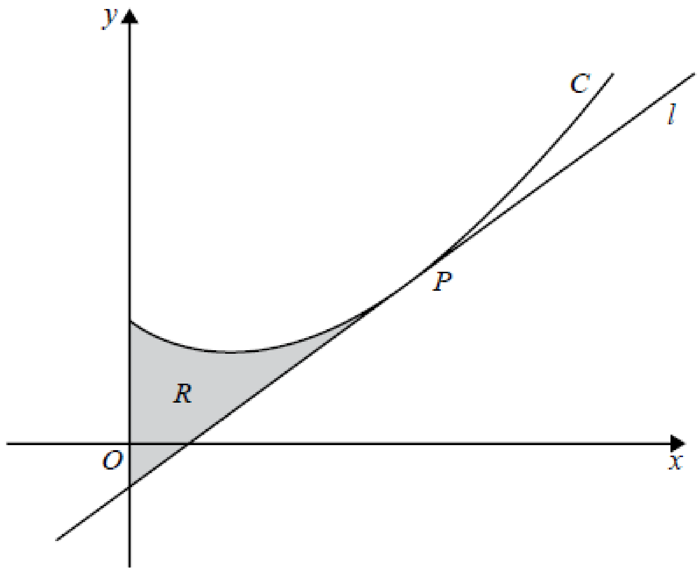

Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{22}P(11-2P), \quad t \geqslant 0, \qquad 0 < P < 5.5$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + C\mathrm{e}^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)