HOW TO ANSWER THE QUIZ

SECTIONS I – III

SECTION I

Directions. Each of the questions is followed by five suggested answers. Select the correct each case.

SECTION II

Directions. For each of the questions **ONE** or **MORE** of the responses given are correct. De of the responses is (are) correct. Then choose

A if **1**, **2** and **3** are correct

B if only **1** and **2** are correct

C if only **2** and **3** are correct

D if only **1** is correct

E if only **3** is correct

Directions Summarized			
A	1	2	3
B	1	2	
C		2	3
D	1		
E			3

SECTION III

Directions. Each of the questions consists of two statements (in some cases following preliminary information). You are required to determine the relationship between these and to choose

A if **1** always implies **2** but **2** does not imply **1**

B if **2** always implies **1** but **1** does not imply **2**

C if **1** always implies **2** and **2** always implies **1**

D if **1** always denies **2** and **2** always denies **1**

E if none of the above relationships holds

Directions Summarized	
A	$1 \Rightarrow 2,\ 2 \nRightarrow 1$
B	$2 \Rightarrow 1,\ 1 \nRightarrow 2$
C	$1 \Leftrightarrow 2$
D	**1** denies **2**, **2** denies **1**
E	None of these

HOW TO ANSWER QUIZ SECTIONS IV AND V

SECTION IV

Directions. Each of the questions consists of a problem followed by four pieces of information. Do not actually solve the problem, but decide whether the problem could be solved if any of the pieces of information were omitted, and choose

A if **1** could be omitted

B if **2** could be omitted

C if **3** could be omitted

D if **4** could be omitted

E if none of the pieces of information could be omitted

Directions Summarized	
A	Omit **1**
B	Omit **2**
C	Omit **3**
D	Omit **4**
E	Omit none

SECTION V

Directions. Each of the questions consists of a problem and two statements, **1** and **2**, in which certain data are given. You are not asked to solve the problem: you have to decide whether the data given in the statements are *sufficient* for solving the problem. Using the data given in the statements, choose

A if EACH statement (i.e. statement **1** ALONE and statement **2** ALONE) is sufficient by itself to solve the problem

B if statement **1** ALONE is sufficient but statement **2** alone is not sufficient to solve the problem

C if statement **2** ALONE is sufficient but statement **1** alone is not sufficient to solve the problem

D if BOTH statements **1** and **2** TOGETHER are sufficient to solve the problem, but NEITHER statement ALONE is sufficient

E if statements **1** and **2** TOGETHER are NOT sufficient to solve the problem, and additional data specific to the problem are needed

Directions Summarized	
A	Either
B	**1**
C	**2**
D	Both
E	Neither

QUIZ TWO

Time allowed: 1 hour

SECTION I

Questions 1–20 **(Twenty questions)**

1. $\sin 5\theta - \sin 9\theta \equiv$

A $-2 \sin 7\theta \cos 2\theta$

B $-2 \sin 2\theta \cos 7\theta$

C $-2 \cos 7\theta \cos 2\theta$

D $2 \sin 2\theta \sin 7\theta$

E $2 \sin 2\theta \cos 7\theta$

2. The modulus of $(1 - i)^6$ is

A 1

B $\sqrt{2}$

C 2

D $2\sqrt{2}$

E 8

3. $\sum_{r=1}^{10} (2r)^2 =$

A 12 100

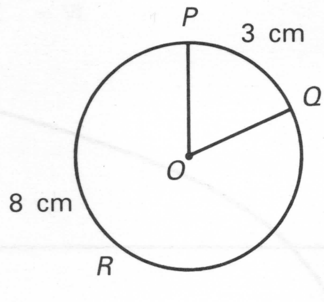
B 3025

C 2870

D 1540

E 770

4.



The minor arc PQ is of length 3 cm. The major arc QRP is of length 8 cm.

$\angle POQ =$

A $\frac{3}{8}$ radians

B $\frac{3\pi}{11}$ radians

C $\frac{6\pi}{11}$ radians

D $\frac{33}{2\pi}$ radians

E $\frac{8\pi}{11}$ radians

5. In a convergent geometric progression the first term is 3 and the sum to infinity is 4. The fourth term of the progression is

A $\frac{3}{4}$

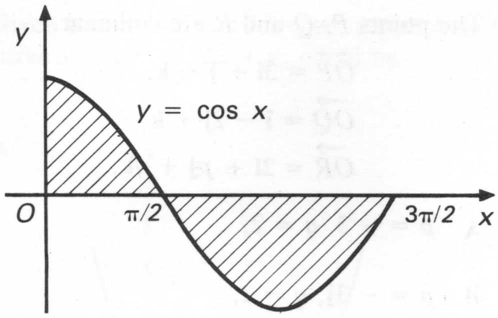
B $\frac{3}{64}$

C $\frac{3}{256}$

D $-\frac{3}{64}$

E $-\frac{3}{256}$

6.



The total area, in square units, of the shaded regions is

A 3

B − 1

C 1

D 2

E − 2

7. The complete solution set of the inequality

$$2|x| > |x - 1|,$$

where $x \in \mathbb{R}$, is

A $\{x : x < -1\}$

B $\{x : x > \frac{1}{3}\}$

C $\{x : -1 < x < \frac{1}{3}\}$

D $\{x : x < -1\} \cup \{x : x > \frac{1}{3}\}$

E $\{x : x < -\frac{1}{3}\} \cup \{x : x > 1\}$

8. The complex number z has modulus 20 and argument $\tan^{-1}(-4/3)$, where $-\pi/2 < \arg z < \pi/2$.

$z =$

A 12 + 16i

B 16 + 12i

C 12 − 16i

D 16 − 12i

E − 16 − 12i

9. An equation of the straight line which passes through the point (1, 0) and through the centre of the circle

$$x^2 + y^2 - 10x + 4y = 0$$

is

A $x - 3y - 1 = 0$

B $x - 2y - 1 = 0$

C $x + 2y - 1 = 0$

D $2x + y + 2 = 0$

E $2x + y - 8 = 0$

10. $\frac{d}{dx} \cos(x^2) =$

A $\sin(x^2)$

B $-\sin(x^2)$

C $\cos 2x$

D $-2x \sin(x^2)$

E $2x \sin(x^2)$

11. $f(x) \equiv (1 - 2x)^{-1} + (1 + x)^{-1}$.
$f(x)$ can be expanded as a series of ascending powers of x if

A $-1 < x < \frac{1}{2}$

B $-1 < x < 1$

C $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$

D $-\frac{1}{2} < x < \frac{1}{2}$

E $-2 < x < 2$

12. Given that $f(x) = e^{-x}$, for $x \in \mathbb{R}^+$, then $f^{-1}(x) =$

A e^x

B $-e^x$

C $\ln x$

D $-\ln x$

E $e^{-1/x}$

13. The equation $2x^2 + 5x - 6 = 0$ has roots α and β.

$\alpha^2 + \beta^2 =$

A $\frac{1}{4}$

B $\frac{13}{4}$

C $\frac{25}{4}$

D $\frac{37}{4}$

E $\frac{49}{4}$

14. The radius of a sphere is increasing at a constant rate. When the radius is 20 cm, the rate of increase of the surface area is 30 $cm^2\ s^{-1}$. At this moment the rate of increase of the volume, in $cm^3 s^{-1}$, is

A 300π

B 300

C 200

D 15

E 3

15. Given that

$$f : x \mapsto \frac{e^x}{1 - e^x},\ x \in \mathbb{R}^+,$$

then $f^{-1} : x \mapsto$

A $\ln\left(\frac{x}{x+1}\right)$

B $\ln\left(\frac{x+1}{x}\right)$

C $\frac{x}{x+1}$

D $\frac{1 - e^x}{e^x}$

E $\frac{e^{-x}}{1 - e^{-x}}$

16. The points P, Q and R are collinear.

$$\overrightarrow{OP} = 3\mathbf{i} + \mathbf{j} - \mathbf{k},$$
$$\overrightarrow{OQ} = \mathbf{i} - 2\mathbf{j} + \mathbf{k},$$
$$\overrightarrow{OR} = 2\mathbf{i} + p\mathbf{j} + q\mathbf{k}.$$

A $p = -3, q = 2,$

B $p = -3\tfrac{1}{2}, q = 2,$

C $p = -\tfrac{1}{2}, q = 0,$

D $p = 3, q = -2,$

E $p = -\tfrac{1}{2}, q = 2$

17. Given the following two statements,
(1) $x^2 < 1$,
(2) $x < 1$,
where $x \in \mathbb{R}$, which one of the following statements is always true?

A (1) $\Rightarrow$ (2) but (2) $\not\Rightarrow$ (1)

B (2) $\Rightarrow$ (1) but (1) $\not\Rightarrow$ (2)

C (1) $\Leftrightarrow$ (2)

D (1) $\not\Rightarrow$ (2) and (2) $\not\Rightarrow$ (1)

E None of the above

18. $\int \ln x \, dx =$

A $\frac{1}{x}$ + constant

B $x \ln x$ + constant

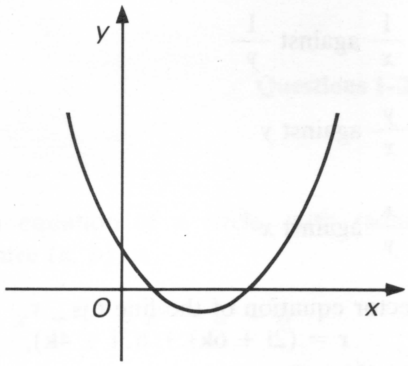
C $x \ln x - x$ + constant

D $\frac{1}{x} \ln x$ + constant

E $x \ln x + x$ + constant

19. Given that $a > 0$ and $b^2 < ac$, a sketch of the curve $y = ax^2 + 2bx + c$, could be

A



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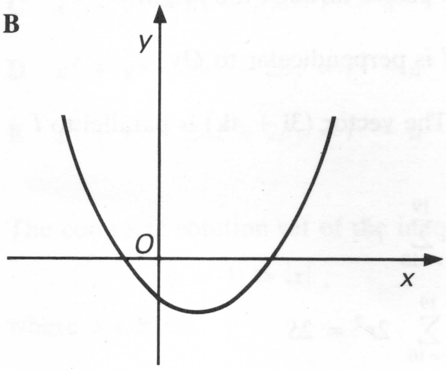

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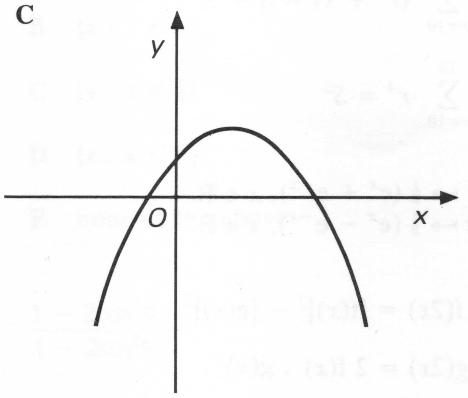

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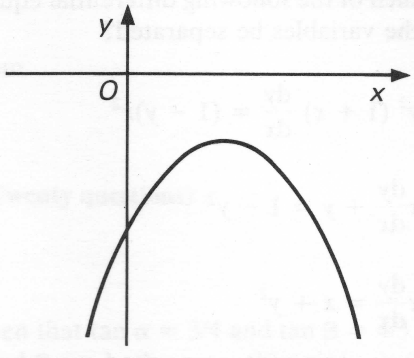

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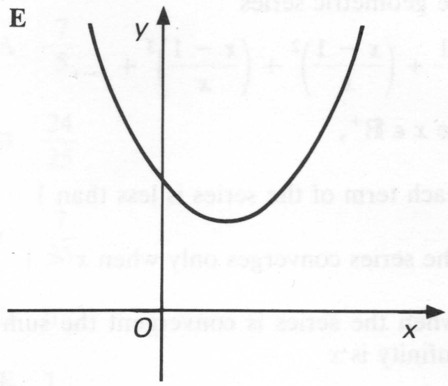

20. The number of different arrangements which can be made using all the letters of the word FOOLS, if the O's are never separated, is

A 120

B 48

C 24

D 20

E 10

SECTION II

Questions 21–30 **(Ten questions)**

21. $z = \dfrac{2 + \text{i}}{2 - \text{i}}$.

1 $|z| = 1$

2 $\text{Re } z = \dfrac{3}{5}$

3 $\arg z = \dfrac{\pi}{4}$

22. The solutions of the equation

$$2x^3 + 6x^2 - 1 = 0$$

can be found from the intersections of the two graphs

1 $y = 2x^3$ and $y = 6x^2 - 1$

2 $y = 6 - \dfrac{1}{x^2}$ and $y = 2x$

3 $y = x^2(x + 3)$ and $y = \frac{1}{2}$

23. In which of the following differential equations can the variables be separated?

1 $y^2 (1 + x) \frac{dy}{dx} = (1 - y)x^2$

2 $x\frac{dy}{dx} + y = 1 - y^2$

3 $x\frac{dy}{dx} = x + y^2$

24. In the geometric series

$$\frac{x-1}{x} + \left(\frac{x-1}{x}\right)^2 + \left(\frac{x-1}{x}\right)^3 + \ldots,$$

where $x \in \mathbb{R}^+$,

1 each term of the series is less than 1

2 the series converges only when $x < 1$

3 when the series is convergent the sum to infinity is x

25. $f : x \mapsto e^x$ and $x,\ y \in \mathbb{R}^+$.

1 $f(x + y) = f(x) . f(y)$

2 $f^{-1}(xy) = f^{-1}(x) + f^{-1}(y)$

3 $f^{-1}(x^y) = y\, f^{-1}(x)$

26. $f(x) \equiv x^6 + 64$.

1 $(x + 2)$ is a factor of $f(x)$

2 $(x - 2)$ is a factor of $f(x)$

3 $(x^2 + 4)$ is a factor of $f(x)$

27. Given that $y = \dfrac{x}{a + bx}$, where a, b are non-zero constants, then a straight line graph obtained by plotting

1 $\frac{1}{x}$ against $\frac{1}{y}$

2 $\frac{y}{x}$ against y

3 $\frac{x}{y}$ against x

28. A vector equation of the line l is

$$\mathbf{r} = (2\mathbf{i} + 6\mathbf{k}) + t(3\mathbf{i} + 4\mathbf{k}),$$

where t is a parameter.

1 l passes through the origin O

2 l is perpendicular to Oy

3 The vector $(3\mathbf{i} + 4\mathbf{k})$ is parallel to l

29. $S = \sum_{r=10}^{19} r^2$.

1 $\sum_{r=10}^{19} 2r^2 = 2S$

2 $\sum_{r=10}^{19} (r^2 + 1) = S + 9$

3 $\sum_{r=10}^{19} r^4 = S^2$

30. $f : x \mapsto \frac{1}{2}(e^x + e^{-x}),\ x \in \mathbb{R}$.
$g : x \mapsto \frac{1}{2}(e^x - e^{-x}),\ x \in \mathbb{R}$.

1 $f(2x) = [f(x)]^2 - [g(x)]^2$

2 $g(2x) = 2\, f(x) . g(x)$

3 $-1 < \dfrac{g(x)}{f(x)} < 1$