Quiz 1

Time allowed: 1 hour

SECTION I

Questions 1–20 **(Twenty questions)**

1. $e^{\frac{1}{2}\ln x} =$

A $\ln(x^2)$

B $\ln(\sqrt{x})$

C x^2

D $\frac{1}{2}x$

E $\sqrt{x}$

2. $\frac{d}{dx}\left(\frac{x-1}{\sqrt{x}}\right) =$

A $2\sqrt{x}$

B $\frac{x+1}{x\sqrt{x}}$

C $\frac{3x-1}{2\sqrt{x}}$

D $\frac{x+1}{2x\sqrt{x}}$

E $\frac{3x-1}{2x\sqrt{x}}$

3. $\cos(\pi + \theta) \equiv$

A $-\cos\theta$

B $\cos\theta$

C $\sin\theta$

D $-\sin\theta$

E $-1 + \cos\theta$

4. The complete solution set of the inequality

$$x^2 + 2x - 15 < 0,$$

where $x \in \mathbb{R}$, is

A $\{x : x > -5\}$

B $\{x : -5 < x < 3\}$

C $\{x : x > 3\}$

D $\{x : -3 < x < 5\}$

E $\{x : x < -5\} \cup \{x : x > 3\}$

5. $\int \tan x \, dx =$

A ln cos x + constant

B $\sec^2 x$ + constant

C ln sec x + constant

D ln cosec x + constant

E ln sin x + constant

6. $\sum_{r=1}^{n} r^2 =$

A $n(n+1)/2$

B $n^2(n+1)^2/4$

C $n(n+1)(n+2)/3$

D $n(n+1)(2n+1)/3$

E $n(n+1)(2n+1)/6$

7. $z^2 + 1 \equiv$

A $(z + 1)(z - 1)$

B $(z + 1)^2$

C $(z + i)^2$

D $(z + i)(z - i)$

E none of the above

8. The first two terms in the binomial expansion of $\frac{1}{3x - 2}$, where $|x| < \frac{2}{3}$, in ascending powers of x are

A $-\frac{1}{2}, -\frac{3x}{4}$

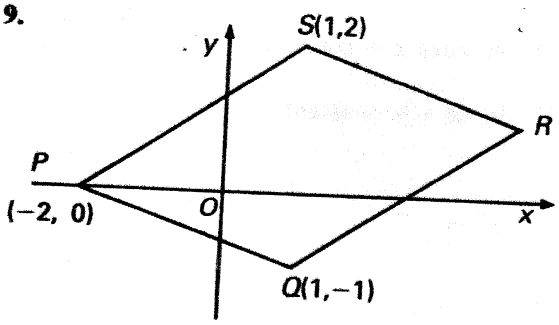
B $-\frac{1}{2}, +\frac{3x}{4}$

C $\frac{1}{2}, -\frac{3x}{4}$

D $\frac{1}{3}, +\frac{2x}{9}$

E $\frac{1}{3x}, +\frac{2}{9x^2}$

9.



PQRS is a parallelogram. The coordinates of *R* are

A (3, 1)

B (4, 1)

C (4, 2)

D (3, 2)

E (2, 1)

10. Given that $x = a$ is an approximation to a root of the equation $f(x) = 0$, then, in general, a closer approximation is given by

A $x = a + \frac{f(a)}{f'(a)}$

B $x = a + \frac{f'(a)}{f(a)}$

C $x = a - \frac{f(a)}{f'(a)}$

D $x = a - \frac{f'(a)}{f''(a)}$

E $x = a - \frac{f'(a)}{f(a)}$

11. $\mathbf{u} = 4\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -6\mathbf{i} + 3\mathbf{j}$.
$\mathbf{u} \cdot \mathbf{v} =$

A $-2\mathbf{i}$

B $-24\mathbf{i} - 9\mathbf{j}$

C -3

D -33

E 30

12. Which one of the following can be seen, BY INSPECTION, NOT to be a factor of

$$6x^4 - 5x^3 - 53x^2 + 45x - 9\ ?$$

A $2x - 1$

B $x - 3$

C $x + 3$

D $3x - 1$

E $4x - 3$

13. $\dfrac{x+7}{x^2-x-6} \equiv$

A $\dfrac{2}{x-3} - \dfrac{1}{x+2}$

B $\dfrac{2}{x+2} - \dfrac{1}{x-3}$

C $\dfrac{9}{5(x-2)} - \dfrac{4}{5(x+3)}$

D $\dfrac{4}{5(x-2)} - \dfrac{9}{5(x+3)}$

E $\dfrac{1}{x+2} + \dfrac{2}{x-3}$

14. Given that $\cos^2 x = \dfrac{16}{25}$, where $x \in \mathbb{R}$ and $\pi \leqslant x \leqslant 2\pi$, then the possible value(s) of $\sin x$ is (are)

A $\dfrac{3}{5}$ only

B $-\dfrac{3}{5}$ only

C $\pm\dfrac{3}{5}$

D $-\dfrac{3}{4}$ only

E $\pm\dfrac{3}{4}$

15. Given that $x = t^2$, $y = t^3$, then $\dfrac{dx}{dy} =$

A $\dfrac{1}{t}$

B $\dfrac{2}{3t}$

C $\dfrac{3t}{2}$

D $\dfrac{2t}{3}$

E $\dfrac{3}{2t}$

16. $\sin\theta + \sqrt{3}\cos\theta \equiv r\cos(\theta + \alpha)$, where $r > 0$ and $-\pi/2 \leqslant \alpha \leqslant \pi/2$.

A $r = 2, \quad \alpha = \pi/6$

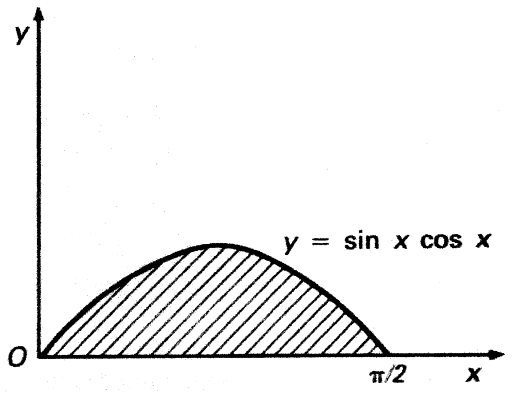
B $r = 2, \quad \alpha = \pi/3$

C $r = 2, \quad \alpha = -\pi/3$

D $r = 2, \quad \alpha = -\pi/6$

E $r = 4, \quad \alpha = -\pi/6$

17.



The area, in square units, of the shaded region is

A $\dfrac{1}{2}$

B $\dfrac{1}{4}$

C 1

D $\pi/2$

E $\dfrac{1}{8}$

18. The roots of the equation $x^2 - 4x + 7 = 0$ are α and β. An equation whose roots are α/β and β/α is

A $7x^2 - 2x - 7 = 0$

B $7x^2 + 2x + 7 = 0$

C $7x^2 - 2x + 7 = 0$

D $7x^2 - 30x + 7 = 0$

E none of the above

19. Which one of the following is an odd function of x?

A $f : x \mapsto |x|^3$

B $f : x \mapsto \sin^2 x$

C $f : x \mapsto (1 - x)^5$

D $f : x \mapsto e^{-x}$

E $f : x \mapsto -\sin 2x$

20. Given that

$$\lg(y + 2) + 2\lg x = 1,$$

then $y =$

A $\frac{1}{x^2} - 2$

B $\frac{5}{x} - 2$

C $\frac{10}{x^2} - 2$

D $\frac{1}{2x} - 2$

E $8 - x^2$

SECTION II

Questions 21–30 **(Ten Questions)**

21. Given that $\tan(x/2) = t$, then

1 $\cos x = \frac{1 - t^2}{1 + t^2}$

2 $\sin x = \frac{2t}{1 - t^2}$

3 $\frac{dx}{dt} = \frac{2}{1 - t^2}$

22. l is the line with equation $4x - 3y = 5$.

1 The gradient of l is $-4/3$

2 l touches the circle $x^2 + y^2 = 1$

3 The area of the finite region enclosed by l and the coordinate axes is $25/24$ units^2

23. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ has centre $(-2, 4)$ and radius 6.

1 $g = -2$

2 $f = 4$

3 $c = -16$

24. $z = \frac{2 - i}{1 + 2i}$.

1 $|z| = 1$

2 $zz^* = 1$

3 $z + z^* = 0$

25. $f(x) \equiv x^3 + 3x - 5$.

1 The equation $f(x) = 0$ has just one real root

2 The equation $f(x) = 0$ has a root in the interval $[1, 2]$

3 The curve $y = f(x)$ has just one asymptote

26. p, q, r are 3 positive unequal integers in geometric progression.

1 $pr = q^2$

2 $\sqrt{(r/p)}$ is the common ratio of the progression

3 $\ln p$, $\ln q$ and $\ln r$ are numbers in arithmetic progression

27. $3^x = 1{\cdot}72$.

1 $x = \log_3 1{\cdot}72$

2 $5{\cdot}16 = 3^{1+x}$

3 $3{\cdot}44 = 3^{2x}$

28. Which of the following relations will give a straight line when $\frac{1}{x}$ is plotted against y?

1 $xy + 3x = 2$

2 $x + y = 2xy$

3 $\frac{2}{x} + y = 3x$

29. f and g are functions of x defined for $x \in \mathbb{R}$. It is necessarily true that

1 $f^{-1} = \frac{1}{f}$

2 $(fg)^{-1} = f^{-1}g^{-1}$

3 $ff^{-1}: x \mapsto x$

30. The plane $x - 2y - 4z = 3$

1 passes through the point $(-1, 1, -1)$

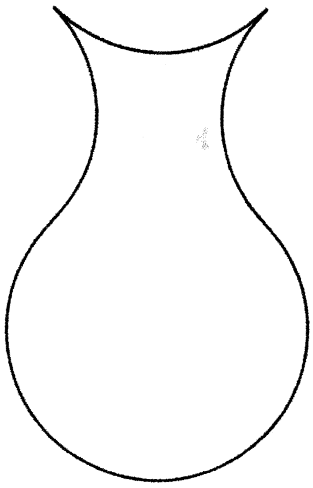
2 meets the x-axis at the point $(3, 0, 0)$

3 is perpendicular to the line $x = t - 2, y = 4 - 2t, z = 7 - 4t$

Problem

Arcs

The shape below is made up of six joined quarter circle arcs, all of equal radius r.



What is the area of the shape in terms of r?

Problem 43

END OF QUIZ