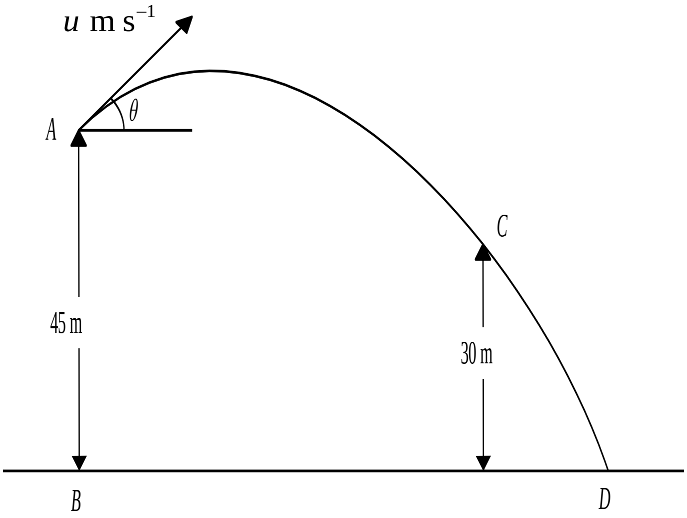
Assignment pi VJM M2 questions
due in Wednesday 31st January
14 questions so start early go to SE's for help

1) **Figure 3**



A particle P is projected from a point A with speed u m s^{-1} at an angle of elevation θ, where $\cos\theta = \frac{4}{5}$. The point B, on horizontal ground, is vertically below A and AB = 45 m. After projection, P moves freely under gravity passing through point C, 30 m above the ground, before striking the ground at the point D, as shown in Figure 3.

Given that P passes through C with speed 24.5 m s^{-1},

(*a*) using conservation of energy, or otherwise, show that u = 17.5, **(4)**

(*b*) find the size of the angle which the velocity of P makes with the horizontal as P passes through C, **(3)**

(*c*) find the distance BD. **(7)**

2) At time t seconds ($t \geq 0$), a particle P has position vector **p** metres, with respect to a fixed origin O, where

$$\mathbf{p} = (3t^2 - 6t + 4)\mathbf{i} + (3t^3 - 4t)\mathbf{j}.$$

Find

(*a*) the velocity of P at time t seconds, **(2)**

(*b*) the value of t when P is moving parallel to the vector **i**. **(3)**

When t = 1, the particle P receives an impulse of (2**i** – 6**j**) N s. Given that the mass of P is 0.5 kg,

(*c*) find the velocity of P immediately after the impulse. **(4)**

3)

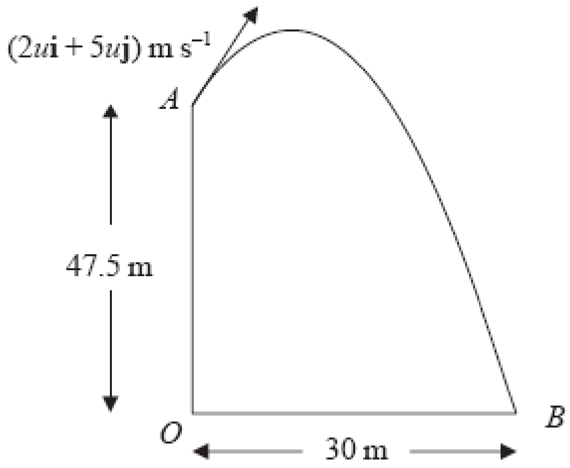


Figure 3

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.*]

A particle P is projected from the point A which has position vector 47.5**j** metres with respect to a fixed origin O. The velocity of projection of P is $(2u\mathbf{i} + 5u\mathbf{j})$ m s^{-1}. The particle moves freely under gravity passing through the point B with position vector 30**i** metres, as shown in Figure 3.

(*a*) Show that the time taken for P to move from A to B is 5 s. **(6)**

(*b*) Find the value of u. **(2)**

(*c*) Find the speed of P at B. **(5)**

4)

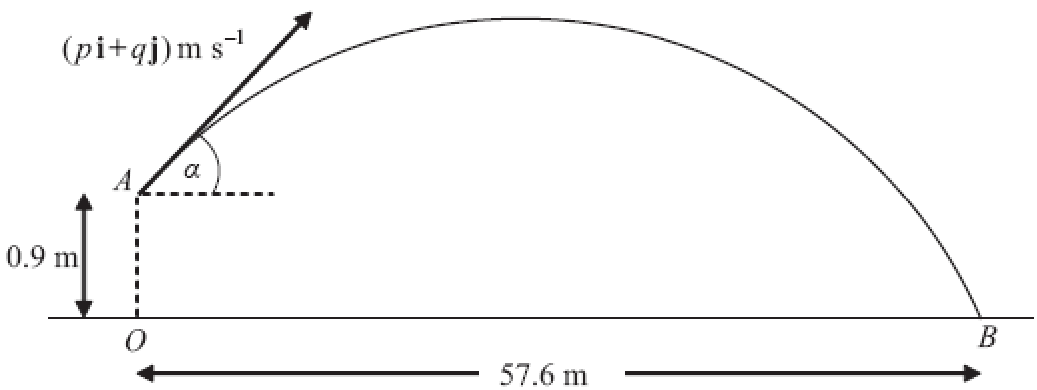


Figure 3

A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, at an angle α above the horizontal. The unit vectors **i** and **j** are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and OB = 57.6 m, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

(*a*) find the value of *p*, **(2)**
(*b*) show that $q = 14.4$, **(3)**
(*c*) find the initial speed of the cricket ball, **(2)**
(*d*) find the exact value of $\tan \alpha$. **(1)**
(*e*) Find the length of time for which the cricket ball is at least 4 m above the ground. **(6)**
(*f*) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic. **(1)**

5) **Figure 1**

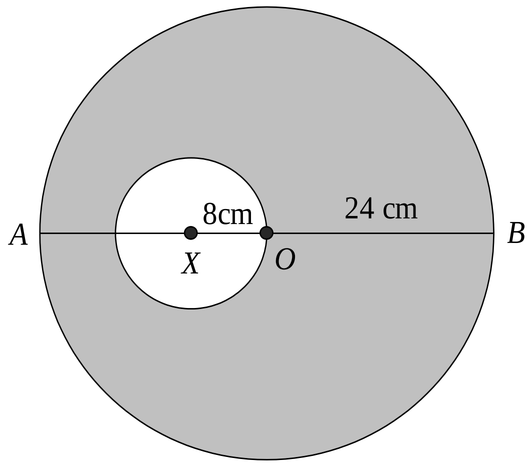


Figure 1 shows a template *T* made by removing a circular disc, of centre *X* and radius 8 cm, from a uniform circular lamina, of centre *O* and radius 24 cm. The point *X* lies on the diameter *AOB* of the lamina and *AX* = 16 cm. The centre of mass of *T* is at the point *G*.

(*a*) Find *AG*. **(6)**

The template *T* is free to rotate about a smooth fixed horizontal axis, perpendicular to the plane of *T*, which passes through the mid-point of *OB*. A small stud of mass $\frac{1}{4}m$ is fixed at *B*, and *T* and the stud are in equilibrium with *AB* horizontal. Modelling the stud as a particle,

(*b*) find the mass of *T* in terms of *m*. **(4)**

6)

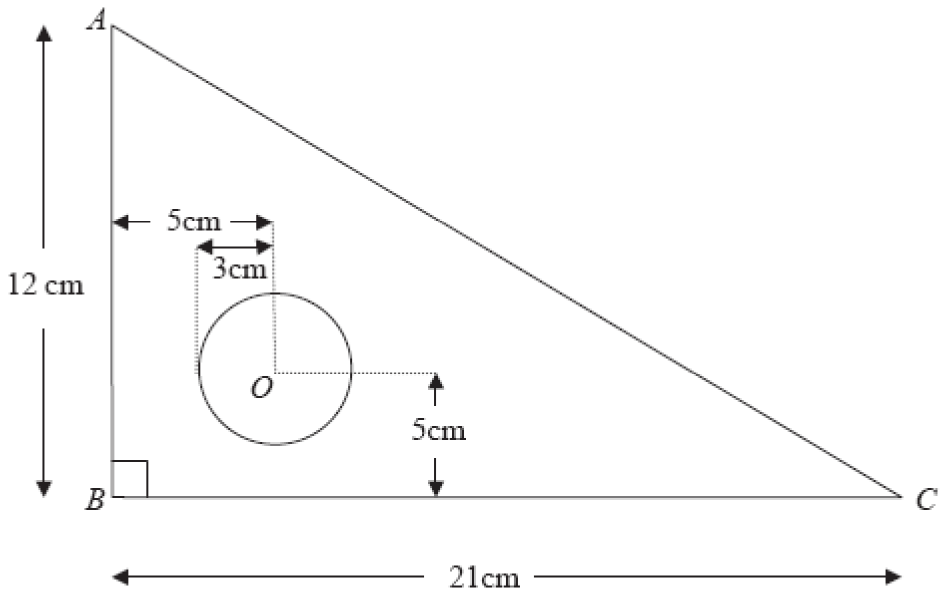


Figure 1

A set square S is made by removing a circle of centre O and radius 3 cm from a triangular piece of wood. The piece of wood is modelled as a uniform triangular lamina ABC, with $\angle ABC = 90°$, $AB = 12$ cm and $BC = 21$ cm. The point O is 5 cm from AB and 5 cm from BC, as shown in Figure 1.

(*a*) Find the distance of the centre of mass of S from

(i) AB,

(ii) BC. **(9)**

The set square is freely suspended from C and hangs in equilibrium.

(*b*) Find, to the nearest degree, the angle between CB and the vertical. **(3)**

7)

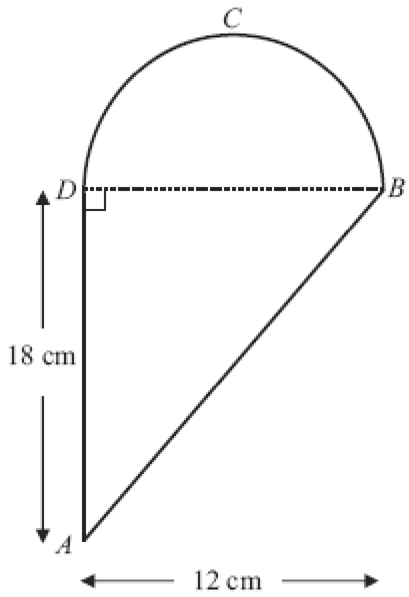


Figure 2

A uniform lamina *ABCD* is made by joining a uniform triangular lamina *ABD* to a uniform semi-circular lamina *DBC*, of the same material, along the edge *BD*, as shown in Figure 2. Triangle *ABD* is right-angled at *D* and *AD* = 18 cm. The semi-circle has diameter *BD* and *BD* = 12 cm.

(*a*) Show that, to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *AD* is 4.69 cm.

(4)

Given that the centre of mass of a uniform semicircular lamina, radius *r*, is at a distance $\frac{4r}{3\pi}$ from the centre of the bounding diameter,

(*b*) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *BD*.

(4)

The lamina is freely suspended from *B* and hangs in equilibrium.

(*c*) Find, to the nearest degree, the angle which *BD* makes with the vertical.

(4)

8) A particle P of mass m is moving in a straight line on a smooth horizontal table. Another particle Q of mass km is at rest on the table. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. After the collision, the speed of P is v and the speed of Q is $3v$. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(*a*) Find, in terms of v only, the speed of P before the collision.

(3)

(*b*) Find the value of k.

(3)

After being struck by P, the particle Q collides directly with a particle R of mass $11m$ which is at rest on the table. After this second collision, Q and R have the same speed and are moving in opposite directions. Show that

(*c*) the coefficient of restitution between Q and R is $\frac{3}{4}$,

(4)

(*d*) there will be a further collision between P and Q.

(2)

9) A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P. The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(*a*) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$.

(5)

(*b*) Find the total kinetic energy lost in the collision.

(5)

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

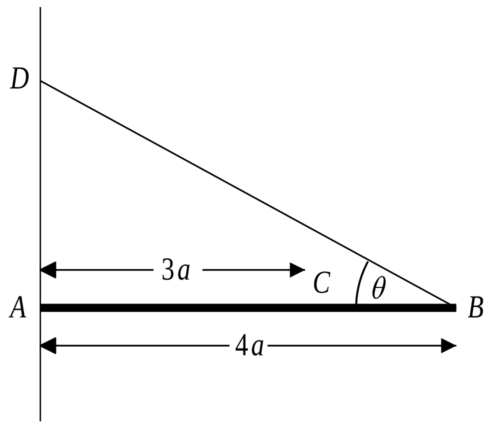
(*c*) Calculate the range of values of e for which there will be a second collision between P and Q.

(7)

10. Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $2u$ and the speed of Q is u. The coefficient of restitution between the particles is e, where $e < 1$. Find, in terms of u and e,
(i) the speed of P immediately after the collision,
(ii) the speed of Q immediately after the collision. **(7)**

11)

Figure 2



A horizontal uniform rod *AB* has mass *m* and length $4a$. The end *A* rests against a rough vertical wall. A particle of mass $2m$ is attached to the rod at the point *C*, where $AC = 3a$. One end of a light inextensible string *BD* is attached to the rod at *B* and the other end is attached to the wall at a point *D*, where *D* is vertically above *A*. The rod is in equilibrium in a vertical plane perpendicular to the wall. The string is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$, as shown in Figure 2.

(*a*) Find the tension in the string.

(5)

(*b*) Show that the horizontal component of the force exerted by the wall on the rod has magnitude $\frac{8}{3}mg$.

(3)

The coefficient of friction between the wall and the rod is μ. Given that the rod is in limiting equilibrium,

(*c*) find the value of μ.

(4)

12)

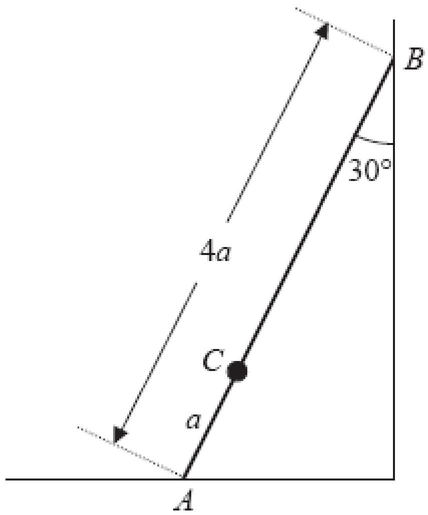


Figure 2

A ladder *AB*, of mass *m* and length $4a$, has one end *A* resting on rough horizontal ground. The other end *B* rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point *C*, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground. **(10)**

13)

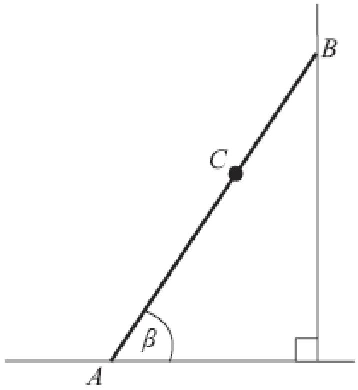


Figure 1

Figure 1 shows a ladder *AB*, of mass 25 kg and length 4 m, resting in equilibrium with one end *A* on rough horizontal ground and the other end *B* against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle *β* with the ground. When Reece, who has mass 75 kg, stands at the point *C* on the ladder, where *AC* = 2.8 m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(*a*) Find the magnitude of the frictional force of the ground on the ladder. **(3)**

(*b*) Find, to the nearest degree, the value of *β*. **(6)**

(*c*) State how you have used the modelling assumption that Reece is a particle. **(1)**

14)

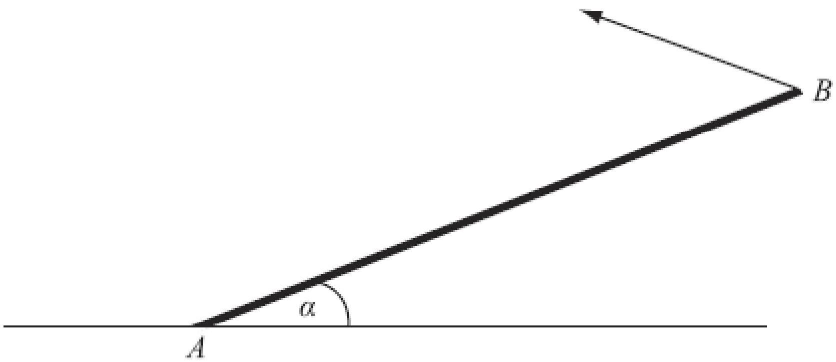


Figure 2

A uniform rod *AB*, of mass 20 kg and length 4 m, rests with one end *A* on rough horizontal ground. The rod is held in limiting equilibrium at an angle *α* to the horizontal, where tan *α* = $\frac{3}{4}$, by a force acting at *B*, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5.

Find the magnitude of the normal reaction of the ground on the rod at *A*.

(7)

Answers

1a) Proof   b) $55°$ c) 60

2 a) $(6\mathrm{t}-6)\mathbf{i}+(9\mathrm{t}^2-4)\mathbf{j}$ b) $\frac{2}{3}$ c) 4**i** – 7**j**

3 a) Proof   b) 3   c) 34.5

4 a) 19.2   b) Proof   c) 24   d) $\frac{3}{4}$ e) 2.5 s

f) e.g. Variable '*g*', Air resistance, Speed of wind, Swing of ball, The ball is not a particle.

5 a) 25   b) $\frac{3}{11}m$

6 a)i) 7.6 ii) 3.7 b) $15°$

7 a) Proof   b) 3.06   c) $23°$

8 a) 8v   b) 3   c) Proof   d) $\frac{9}{8}V > V\ hence\ further\ collision$

9 a) Proof   b) $\frac{9}{20}mu^2$ c) $e > \frac{1}{4}$

10 i) u(1 - e)   ii) u(1 + 2e)

11 a) $\frac{10}{3}mg$ b) Proof   c) $\frac{3}{8}$

12 $\frac{5}{16\sqrt{3}}$

13 a) 44g   b) $56°$ c) So that Reece's weight acts directly at the point *C*

14 160 N