A2 Assignment theta Cover Sheet

| Question |  | Oٍ | 会 |  | Topic | Comment |
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| 高 | Aa |  |  |  | C4 Integration as the reverse of differentiation | $\frac{1}{4} e^{x}+\frac{3}{4} x+c$ |
|  | Ab |  |  |  | C4 Integration as the reverse of differentiation | $3 e^{x}-2 \ln \|x\|+c$ |
|  | Ac |  |  |  | C4 Integration as the reverse of differentiation | $-\frac{3}{4} \ln \|x\|+\mathrm{c}$ |
|  | Ba |  |  |  | C3 Rcos | $\mathrm{R}=5, \alpha=0.64$ |
|  | Bb |  |  |  | C3 Rcos | $\mathrm{R}=13, \alpha=0.39$ |
|  | Bc |  |  |  | C3 Rcos | $\mathrm{R}=\sqrt{2}, \alpha=0.79$ |
|  | Ca |  |  |  | C3 Sketch, domain and range | $\mathrm{f} \in \mathrm{R}: \mathrm{f}=1$ |
|  | Cb |  |  |  | C3 Sketch, domain and range | $\mathrm{g} \in \mathrm{R}: \mathrm{g}<1$ |
|  | Cc |  |  |  | C3 Sketch, domain and range | $h \in R$ |
|  | Da |  |  |  | C3 Sketching arccos, arcsin, arctan, domain and range | Use Autograph, your graphical calculator or Desmos to check |
|  | Db |  |  |  | C3 Sketching arccos, arcsin, arctan, domain and range | Use Autograph, your graphical calculator or Desmos to check |
|  | Dc |  |  |  | C3 Sketching arccos, arcsin, arctan, domain and range | Use Autograph, your graphical calculator or Desmos to check |
|  | TT3A |  |  |  | Trig | $\begin{aligned} \cos (\theta-\theta) & \equiv \cos \theta \cos \theta+\sin \theta \sin \theta \\ & \Rightarrow \sin ^{2} \theta+\cos ^{2} \theta \equiv 1 \text { as } \cos \theta=0 \end{aligned}$ |
|  | TT3Bi |  |  |  | Trig | $\sin 35^{\circ}$ |
|  | TT3Bii |  |  |  | Trig | $\cos 7 \theta$ |
|  | TTCBiii |  |  |  | Trig | $\tan 5 \theta$ |
|  | TT3Ci |  |  |  | Differentiation | $\mathrm{e}^{2 x}(2 \cos x-\sin x)$ |
|  | TT3Cii |  |  |  | Differentiation | $\mathrm{e}^{x} \sec 3 x(1+3 \tan 3 x)$ |
|  | TT3Ciii |  |  |  | Differentiation | $\frac{\mathrm{e}^{\sin x}\left(\cos ^{2} x+\sin x\right)}{\cos ^{2} x}$ |
|  | TT3Di |  |  |  | Trig | $\frac{1}{\sqrt{1+q^{2}}}$ |
|  | TT3Dii |  |  |  | Trig | $\frac{\frac{1}{q}+1}{1-\frac{1}{q}}$ |
|  | TT3Diii |  |  |  | Trig | $\frac{q}{q+1}$ |
| $\begin{aligned} & \text { n } \\ & \text { B } \\ & 3 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | 1a |  |  |  | C4 partial fractions | $\frac{4}{(2 x+1)}-\frac{1}{(x-3)}$ |
|  | 1b |  |  |  | C4 partial fractions | $\frac{1}{(x+2)}+\frac{1}{(x+2)^{2}}+\frac{1}{(x+2)^{3}}$ |
|  | 1c |  |  |  | C4 partial fractions | $1-\frac{2}{(x-2)}+\frac{3}{(x+1)}$ |
|  | 2 |  |  |  | C4 Binomial expansion (simple) | $2-\frac{9}{4} x-\frac{81}{64} x^{2}-\frac{729}{512} x^{3}$ |
|  | 3a |  |  |  | C4 Binomial expansion (simple) | $\frac{1}{2}\left[1+\frac{3}{8} x+\frac{27}{128} x^{2}+\ldots\right]$ |



| $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | $\varepsilon$ | $\zeta$ | $\eta$ | $\theta$ | $\imath$ | $\kappa$ | $\lambda$ | $\mu$ | $\nu$ | $\xi$ | $o$ | $\pi$ | $\rho$ | $\sigma$ | $\tau$ | $\nu$ | $\varphi$ | $\chi$ | $\psi$ | $\omega$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

"Music is the pleasure the human soul experiences from counting without being aware that it is counting"
G W Leibnitz

## A2 Maths with Mechanics Assignment $\Theta$ (theta) Due in the week after Reading week w/b 21/11

## Drill

Part A Integrate* the following functions with respect to $x$ :
(a) $\frac{\mathrm{e}^{x}+3}{4}$
(b) $\frac{3 x \mathrm{e}^{x}-2}{x}$
(c) $-\frac{3}{4 x}$
*always use the correct notation when integrating and the constant for indefinite integration
Part B Write in the form indicated giving $\alpha$ as an acute angle in radians to 2 dp :
(a) $4 \sin x-3 \cos x$
$R \sin (x-\alpha)$
(b) $\quad 12 \sin x+5 \cos x$ $R \sin (x+\alpha)$
(c) $\cos x+\sin x$
$R \cos (x-\alpha)$

Part C Sketch and state the ranges of the following functions (defined on $\mathbb{R}$ ): show asymptotes clearly
(a)
$f(x)=\frac{1}{x+2}+1$
(b) $\quad g(x)=1-e^{2 x}$
(c) $\quad h(x)=\ln (1+x)$

Part D Sketch the following functions stating the domain and range in each case.
(a) $y=2-\arcsin (3 x)$
(b) $y=3+2 \arccos x$
(c) $y=-\arctan \left(\frac{1}{2} x\right)$
*note in the specifications arcsin is used, not $\sin ^{-1} \mathbf{x}$ as on your calculators

## FOCUS from C3 Mock exam

## Learning trig formula is vital unless you know the trig formula you will not recognise them in questions

TT3A) Using the expansion of $\cos (A-B)$ with $A=B=\theta$, show that $\sin ^{2} \theta+\cos ^{2} \theta \equiv 1$.

TT3B) Express the following as a single sine, cosine or tangent:
i) $\sin 15^{\circ} \cos 20^{\circ}+\cos 15^{\circ} \sin 20^{\circ}$
ii) $\cos 4 \theta \cos 3 \theta-\sin 4 \theta \sin 3 \theta$
iii) $\frac{\tan 2 \theta+\tan 3 \theta}{1-\tan 2 \theta \tan 3 \theta}$

## Write these out as many times as you need to remember them

Trig Identity table
$\sin 2 x=$
$\cos 2 x=$
$\cos 2 x=$
$\cos 2 x=$
$\tan 2 x=$
$\sec ^{2} x=$
$\operatorname{cosec}^{2} x=$
$\sin ^{2} x=$
$\cos ^{2} x=$
$\tan (A-B)=$

Trig Identity table
$\sin 2 x=2 \sin x \cos x$
$\cos 2 x=2 \cos ^{2} x-1$
$\cos 2 x=1-2 \sin ^{2} x$
$\cos 2 x=\cos ^{2} x-\sin ^{2} x$
$\tan 2 x=\frac{2 \tan x}{1-\tan ^{2} x}$
$\sec ^{2} x=1+\tan ^{2} x$
$\operatorname{cosec}^{2} x=1+\cot ^{2} x$
$\sin ^{2} x=\frac{1}{2}-\frac{1}{2} \cos 2 x$
$\sin (A-B)=$
$\cos (A-B)=$
$\sin P+\sin Q=$
$\sin P-\sin Q=$
$\cos P+\cos Q=$
$\cos P-\cos Q=$
$\cos ^{2} x=\frac{1}{2}+\frac{1}{2} \cos 2$
$\tan (A-B)=\frac{\tan A-\tan B}{1+\tan A \tan B}$
$\sin (A-B)=\sin A \cos B-\cos A \sin B$
$\cos (A-B)=\cos A \cos B+\sin A \sin B$
$\sin P+\sin Q=2 \sin \left(\frac{P+Q}{2}\right) \cos \left(\frac{P-Q}{2}\right)$
$\sin P-\sin Q=2 \cos \left(\frac{P+Q}{2}\right) \sin \left(\frac{P-Q}{2}\right)$
$\cos P+\cos Q=2 \cos \left(\frac{P+Q}{2}\right) \cos \left(\frac{P+Q}{2}\right)$
$\cos P-\cos Q=-2 \sin \left(\frac{P+Q}{2}\right) \sin \left(\frac{P-Q}{2}\right)$

TT3C) Find the function $\mathrm{f}^{\prime}(x)$ where $\mathrm{f}(x)$ is
i) $\mathrm{e}^{2 x} \cos x$
ii) $\mathrm{e}^{x} \sec 3 x$
iii) $\frac{e^{\sin x}}{\cos x}$

TT3D) Given cotx $=q$ find in terms of $q$ i) $\sin x$ ii) $\tan (x-45)$ iii) $\cot 2 x$

## Current work: Partial fractions, binomial expansion

1. Express the following as partial fractions
(a) $f(x)=\frac{2 x-13}{(2 x+1)(x-3)}$
(b) $f(x)=\frac{x^{2}+5 x+7}{(x+2)^{3}}$
(c) $f(x)=\frac{x^{2}-10}{(x-2)(x+1)} *$

* this is an improper fraction

2. Use the binomial theorem to expand

$$
\sqrt{ }(4-9 x), \quad|x|<\frac{4}{9},
$$

in ascending powers of $x$, up to and including the term in $x^{3}$, simplifying each term.
3. (a) Expand $\frac{1}{\sqrt{4-3 x}}$, where $|x|<\frac{4}{3}$, in ascending powers of $x$ up to and including the term in $x^{2}$. Simplify each term.
(b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{4-3 x}}$ as a series in ascending powers of $x$.

## Consolidation

4. 

a) $\frac{\cos A}{\sin B}-\frac{\sin A}{\cos B} \equiv \frac{2 \cos (A+B)}{\sin 2 B}$
b) $\tan \frac{A}{2}+\cot \frac{A}{2} \equiv 2 \operatorname{cosec} A$
5. Prove that the equation $\frac{4 x+3}{2 x-1}+\frac{6 x+1}{2 x+1}=3$ has no real solutions.
6.


The diagram shows a sketch of the graph of $y=f(x)$.
The curve has a minimum at the point $A(1,-1)$ passed through $x$-axis at the origin, and the points $B(2,0)$ and C $(5,0)$; the asymptotes have equations $x=3$ and $y=2$.
(a) Sketch on separate axes, the graph of
(i) $y=|f(x)|$
(ii) $y=-f(x+1)$
(iii) $y=f(-2 x)$
(b) State the number of solutions to the equation
(i) $\quad 3|f(x)|=2$
(ii) $\quad 2|f(x)|=3$
7. Find the equations of the tangent and the normal to $y=\frac{x}{1-x}$ at the point $\left(\frac{1}{2}, 1\right)$. Where does the normal meet the curve again?
8. Find the coordinates of the stationary points on $y=\left(x^{2}-x-2\right)^{4}$
9. Given that $y=x e^{2 x}$, show that this curve has only one stationary point, find its coordinates and determine its nature.
10. Find the x coordinate of points on the curve $y=\frac{3 x^{2}+2}{2 x-3}$, where the gradient at these points is parallel to the line $y-x=0$.
11. A curve has the equation $x^{2}+2 y^{2}-x+4 y=6$
a) Show that $\frac{d y}{d x}=\frac{1-2 x}{4(y+1)}$
b) Find an equation for the normal to the curve at the point (1, -3 )

## Mechanics : M1 Practice (Preparation for M2)

12. A ball is thrown vertically upwards from a height 1.6 m above the ground, with a speed of $7 \mathrm{~m} \mathrm{~s}^{-1}$. Find:
a) The maximum height above the ground.
b) The speed when it hits the ground.

## Challenge Question



A company logo has centrally-symetric white cross of width $\sqrt{2}$ on a dark circle. The dark corner pieces have side length 1 as indicated. What is the total area of the corners?

Preparation: Learning Integration techniques is a vital part of the C4 module Read about Integration using Trig and Partial Fractions old C4 textbook pages 82-94 and new textbook pages 87-100.

