A2 Assignment zeta Cover Sheet **Name:**

Question		Done	Backpack	Ready?	Topic	Answers
Drill	Aa				C3 Differentiation all methods	$\frac{-1}{(x-1)^2}$
	Ab				C3 Differentiation all methods	$\frac{1-x+x\ln 2x}{x(1-x)^2}$
	Ac				C3 Differentiation all methods	$\frac{e^x(2x-3)}{(2x+1)^3}$
	Ad				C3 Differentiation all methods	$2x\ln x+x$
	Ae				C3 Differentiation all methods	$\frac{4}{1+4x}$
	Af				C3 Differentiation all methods	$3x^2\sec^2 x^3$
	Ag				C3 Differentiation all methods	$-\sin x e^{\cos x}$
	Ah				C3 Differentiation all methods	$-6e^x\left(3-e^x\right)^5$
	Ba				C3 Modulus function	Check on google inc asymptotes
	Bb				C3 Modulus function	Check on google inc asymptotes
	Bc				C3 Modulus function	Check on google inc asymptotes
	Ca				C3 Sketch and find range	$\mathrm{f}(x)\geq 5$
	Cb				C3 Sketch and find range	$\mathrm{f}(x)\leq 0$
	Cc				C3 Sketch and find range	$-\infty<\mathrm{f}(x)<\infty$ or f(x) ε $\mathbb{R}$
	Da				C3 Integration by inspection	$-\frac{1}{2}\operatorname{cosec}2x+c$
	Db				C3 Integration by inspection	$\frac{1}{2}\ln\lvert 2x-1\rvert+c$
	Dc				C3 Integration by inspection	$e^{x^3}+c$
	TT1A				C3 Algebraic Division & ln & e solve	a) Check! b) $x=\frac{3-e}{2e-1}$
	TT1B				C3 Factor Formula Trig Proof & solve	a) Proof B) $x = 0, \pi/4, \pi/3, 2\pi/3, 3\pi/4$
	TT1C				C3 differentiation & area of triangle	a) (4,0) B) 1 c) $x=4e^{-\frac{2}{5}}$
oli	1a				C3 Rcos(x-a) max and min	Min f(x) = 7, max f(x) = 17

	1b				C3 Rcos(x-a) max and min	Min f(x) = 5, max f(x) = 9
	1c				C3 Rcos(x-a) max and min	Min f(x) = 1/7, max f(x) = 1/3
	2a				C3 Rcos(x-a) min	$R=\sqrt{2}, \alpha=\frac{\pi}{4}$ min $-\sqrt{2}, \theta=\frac{5\pi}{4}$
	2b				C3 Rcos(x-a) min	$R=13, \alpha=1.18$ min $-13, \theta=1.96$
	2c				C3 Rcos(x-a) min	$R=2\sqrt{3}, \alpha=\frac{\pi}{3}$ min $-2\sqrt{3}, \theta=\frac{7\pi}{6}$
	2d				C3 Rcos(x-a) min	$R=\sqrt{58}, \alpha=\arctan\frac{7}{3}$ min $-\sqrt{58}, \theta=5.88$
	3				C3 Rcos(x-a) with solving	R = 13, α = 0.3948. x = 0.963 or 2.968
	4a				C3 Trig proof	PROOF
	4b				C3 Trig proof	PROOF
	5a				C3 Inverse	$\frac{1}{3}\left(e^{x}+6\right)$
	5b				C3 Domain and range of inverse	sketch f(x) to find range of f equals domain of f^{-1}
	5c				C3 Solve function	8.70
	6				C3 Find normal	$2y+x-2=0$
	7				C3 Minimum points	$(\ln 2, 2-\ln 4)$ min by showing second derivative is positive
	8a				C3 Trig solve	2.7, 5.8 radians
	8b				C3 Trig solve	0, π, 2π, 2π/3, 4π/3 radians
	9				M1 Impulse/Momentum vectors	$5.81 ms^{-1}$
	10					7
					PAST PAPER !!!	PAST PAPER !!!

α	β	γ	δ	ε	$\boxed{\zeta}$	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"Perhaps the greatest paradox of all is that there are paradoxes in mathematics"

J Newman

A2 Maths with Mechanics Assignment ζ (zeta)

(14 questions including drill and challenge plus a **C3 past paper*** to do in the week's break found at the end of this assignment)

Due in after the half term break w/b 31/10

Maths Trip: Maths In Action University Lectures in London. £20 a ticket (10 tickets available) 15th November
Maths Trip: Maths In Action University Lectures in London. £20 a ticket (10 tickets available) 14th December

Drill

Part A Differentiate the following functions with respect to *x*:

(a) $\dfrac{x}{x-1}$ (c) $\dfrac{e^x}{(2x+1)^2}$ (c) $\sin\left(e^x\right)$ (d) $x^2 \ln x$

(e) $\ln(1+4x)$ (f) $\tan\left(x^3\right)$ (g) $e^{\cos x}$ (h) $\left(3-e^x\right)^6$

Part B For each of these functions sketch $f\left(|x|\right)$ and $|f(x)|$

(a) $y=\ln(x+1)$ (b) $y=1-e^x$ (c) $y=1-\dfrac{1}{x+2}$

Part C Sketch the following functions where each function is defined on domain, $x \in \mathbb{R}$..
State the range of each function.

(a) $f(x)=(x+1)^2+4, \quad x \geq 0$ (b) $f(x)=1-e^x, \quad x \geq 0$ (c) $f(x)=3\ln x, \quad x>0$

Part D Integrate the following functions by working out what has been differentiated:

(a) $\int \operatorname{cosec} 2x \cot 2x \, dx$ (b) $\int \dfrac{1}{2x-1} \, dx$ (c) $\int 3x^2 e^{x^3} \, dx$

TT1 FOCUS:

A)

(a) Simplify

$$\frac{x^2+7x+12}{2x^2+9x+4}.$$ **(3)**

(b) Solve the equation

$$\ln(x^2+7x+12)-1=\ln(2x^2+9x+4),$$

giving your answer in terms of e. **(4)**

B)

(a) Use the identities for $\sin(A+B)$ and $\sin(A-B)$ to prove that

$$\sin P + \sin Q \equiv 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}. \qquad \textbf{(4)}$$

(b) Find, in terms of π, the solutions of the equation

$$\sin 5x + \sin x = 0,$$

for x in the interval $0 \le x < \pi$. **(5)**

C)

The curve with equation $y = x^{\frac{5}{2}} \ln\frac{x}{4}$, $x > 0$ crosses the x-axis at the point P.

(a) Write down the coordinates of P. **(1)**

The normal to the curve at P crosses the y-axis at the point Q.

(b) Find the area of triangle OPQ where O is the origin. **(6)**

The curve has a stationary point at R.

(c) Find the x-coordinate of R in exact form. **(3)**

Current Work:

1. Write down the maximum and minimum values of the following functions

(a) $f(x) = 12 + 5\sin x$ (b) $f(x) = 7 - 2\sin(2x+\pi)$

(c) $f(x) = \dfrac{2}{10 - 4\sin 2x}$ hint: think about max and min values of $\sin x$

2. Express the following in the form $R\sin(\theta \pm \alpha)$ or $R\cos(\theta \pm \alpha)$ as appropriate (with α in radians) and hence find the **minimum** value of the function, and the first positive value of θ for which it occurs: check using your graphic calculator

(a) $\cos\theta + \sin\theta$ [use $R\cos(\theta - \alpha)$]

(b) $5\cos\theta - 12\sin\theta$ [use $R\cos(\theta + \alpha)$]

(c) $\sqrt{3}\sin\theta + 3\cos\theta$ [use $R\sin(\theta + \alpha)$]

(d) $3\sin\theta - 7\cos\theta$ [use $R\sin(\theta - \alpha)$]

3. Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Hence solve the equation $12\sin x - 5\cos x = 7$ for $0 < x < 2\pi$ giving x correct to 3 decimal places.

Consolidation:

4. Prove the following identities:

(a) $\frac{1}{\operatorname{cosec} x - 1} + \frac{1}{\operatorname{cosec} x + 1} \equiv 2\sec x \tan x$

(b) $\frac{\sin A}{\sin B} + \frac{\cos A}{\cos B} \equiv \frac{2\sin(A+B)}{\sin 2B}$

5. The function f is given by f: $x \rightarrow \ln(3x - 6)$, $x \in \mathbb{R}$, $x > 2$.

(a) Find $f^{-1}(x)$.
(b) Write down the domain of f^{-1} and the range of f^{-1}.
(c) Find, to 3 significant figures, the value of x for which $f(x) = 3$.

6. Find the equation of the normal to the curve $y = e^x(\cos x + \sin x)$ at the point (0,1)

7 A curve has equation $y = e^x - 2x$. Find the coordinates of the turning point in terms of natural logarithms, and show that it is a minimum point.

8. Solve the following equations in the interval $0 \le x \le 2\pi^c$ giving x in terms of π or to 1dp as appropriate:

(a) $\cos\left(x - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}\sin x$ (b) $\sin 2x + \sin x = 0$

M1 Practice (Preparation for M2)

9 A particle of mass 5 kg is moving with velocity (3i + 4j) ms^{-1} when it is given an impulse of (2i + 6j) Ns. Find the speed of the particle after the impact.

10 If $x^2 - 3x + 1 = 0$, what is the value of $x^2 + \left(\frac{1}{x}\right)^2$

Preparation: **Start of C4**, **Read*** about Partial Fractions
C4 new textbook pages1-9, and old C4 textbook pages 1-8

* you are not expected to work through questions in this preparation section but read the textbook, making notes if you wish, to help you understand the topic.

<u>PAST PAPER NEXT PAGE</u>

Complete this past paper (C3 June 2015) in 1 hour 30mins
If it takes longer draw a line under where you got to in the time allowed and continue.

- **Do it under exam conditions.**
  Do not use the mark scheme until you have done the whole paper
- **Mark it yourself using the mark scheme on the VLE and write down your % on the paper.**

- **Do your corrections in another colour.**
- **Hand it to your teacher with your assignment**

C3 June 2015 (9 questions 75 marks)

1. Given that

$$\tan \theta° = p, \text{ where } p \text{ is a constant, } p \neq \pm 1,$$

use standard trigonometric identities, to find in terms of p,

(a) $\tan 2\theta°$,

(2)

(b) $\cos \theta°$,

(2)

(c) $\cot (\theta - 45)°$.

(2)

Write each answer in its simplest form.

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2. Given that

$$f(x) = 2e^x - 5, \qquad x \in \mathbb{R},$$

(a) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$,

(ii) $y = |f(x)|$.

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(6)

(b) Deduce the set of values of x for which $f(x) = |f(x)|$.

(1)

(c) Find the exact solutions of the equation $|f(x)| = 2$.

(3)

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3. $$g(\theta) = 4\cos 2\theta + 2\sin 2\theta.$$

Given that $g(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the exact value of R and the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $-90° < \theta < 90°$,

$$4\cos 2\theta + 2\sin 2\theta = 1,$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(*c*) state the range of possible values of k.

(2)

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4. Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \qquad 0 \leq t \leq T.$$

(*a*) State the value of θ when $t = 0$.

(1)

Given that the temperature of the water in the kettle is 70 °C when $t = 40$,

(*b*) find the exact value of λ, giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers.

(4)

When $t = T$, the temperature of the water reaches 100 °C and the kettle switches off.

(*c*) Calculate the value of T to the nearest whole number.

(2)

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5. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2.$$

Given that P has (x, y) coordinates $\left(p, \dfrac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

6.

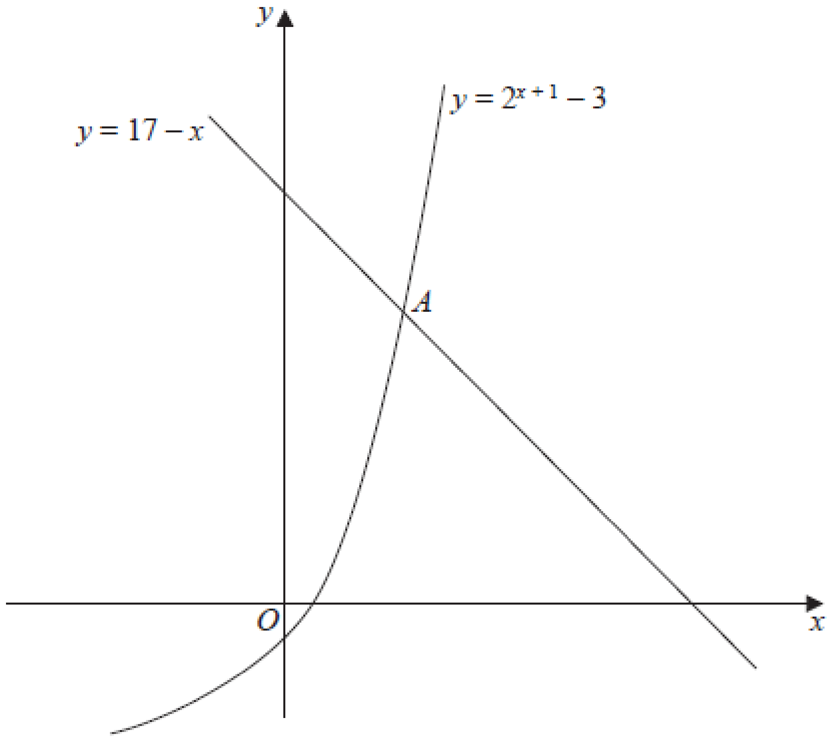

Figure 1

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A.

(*a*) Show that the x-coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1.$$

(3)

(*b*) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \qquad x_0 = 3,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(*c*) Use your answer to part (*b*) to deduce the coordinates of the point A, giving your answers to one decimal place.

(2)

7.

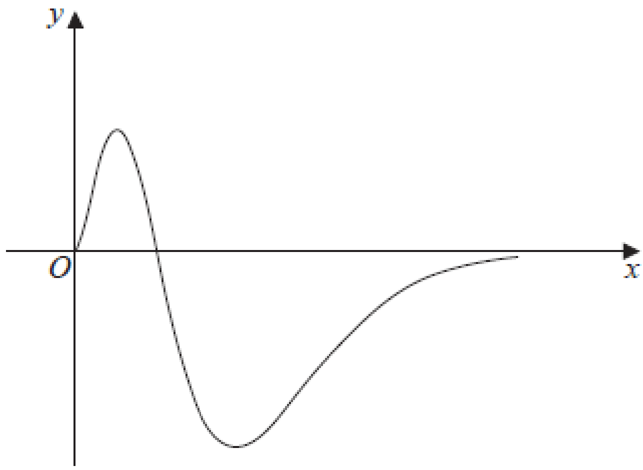

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \qquad x \geq 0,$$

(*a*) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(*b*) Hence find the range of g.

(6)

(*c*) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

8. (*a*) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A|}, \qquad A \neq \frac{(2n+1)\pi}{4}, \quad n \in \mathbb{Z},$$

(5)

(*b*) Hence solve, for $0 \leq \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2},$$

Give your answers to 3 decimal places.

(4)

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9. Given that k is a **negative** constant and that the function $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \qquad x \geq 0,$$

(3)

(*b*) Hence find $\mathrm{f}'(x)$, giving your answer in its simplest form.

(3)

(*c*) State, with a reason, whether $\mathrm{f}(x)$ is an increasing or a decreasing function.
Justify your answer.

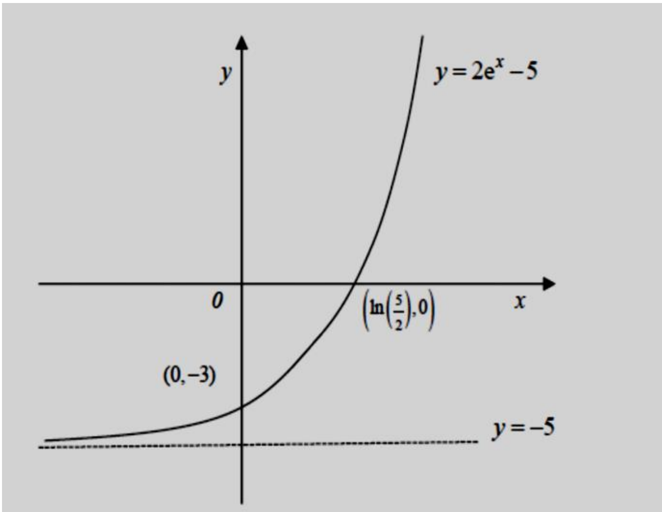
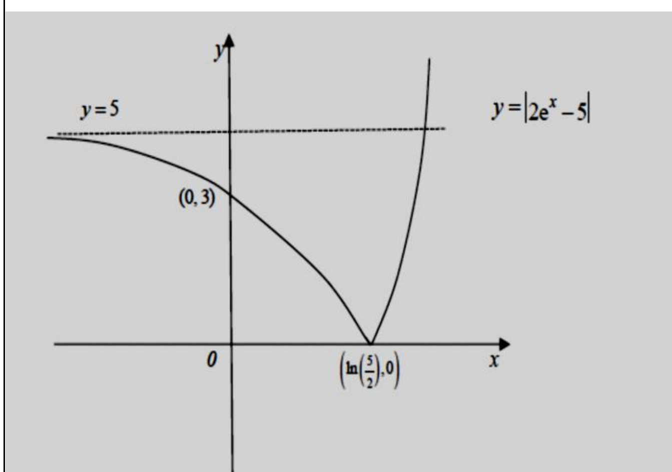
(2)

TOTAL FOR PAPER: 75 MARKS

END

Finished and done all the questions?
Now mark it using the MARK SCHEME on the next page. ONLY look at this once you have completed the full 1 hour 30 mins exam

Question Number	Scheme	Marks
1.(a)	$\tan 2\theta^\circ = \dfrac{2\tan\theta^\circ}{1-\tan^2\theta^\circ} = \dfrac{2p}{1-p^2}$	M1A1 **(2)**
(b)	$\cos\theta^\circ = \dfrac{1}{\sec\theta^\circ} = \dfrac{1}{\sqrt{1+\tan^2\theta^\circ}} = \dfrac{1}{\sqrt{1+p^2}}$	M1A1 **(2)**
(c)	$\cot\ \theta-45\ {}^\circ = \dfrac{1}{\tan\ \theta-45\ {}^\circ} = \dfrac{1+\tan\theta^\circ\tan 45^\circ}{\tan\theta^\circ-\tan 45^\circ} = \dfrac{1+p}{p-1}$	M1A1 **(2)** **(6 marks)**

Question Number	Scheme		Marks
2.(ai)		Shape	B1
		$\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$	B1
		$y=-5$	B1
			(3)
(aii)		Shape inc cusp	B1ft
		$\left(\ln\left(\frac{5}{2}\right),0\right)$ *and* $(0,3)$	B1ft
		$y=5$	B1ft
			(3)
(b)	$x \geqslant \ln\left(\frac{5}{2}\right)$		B1 ft **(1)**
(c)	$2e^x-5=-2 \Rightarrow \; x=\ln\left(\frac{3}{2}\right)$		M1A1
	$x=\ln\left(\frac{7}{2}\right)$		B1
			(3)

Question Number	Scheme	Marks
		(10 marks)

Question Number	Scheme	Marks
3(a)	$4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°... = \text{awrt } 26.57°$	M1A1
		(3)
(b)	$\sqrt{20}\cos\ 2\theta - 26.6\ = 1 \Rightarrow \cos\ 2\theta - 26.57\ = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow\ 2\theta - 26.57\ = +77.1.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 51.8°$	A1
	$2\theta - 26.57 = '-77.1...' \Rightarrow \theta = -\text{awrt } 25.3°$	ddM1A1
		(5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)
		(10 marks)
4(a)	$\theta = 20$	B1
		(1)
(b)	Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$	
	$\Rightarrow e^{-40\lambda} = 0.5$	M1A1
	$\Rightarrow \lambda = \frac{\ln 2}{40}$	M1A1
		(4)
(c)	$\theta = 100 \Rightarrow T = \frac{\ln 0.2}{-\text{their}\ '\lambda'}$	M1

Question Number	Scheme	Marks
	$T =$ awrt 93	A1
		(2)
		(7 marks)

Question Number	Scheme	Marks
5.(a)	$p = 4\pi^2$ or $2\pi^{2}$	B1
		(1)
(b)	$x = (4y - \sin 2y)^2 \Rightarrow \dfrac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	M1A1
	Sub $y = \dfrac{\pi}{2}$ into $\dfrac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	
	$\Rightarrow \dfrac{dx}{dy} = 24\pi$ (= 75.4) / $\dfrac{dy}{dx} = \dfrac{1}{24\pi} = 0.013$	M1
	Equation of tangent $y - \dfrac{\pi}{2} = \dfrac{1}{24\pi}\ x - 4\pi^2$	M1
	Using $y - \frac{\pi}{2} = \frac{1}{24\pi}\ x - 4\pi^2$ with $x = 0 \Rightarrow y = \dfrac{\pi}{3}$ cso	M1, A1
		(6)
		(7 marks)
6.(a)	$2^{x+1} - 3 = 17 - x \Rightarrow 2^{x+1} = 20 - x$	M1
	$(x+1)\ln 2 = \ln(20 - x) \Rightarrow x = ...$	dM1
	$x = \dfrac{\ln(20 - x)}{\ln 2} - 1$	A1*
		(3)
(b)	Sub $x_0 = 3$ into $x_{n+1} = \dfrac{\ln(20 - x_n)}{\ln 2} - 1, \Rightarrow$ $x_1 = 3.087$ (awrt)	M1A1
	$x_2 = 3.080,\ x_3 = 3.081$ (awrt)	A1
		(3)
(c)	$A = 3.1, 13.9$ cao	M1,A1
		(2)
		(8 marks)

Question Number	Scheme	Marks
7.(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)e^{-2x}$	
	$g'(x)=\left(x^2-x^3\right)\times -2e^{-2x}+\left(2x-3x^2\right)\times e^{-2x}$	M1 A1
	$g'(x)=\left(2x^3-5x^2+2x\right)e^{-2x}$	A1
		(3)
(b)	Sets $\left(2x^3-5x^2+2x\right)e^{-2x}=0\Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0\Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2},2$ into $g(x)=\left(x^2-x^3\right)e^{-2x}\Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8e},\quad g(2)=-\frac{4}{e^4}$	dM1,A1
	Range $-\frac{4}{e^4}\leqslant g(x)\leqslant\frac{1}{8e}$	A1
		(6)
(c)	Accept $g(x)$ is NOT a ONE to ONE function	
	Accept $g(x)$ is a MANY to ONE function	B1
	Accept $g^{-1}(x)$ would be ONE to MANY	**(1)**
		(10 marks)

Question Number	Scheme	Marks
8(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	M1
	$= \dfrac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \dfrac{(\cos A+\sin A)(\cos A+\sin A)}{(\cos A+\sin A)(\cos A-\sin A)}$	M1
	$= \dfrac{\cos A+\sin A}{\cos A-\sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta = awrt\ 2.820, 5.961$	dM1A1 **(4)**
		(9 marks)

Question Number	Scheme	Marks
9.(a)	$x^2-3kx+2k^2=(x-2k)(x-k)$	B1
	$2-\dfrac{(x-5k)(x-k)}{(x-2k)(x-k)}=2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$=\dfrac{x+k}{(x-2k)}$	A1*
		(3)
(b)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$	
	$\Rightarrow \mathrm{f}'(x)=\dfrac{(x-2k)\times 1-(x+k)\times 1}{(x-2k)^2}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}$	A1
		(3)
(c)	If $\mathrm{f}'(x)=\dfrac{-Ck}{(x-2k)^2}\Rightarrow \mathrm{f}(x)$ is an increasing function as $\mathrm{f}'(x)>0$,	M1
	$\mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}>0$ for all values of x as $\dfrac{\text{negative}\times\text{negative}}{\text{positive}}=\text{positive}$	A1
		(2)
		(8 marks)