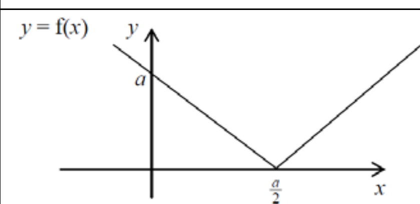
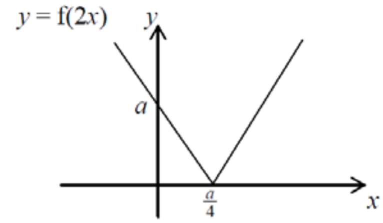
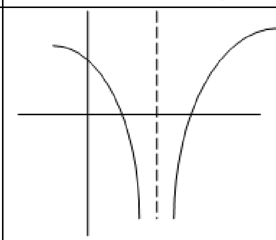
A2 Assignment gamma Cover Sheet **Name:**

Question		Done	Back pack		Topic	Answers
Drill	Aa)				differentiation	$-x\sin x+\cos x$
	Ab)				differentiation	$2x\sec 3x+3x^2\sec 3x\tan 3x$
	Ac)				differentiation	$\dfrac{2x\sec^2 2x-\tan 2x}{x^2}$
	Ad)				differentiation	$3\sin^2 x\cos^2 x-\sin^4 x$
	Ba)				differentiation	$\dfrac{2x\tan x-x^2\sec^2 x}{\tan^2 x}$
	Bb)				differentiation	$\dfrac{1+\sin x}{\cos^2 x}$
	Bc)				differentiation	$e^{2x}(2\cos x-\sin x)$
	Bd)				differentiation	$e^x\sec 3x\,(1+3\tan 3x)$
	Ca)				differentiation	$\dfrac{3\cos 3x-\sin 3x}{e^x}$
	Cb)				differentiation	$e^x\sin x(\sin x+2\cos x)$
	Cc)				differentiation	$\dfrac{\tan x-x\sec^2 x\ln x}{x\tan^2 x}$
	Cd)				differentiation	$\dfrac{e^{\sin x}(\cos^2 x+\sin x)}{\cos^2 x}$
Current Work	1)				Algebraic fractions	$\dfrac{2}{x+2}$
	2a)				Exponential functions	$(0, e^3-1)$, $(-\frac{3}{2}, 0)$, $y=-1$
	2b)					$a=2$, $b=-1.5$
	3a)				Trig	Proof
	3b)					$x=2.4, 5.5$ $x=0.5, 3.6$ (1d.p)
	4a)				Diffn normal to the curve	$3\sin^2 x+6x\sin x\cos x$
	4b)					$x+3y-5\pi=0$
	4c)					$\dfrac{25\pi^2}{6}$
	5a)				Functions	Proof
	5b)					Proof
	5c)					$2\pm\sqrt{19}$
	6)				Algebraic functions	$\dfrac{(x-3)(x-5)}{(x-3)(x+3)}\times\dfrac{2x(x+3)}{(x-5)^2}$ (3 × factorising) $=\dfrac{2x}{x-5}$
	7a)				Functions	

Updated: 28/09/2017

	7b)					
	7c)					$-(2x - a) = \frac{1}{2}x$ when $x = 4$, $\Rightarrow a - 8 = 2$ $\therefore a = 10$ $2x - a = \frac{1}{2}x$ when $x = 4$, $\Rightarrow 8 - a = 2$ $\therefore a = 6$
	8)				Trig	$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $\left(\text{or } \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta} \text{ or equivalent}\right)$ $\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \frac{\cos 2\theta}{1} = \cos 2\theta$
	9a)				Functions	$\mathrm{f}''(x) = 2x - 2x{-3}$ $= 8 - \frac{6}{24} = 7\frac{31}{32}$ (7.97)
	9b)					$\mathrm{f}(x) = \frac{1}{3}x^3 - 2x - \frac{1}{x}$ (+C) $0 = 9 - 6 - \frac{1}{3} + C$ $C = -\frac{8}{3}$ (or –2.67)
	9c)					f\`($x$) > 0 needed, or f\`(x) ≥ 0, or "as x increases, f(x) increases" f\`(x) = $(x - \frac{1}{x})^2$, > 0 always, or ≥ 0 always
	10a)				Functions	$y = \ln(3x - 6) \Rightarrow 3x - 6 = e^y$ $\Rightarrow x = \frac{e^y + 6}{3}$; $\{\mathrm{f}^{-1}(x)\} = \frac{e^x + 6}{3}$
	10b)					Domain: $x \in \Re$ Range: $\mathrm{f}^{-1}(x) > 2$
	10c)					Attempting to find $\mathrm{f}^{-1}(3)$ $\left[= \frac{e^3 + 6}{3}\right]$; $= 8.70$
	10d)					 ln curve passing through $y = 0$ Symmetry in $x = k$, $k > 0$ All correct and asymptote at $x = 2$ labelled
	10e)					Meets y-axis: ($x = 0$), $y = \ln 6$

						Meets x-axis: $x = \frac{5}{3}$, (0) ; $x = \frac{7}{3}$, (0)
Challenge						$V = \int_0^{10} \pi y^2 dx$ $= \pi \int_0^{10} \frac{x}{x^2+1} dx$ $= \frac{1}{2}\pi[\ln(x^2+1)]_0^{10}$ $= \frac{1}{2}\pi \ln 101$ = 7.25 cubic units (3 s.f.)

α	β	**γ**	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"Mathematics is indeed dangerous in that it absorbs students to such a degree that it dulls their senses to everything else"

P Kraft

A2 Maths with Mechanics Assignment γ (gamma)

due w/b 9/10

Drill

Part A Find the function $f'(x)$ where f (x) is:

a) $x\cos x$ b) $x^2\sec 3x$ c) $\frac{\tan 2x}{x}$ d) $\sin^3 x\cos x$

Part B Find the function $f'(x)$ where f (x) is:

a) $\frac{x^2}{\tan x}$ b) $\frac{1+\sin x}{\cos x}$ c) $e^{2x}\cos x$ d) $e^x\sec 3x$

Part C Find the function $f'(x)$ where f (x) is:

a) $\frac{\sin 3x}{e^x}$ b) $e^x\sin^2 x$ c) $\frac{\ln x}{\tan x}$ d) $\frac{e^{\sin}}{\cos x}$

Current work

1. Express $\dfrac{7x}{x^2-3x-10}+\dfrac{5}{5-x}$ as a single fraction in its simplest terms.

2. $f(x) = e^{2x+3} - 1,\ x \in \mathbb{R}$.

a) Sketch the curve with equation $y = f(x)$, showing the coordinates of any points at which the curve meets the coordinate axes and the equation of the asymptote.

The curve with equation $y = f(x)$ has a gradient of 8 at the point P.
The x-coordinate of P is $\ln a + b$, where $a \in \mathbb{Z}$ and $b \in \mathbb{Q}$.

b) Find the value of a and the value of b.

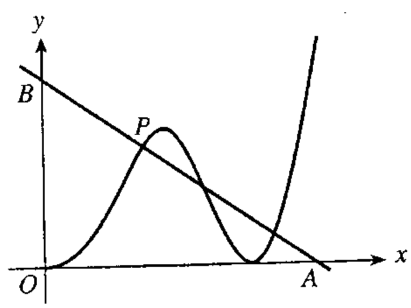
3. a) Use the definition of $\cot x$ in terms of $\sin x$ and $\cos x$, to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$$

Given that $\mathrm{f}(x) = \cot x + 3$,

b) solve $\mathrm{f}(x) + \mathrm{f}'(x) = 0$ for $0 \le x < 2\pi$, giving your answers to one decimal place where appropriate.

4.



The diagram shows part of the curve with equation $y = 3x \sin^2 x$.

a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The point P on the curve has x-coordinate $\frac{\pi}{2}$. The normal to the curve at the point P cuts the x-axis at the point A and the y-axis at the point B. O is the origin.

b) Find an equation for the normal to the curve at P, giving your answer in the form $ax + by + c = 0$, where $a, b \in \mathbb{Z}$.

c) Find the exact value of the area of the triangle OAB.

5.

$$\mathrm{f}: x \to \frac{a}{x}, \quad x \in \mathbb{R}, x \neq 0,$$

$$\mathrm{g}: x \to ax + k, \quad x \in \mathbb{R},$$

where a and k are positive constants.

a) Sketch the curve with equation $y = |\mathrm{f}(x)|$.

The line $y = \mathrm{g}(x)$ is a tangent to the curve $y = |\mathrm{f}(x)|$ at the point P, and cuts the curve $y = |\mathrm{f}(x)|$ at the point Q.

b) Show that $k = 2a$.

Given that $a = 3$,

c) solve the equation $fg(x) = g^{-1}(x)$, giving your answer in the form $a + b\sqrt{c}$ where a, b and c are integers.

6. Express as a single fraction in its simplest form

$$\frac{x^2 - 8x + 15}{x^2 - 9} \times \frac{2x^2 + 6x}{(x-5)^2}.$$

7. The function f is defined by

$$f : x \to |2x - a|, \quad x \in \mathbb{R},$$

where a is a positive constant.

(a) Sketch the graph of $y = f(x)$, showing the coordinates of the points where the graph cuts the axes.

(b) On a separate diagram, sketch the graph of $y = f(2x)$, showing the coordinates of the points where the graph cuts the axes.

(c) Given that a solution of the equation $f(x) = \frac{1}{2}x$ is $x = 4$, find the two possible values of a.

8. Prove that

$$\frac{1 - \tan^2\theta}{1 + \tan^2\theta} \equiv \cos 2\theta.$$

9. The function f, defined for $x \in \mathbb{R}$, $x > 0$, is such that

$$f'(x) = x^2 - 2 + \frac{1}{x^2}.$$

(a) Find the value of $f''(x)$ at $x = 4$.

(b) Given that $f(3) = 0$, find $f(x)$.

(c) Prove that f is an increasing function.

10. The function f is given by

$$f: x \mapsto \ln(3x - 6), \quad x \in \mathbb{R}, \quad x > 2.$$

(a) Find $f^{-1}(x)$.

(b) Write down the domain of f^{-1} and the range of f^{-1}.

(c) Find, to 3 significant figures, the value of x for which f(x) = 3.

The function g is given by

$$g: x \mapsto \ln|3x - 6|, \quad x \in \mathbb{R}, \quad x \neq 2.$$

(d) Sketch the graph of $y = g(x)$.

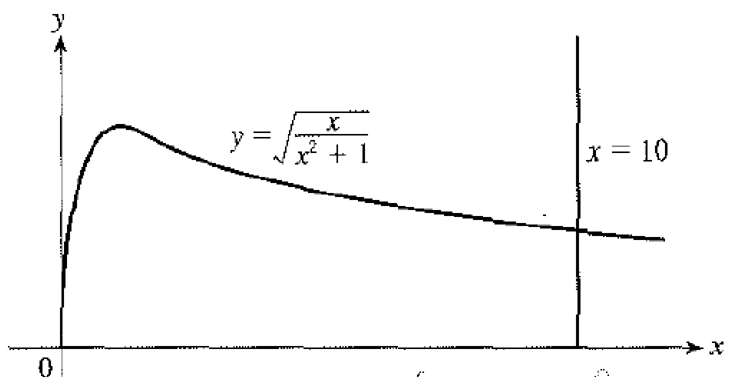
(e) Find the exact coordinates of all the points at which the graph of $y = g(x)$ meets the coordinate axes.

Are you up for a challenge? Then try this question:

The area enclosed by the curve

$$y = \sqrt{\frac{x}{x^2 + 1}}$$

the x-axis and the line $x = 10$ is rotated through 360° about the x-axis.



Use the formula the volume of a solid $= \int_0^{10} \pi y^2 dx$ to find the volume of the solid generated, giving the answer correct to 3 significant figures.