A2 Assignment Psi Cover Sheet **Name:**

Question		Done	BP	Ready	Topic	Comment
Drill	Aa				C4 Vectors – find unit vector	
	Ab				C4 Vectors – find unit vector	
	Ac				C4 Vectors – find unit vector	
	Ba				C4 Integration – sin^2 2x	
	Bb				C4 Integration – partial fractions	
	Bc				C4 Integration – expand and trig identities	
	Ca				C4 Binomial - & state validity	
	Cb				C4 Binomial - & state validity	
	Cc				C4 Binomial - & state validity	
	Da				C3 Transformations – sketch f(x), |f(x)|, f(|x|)	
	Db				C3 Transformations – sketch f(x), |f(x)|, f(|x|)	
	Dc				C3 Transformations – sketch f(x), |f(x)|, f(|x|)	
D1 Consolidation	1a				M2 Collisions – find e	
	1b				M2 Collisions – third collision, show no additional	
	2a				M2 Statics – Find Thrust	
	2b				M2 Statics – Find force at hinge	
	2c				M2 Statics – mass added. Show hinge force horiz	
	3a				M2 Statics – show reaction at peg 16/5 mg	
	3b				M2 Statics – show mu >= 48/61	
	3c				M2 Statics – state how used info peg is smooth	
	4a				M2 Collisions – Find speeds of A and B after coll	
	4b				M2 Collisions – show ¼ < e < 9/16	
	4c				M2 Collisions – e = ½. Find total kinetic lost	
	5a				M2 Statics – find length of rod	
	5b				M2 Statics – find reaction at hinge	
	6ai				M2 Collisions – show speed of P after is u/6 (5e-1)	
	6aii				M2 Collisions – Find speed of Q after collision	
	6b				M2 Collisions – third particle, show further coll	
	6c				M2 Collisions – show if further collision A and B	
	7				C4 Binomial – expand √(4 – 9x)	
	8a				C4 Integration – Find area beneath y = 3sin(x/2)	
	8b				C4 Integration – Find volume by rotation about x	
	9				C4 Binomial – expand $(3 + 2x)^{-3}$	
	10a				C4 Integration – fill in trap rule table	
	10b				C4 Integration – use trapezium rule 4 strips	
	10c				C4 Integration – volume by rotation about x	
C4 Consolidation	11a				C4 Binomial – Expand 1 / √(4 – 3x)	
	11b				C4 Binomial – hence expand (x + 8) / √(4 – 3x)	
	12a				C4 Parametrics – find t at P(4, 2√3)	
	12b				C4 Parametrics – show normal equation	
	12c				C4 Parametrics – show area R is	
	12d				C4 Parametrics – Find exact area R	
	13a				C4 Diff Equations – write diff eq and solve	
	13b				C4 Diff Equations – revised model, solve diff eq	
	13c				C4 Diff Equations – state why model unsuitable	
	14a				C4 Diff Equations – find temp diff in ends of rod	
	14b				C4 Diff Equations – set up diff eq	
	14c				C4 Diff Equations – solve diff eq and proof	

"All the great theorems were discovered after midnight" A. Mathesis

A2 Maths with Mechanics Assignment ψ (psi).

16 questions including drill, exam style questions on the final topics and exam preparation techniques

Due in w/b 20/3.This is the last assignment before the revision schedule. After this assignment you will be required to work on 3 past papers each week. Details are given on assignment omega.

You will also find topic based revision assignments on the VLE. Ask your teacher if, when and how to use these. Don't forget Catch up on the VLE is a good resource.

Please note the C4 mock exam is w/b 20th March

Drill

Part A Find a unit vector in the direction of these vectors:

(a) 3**i** + 2**j** + **k,** (b) **2i** + 3**j** + 3**k,** (c) 2**i** + **j** – 2**k,**

Part B Integrate with respect to *x*:

(a) $\sin^2 2x$ (b) $\frac{x+2}{x^2-x}$ (c) $(2sin4x + 3cos4x)^2$

Part C *Expand the following in ascending powers of x up to and including the term in x cubed. State the values of x for which the expansion is valid:*

(a) $(2+x)^{-\frac{1}{2}}$ (b) $(1-x)^{-3}$ (c) $(4-2x)^{\frac{1}{2}}$

Part D For each of the following functions, sketch $f(x)$, $|f(x)|$ and $f(|x|)$:

(a) $f(x)=3-e^{-2x}$ (b) $f(x)=2-e^{-x}$ (c) $f(x)=e^{-2x}-4$

Current work

M2 Collisions and Statics

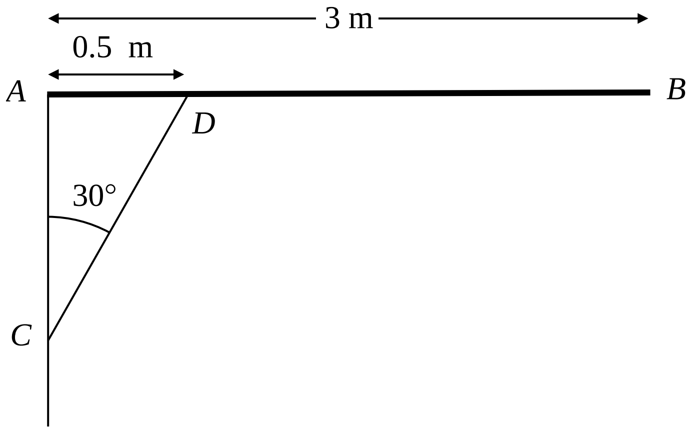
1. Two small spheres *A* and *B* have mass 3*m* and 2*m* respectively. They are moving towards each other in opposite directions on a smooth horizontal plane, both with speed 2*u*, when they collide directly. As a result of the collision, the direction of motion of *B* is reversed and its speed is unchanged.

(a) Find the coefficient of restitution between the spheres.

Subsequently, B collides directly with another small sphere C of mass $5m$ which is at rest. The coefficient of restitution between B and C is $\frac{3}{5}$.

(b) Show that, after B collides with C, there will be no further collisions between the spheres.

2. **Figure 2**



A uniform pole AB, of mass 30 kg and length 3 m, is smoothly hinged to a vertical wall at one end A. The pole is held in equilibrium in a horizontal position by a light rod CD. One end C of the rod is fixed to the wall vertically below A. The other end D is freely jointed to the pole so that $\angle ACD = 30°$ and $AD = 0.5$ m, as shown in Figure 2. Find

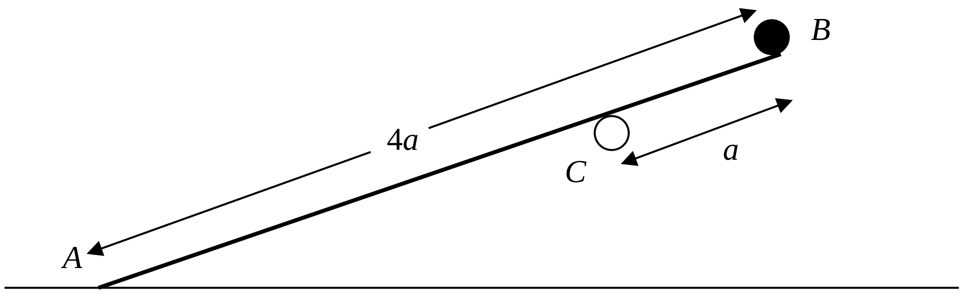
(a) the thrust in the rod CD,

(b) the magnitude of the force exerted by the wall on the pole at A.

The rod CD is removed and replaced by a longer light rod CM, where M is the mid-point of AB. The rod is freely jointed to the pole at M. The pole AB remains in equilibrium in a horizontal position.

(c) Show that the force exerted by the wall on the pole at A now acts horizontally.

3. **Figure 2**



A wooden plank AB has mass $4m$ and length $4a$. The end A of the plank lies on rough horizontal ground. A small stone of mass m is attached to the plank at B. The plank is resting on a small smooth horizontal peg C, where $BC = a$, as shown in Figure 2. The plank is in equilibrium making an angle α with the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the plank and the ground is μ. The plank is modelled as a uniform rod lying in a vertical plane perpendicular to the peg, and the stone as a particle.

Show that

(*a*) the reaction of the peg on the plank has magnitude $\frac{16}{5}\ mg$,

(*b*) $\mu \geq \frac{48}{61}$.

(*c*) State how you have used the information that the peg is smooth.

4. Two particles A and B move on a smooth horizontal table. The mass of A is m, and the mass of B is $4m$. Initially A is moving with speed u when it collides directly with B, which is at rest on the table. As a result of the collision, the direction of motion of A is reversed. The coefficient of restitution between the particles is e.

(*a*) Find expressions for the speed of A and the speed of B immediately after the collision.

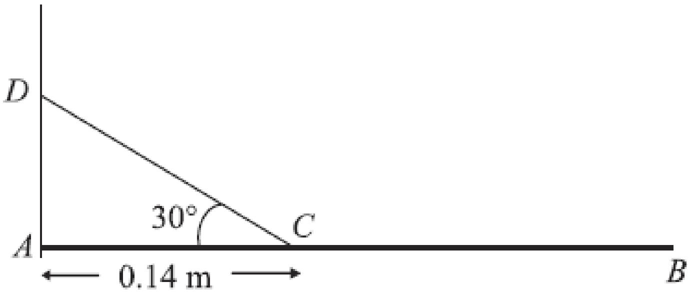
In the subsequent motion, B strikes a smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{4}{5}$. Given that there is a second collision between A and B,

(*b*) show that $\frac{1}{4} < e < \frac{9}{16}$.

Given that $e = \frac{1}{2}$,

(*c*) find the total kinetic energy lost in the first collision between A and B.

5. **Figure 3**



A uniform beam AB of mass 2 kg is freely hinged at one end A to a vertical wall. The beam is held in equilibrium in a horizontal position by a rope which is attached to a point C on the beam, where $AC = 0.14$ m. The rope is attached to the point D on the wall vertically above A, where $\angle ACD = 30°$, as shown in Figure 3. The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 63 N.

Find

(*a*) the length of AB,

(*b*) the magnitude of the resultant reaction of the hinge on the beam at A.

6. Two small spheres P and Q of equal radius have masses m and $5m$ respectively. They lie on a smooth horizontal table. Sphere P is moving with speed u when it collides directly with sphere Q which is at rest. The coefficient of restitution between the spheres is e, where $e > \frac{1}{5}$.

(*a*) (i) Show that the speed of P immediately after the collision is $\frac{u}{6}(5e - 1)$.

(ii) Find an expression for the speed of Q immediately after the collision, giving your answer in the form λu, where λ is in terms of e.

Three small spheres A, B and C of equal radius lie at rest in a straight line on a smooth horizontal table, with B between A and C. The spheres A and C each have mass $5m$, and the mass of B is m. Sphere B is projected towards C with speed u. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.

(*b*) Show that, after B and C have collided, there is a collision between B and A.

(*c*) Determine whether, after B and A have collided, there is a further collision between B and C.

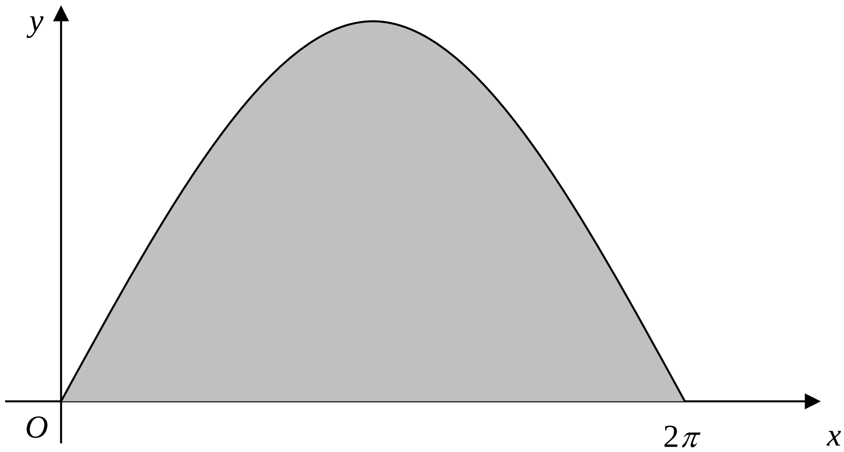
C4 Binomial expansion, volumes of revolution and parametric equations:

7. Use the binomial theorem to expand

$$\sqrt{(4 - 9x)}, \qquad |x| < \frac{4}{9},$$

in ascending powers of x, up to and including the term in x^3, simplifying each term.

8. **Figure 3**



The curve with equation $y = 3 \sin \frac{x}{2}$, $0 \leq x \leq 2\pi$, is shown in Figure 1. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.
This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

9.

$$f(x) = (3 + 2x)^{-3}, \quad |x| < \tfrac{3}{2}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3.

Give each coefficient as a simplified fraction.

10.

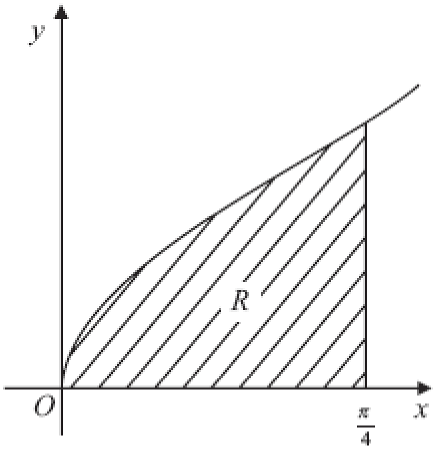


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

(a) Given that $y = \sqrt{(\tan x)}$, copy and complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places.

The region R is rotated through 2π radians around the x-axis to generate a solid of revolution.

(c) Use integration to find an exact value for the volume of the solid generated.

11.

(a) Expand $\dfrac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term.

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\dfrac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x.

12.

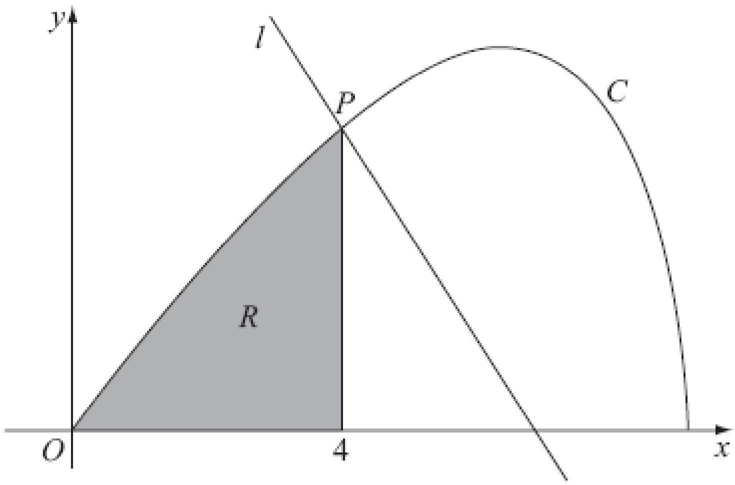


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \le t \le \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t \;\, \mathrm{d}t$.

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

13. In a certain pond, the rate of increase of the number of fish is proportional to the number of fish, n, present at time t.

a) Assuming that n can be regarded as a continuous variable, write down a differential equation relating n and t, and hence show that $n = Ae^{kt}$

b) In a revised model, it is assumed also that fish are removed from the pond, by anglers and by natural wastage, at the constant rate of p per unit time, so that

$$\frac{dn}{dt} = kn - p$$

Given that k = 2, p = 100 and that initially there were 500 fish in the pond, solve this differential equation, expressing n in terms of t

c) Give a reason why this revised model is not satisfactory for large values of t

14. A metal rod is 60cm long and is heated at one end. The temperature at a point on the rod at distance x cm from the heated end is denoted by T ^{o}C. At a point half way along the rod, T = 290 and $\frac{dT}{dx} = -6$

a) In a simple model for the temperature of the rod, it is assumed that $\frac{dT}{dx}$ has the same value at all points on the rod. For this model, express T in terms of x and hence determine the temperature difference between the ends of the rod.

b) In a more refined model, the rate of change of T with respect to x is taken to be proportional to x. Set up a differential equation for T, involving a constant of proportionality k.

c) Solve the differential equation and hence show that, in this refined model, the temperature along the rod is predicted to vary from 380 ^{o}C to 20 ^{o}C.

Exam preparation techniques:
How do you make sure you answer all the questions in the given time?
How do you check your answers are correct?
How do you analyse your exam paper practise?
How do you ensure you don’t keep doing the parts of a paper you can do and ignoring the parts you can’t do in every paper.
Have you updated the survival kit?

The pitfalls to avoid:
Spending too long on one question
Not reading the question properly
Not turning over the last page and so missing the last question completely!
Your work is so untidy or your writing and diagrams are so small that marks are

missed by the person marking your script
Not remembering formulae correctly and having a guess.

Some questions for you
Do accuracy marks depend on method marks?
What is a B1 mark?
If you give two solutions and don't cancel one but draw a line under it, do they mark both and take the best result?
Do you know the accuracy required when it says give answers to an appropriate degree of accuracy?
Can you write in pen or pencil?
How many marks constitute an A*?
Have you double checked the exam dates and time?

Finally well done for working so hard on these assignments!
Keep up the pace now by sticking to the exam practice schedule (on assignment omega), get lots of help on the questions you can't do and the maths team looks forward to sharing your success when the results come through in August!

Drill Answers
Part A
(a) $\frac{1}{\sqrt{14}}(3i+2j+k)$ (b) $\frac{1}{\sqrt{22}}(2\mathbf{i}+3\mathbf{j}+3\mathbf{k})$ (c) $\frac{1}{3}(2i+j-2k)$

Part B
(a) $\frac{1}{2}x-\frac{1}{8}\sin 4x+c$ (b) $3\ln|x-1|-2\ln|x|+c$
(c) $\frac{13}{2}x+\frac{5}{16}sin8x-\frac{3}{4}cos8x+c$

Part C
(a) $\frac{1}{\sqrt{2}}\left[1-\frac{1}{4}x+\frac{3}{32}x^2-\frac{5}{128}x^3\right], \quad |x|<2$ (b) $1+3x+6x^2+10x^3, \quad |x|<1$
(c) $2-\frac{1}{2}x-\frac{1}{16}x^2-\frac{1}{64}x^3 \qquad valid\ |x|<2$

Part D *Check using your graphical calculator or autograph*

M2Answers
$(1a)$ $\frac{2}{3}$ b) Va = 2/3 u direction reversed, Vb = 2/7u direction reversed, Vc = 32u/35 opposite direction to A and B, therefore no further collision

$(2a)$ 1020 N $(2b)$ 778 N
c) since new T = 588N, resolving up and down, find Va = 0 N

(3) proof

(4a) $\left(\frac{1+e}{5}\right)u$, $\left(\frac{4e-1}{5}\right)u$ (4c) $\frac{3}{10}mu^2$

(5a) 45 cm (5b) 55.8 N

(6a) $v = \frac{1}{6}(1-5e)u$, so speed = $\frac{1}{6}(5e-1)u$ $w = \frac{1}{6}(1+e)u$

(6b) After B hits C, velocity of B $= "v" = \frac{1}{6}\left(1-5.\frac{4}{5}\right)u = -\frac{1}{2}u$

velocity $< 0 \Rightarrow$ change of direction $\Rightarrow B$ hits A

(6c) velocity of C after $= \frac{3}{10}u$

velocity of B after $= \frac{1}{4}u$

Travelling in the same direction but $\frac{1}{4} < \frac{3}{10} \Rightarrow$ no second collision

C4 Answers

(7) $2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3$ (8a) 12 (8b) $9\pi^2$ or 88.8264....

(9) $\frac{1}{27} - \frac{2}{27}x + \frac{8}{81}x^2 - \frac{80}{729}x^3$

(10a) 0.44600, 0.64359, 0.81742 (10b) 0.4726 (10c) $\pi \ln\sqrt{2}$

(11a) $\frac{1}{2}[1 + \frac{3}{8}x + \frac{27}{128}x^2 + ...]$ (11b) $4 + 2x + \frac{33}{32}x^2$

(12a) $t = \frac{\pi}{3}$, $0 \le t \le \frac{\pi}{2}$ (12d) $\frac{64}{3} - 8\sqrt{3}$

13) a) proof b) n = 450 e^{2t} + 50 c) number of fish would be infinite, unrealistic

14) a) T = -6x + 470, 360 ^{o}C b) $\frac{dT}{dx} = -kx$ c) $T = -\frac{x^2}{10} + 380$