A2 Assignment Chi Cover Sheet **Name:**

Question		Done	BP	Ready	Topic	Comment
Drill	Aa				C4 Vectors – make line equation	**r** = 7**i** + 2**j** + **k** + λ(– 4**i** – 6**j**)
	Ab				C4 Vectors – make line equation	**r** = **2i** + 3**j** + 3**k**+**s**(– **i** – 6**j** – **k**)
	Ac				C4 Vectors – make line equation	**r** = 2**i** + **j** – 2**k** +t(3**i** – 2**j** + 3**k**)
	Ba				C4 Integration - 3lnx	$3x\ln x - 3x$ + c
	Bb				C4 Integration – improper fraction	$\frac{x^2}{2} + 2x + 4\ln\|x-2\|$ + c
	Bc				C4 Integration – expand and trig identities	$\frac{5}{2}x + \frac{3}{4}sin2x - cos2x + c$
	Ca				C4 Parametrics – find dy/dx	$-\frac{2}{t^3}$
	Cb				C4 Parametrics – find dy/dx	$\operatorname{cosec} t$
	Cc				C4 Parametrics – find dy/dx	$\frac{\sin t + t\cos t}{\cos t - t\sin t}$
	Da				C4 Vectors – Find unit vector	$\frac{(2\mathbf{i} - 14\mathbf{j} + 5\mathbf{k})}{15}$
	Db				C4 Vectors – parallel & perp vectors, find lambda	(i) 4.5 (ii) -2
	Dc				C4 Vectors – acute angle between ***i*** and line	24.1 degrees
M2 Consolidation	1				M2 Kinematics – find distance from O	9
	2a				M2 Collisions – find impulse	5.8
	2b				M2 Collisions – find angle impulse to ***i***	31 degrees
	2c				M2 Collisions – find kinetic energy gained by imp	35 J
	3a				M2 COM – remove triangle, find Com from AD	19a/15
	3b				M2 COM – mass added. Find mass if horizontal sus	m=7M/45
	4a				M2 Projectiles – prove uT = 10	
	4b				M2 Projectiles – find value of u	7
	4c				M2 Projectiles – find tanϕ when hits floor	7/4
	5a				M2 Power – up slope, find power	324W
	5b				M2 Work – WEP down slope, find v	$9.3ms^{-1}$
	5c				M2 Power – Find resistance	32N
	5d				M2 Power – find acceleration down slope	$0.59ms^{-1}$
	6a				M2 Kinematics Vectors – show acc constant	
	6b				M2 Kinematics Vectors – find distance from O	$5\sqrt{5}$
	7a				M2 Kinematics – SPLIT FUNCTION, find speed	9 m s^{-1}
	7b				M2 Kinematics – SPLIT FUNCTION, find speed	13.5 m s^{-1}
	8a				M2 Work – WEP work done by cyclist	2168 J
	8b				M2 Power – given a, find power	300 W
	9a				M2 COM – semi-circle remove from square	
	9b				M2 COM – suspended from D, find angle	
	10a				M2 Projectiles - proof	
	10b				M2 Projectiles – find horizontal distance	21.8 m
	10c				M2 projectiles – find time of flight	2.2 s
C4 consolidation	12a				C4 Diff Equations – write down diff equation	$\frac{dV}{dt} = 20 - kV$
	12b				C4 Diff Equations – solve in terms of k	V = $\frac{20}{k} - \frac{20}{k}e^{-kt}$
	12c				C4 Diff Equations – find V @ 10 seconds	108 cm^3 (3.s.f)
	13a				C4 Diff Equations – find dv/dt	$\frac{dV}{dt} = 4\pi r^2$
	13b				C4 Diff Equations – find dr/dt	$\frac{dr}{dt} = \frac{250}{\pi(2t+1)^2 r^2}$

	13c				C4 Diff Equations – solve diff equation	$V = 500 - \frac{500}{2t+1}$
	13di				C4 Diff Equations – find radius @ t = 5	r = 4.77
	13dii				C4 Diff Equations – find rate of radius @ t = 5	$\frac{dr}{dt} = 0.0289 \ldots$

"Nature laughs at the difficulties of integration" P. Laplace

A2 Maths with Mechanics Assignment χ (chi).

The "Omega" assignment will be a revision schedule showing you which papers you need to complete.

w/b 13/3

Drill

Part A The points *A* and *B* have direction vectors **a** and **b** respectively. Find the vector equation of the line through *A* and *B* in each of the following:

(a) **a** = 7**i** + 2**j** + **k**, **b** = 3**i** – 4**j** + **k**

(b) **a** = 2**i** + 3**j** + 3**k**, **b** = **i** – 3**j** + 2**k**

(c) **a** = 2**i** + **j** – 2**k**, **b** = 5**i** – **j** + **k**

Part B Integrate the following with respect to *x*:

(a) $3\ln x$ (b) $\frac{x^2}{x-2}$ (c) $(\sin x + 2\cos x)^2$

Part C Find $\frac{dy}{dx}$ for each of the following where *t is a* parameter; *a* and *c* are constants.

(a) $x = ct,\ y = \frac{c}{t^2}$ (b) $x = a\sec t,\ y = a\tan t$ (c) $x = t\cos t,\ y = t\sin t$

Part D

(a) Find a unit vector in the direction 2**i** -14**j** +5**k**

(b) If **a**=2**i** + 3**j** and **b**= 3**i** + λ**j** find λ if (i)**a** and **b** are parallel (ii) **a** and **b** are perpendicular

(c) Find to the nearest tenth of a degree, the acute angle between the x axis and the line with equation **r** = 2**i** +λ(5**i** + 2**j** - **k**)

C4 parametric integration using the correct limits

Watch the video from YouTube

http://mathispower4u.wordpress.com/2013/04/26/integration-application-area-using-parametric-equations-ellipse/

M2 consolidation

1. A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where $v = 6t - 2t^2$. When $t = 0$, P is at the origin O. Find the distance of P from O when P comes to instantaneous rest after leaving O.

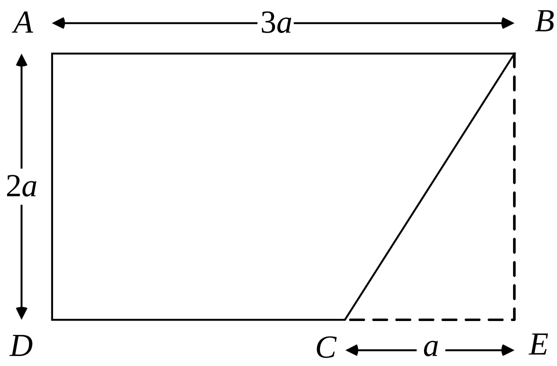
2. A tennis ball of mass 0.2 kg is moving with velocity $(-10\mathbf{i})$ m s^{-1} when it is struck by a tennis racket. Immediately after being struck, the ball has velocity $(15\mathbf{i} + 15\mathbf{j})$ m s^{-1}. Find

(*a*) the magnitude of the impulse exerted by the racket on the ball,

(*b*) the angle, to the nearest degree, between the vector **i** and the impulse exerted by the racket,

(*c*) the kinetic energy gained by the ball as a result of being struck.

3. **Figure 1**



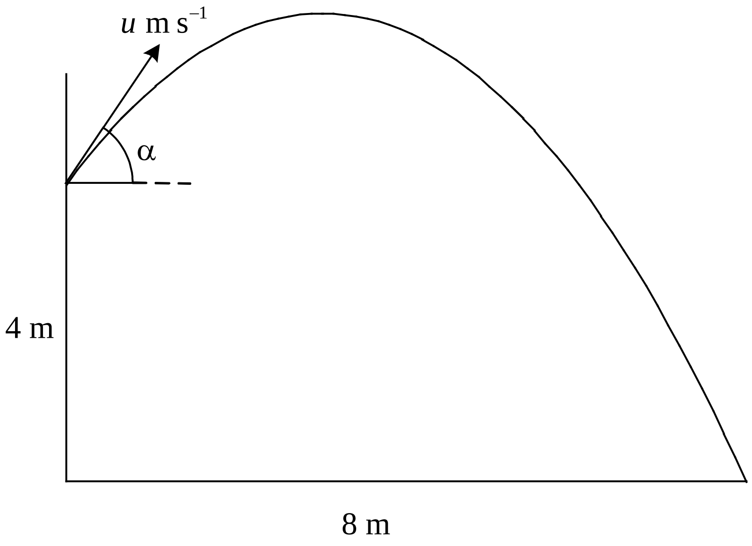
A uniform lamina $ABCD$ is made by taking a uniform sheet of metal in the form of a rectangle $ABED$, with $AB = 3a$ and $AD = 2a$, and removing the triangle BCE, where C lies on DE and $CE = a$, as shown in Fig. 1.

(*a*) Find the distance of the centre of mass of the lamina from AD.

The lamina has mass M. A particle of mass m is attached to the lamina at B. When the loaded lamina is freely suspended from the mid-point of AB, it hangs in equilibrium with AB horizontal.

(*b*) Find m in terms of M.

4. **Figure 3**



A ball is thrown from a point 4 m above horizontal ground. The ball is projected at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$. The ball hits the ground at a point which is a horizontal distance 8 m from its point of projection, as shown in Fig. 3.

The initial speed of the ball is u m s^{-1} and the time of flight is T seconds.

(a) Prove that $uT = 10$.

(b) Find the value of u.

As the ball hits the ground, its direction of motion makes an angle ϕ with the horizontal.

(c) Find $\tan\phi$.

5. A girl and her bicycle have a combined mass of 64 kg. She cycles up a straight stretch of road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{14}$. She cycles at a constant speed of 5 m s^{-1}. When she is cycling at this speed, the resistance to motion from non-gravitational forces has magnitude 20 N.

(*a*) Find the rate at which the cyclist is working.

She now turns round and comes down the same road. Her initial speed is 5 m s^{-1}, and the resistance to motion is modelled as remaining constant with magnitude 20 N. She free-wheels down the road for a distance of 80 m. Using this model,

(*b*) find the speed of the cyclist when she has travelled a distance of 80 m.

The cyclist again moves down the same road, but this time she pedals down the road. The resistance is now modelled as having magnitude proportional to the speed of the cyclist. Her initial speed is again 5 m s^{-1} when the resistance to motion has magnitude 20 N.

(*c*) Find the magnitude of the resistance to motion when the speed of the cyclist is 8 m s^{-1}.

The cyclist works at a constant rate of 200 W.

(*d*) Find the magnitude of her acceleration when her speed is 8 m s^{-1}.

6. The velocity $\mathbf{v}$ m s^{-1} of a particle P at time t seconds is given by

$$\mathbf{v} = (3t - 2)\mathbf{i} - 5t\mathbf{j}.$$

(*a*) Show that the acceleration of P is constant.
At $t = 0$, the position vector of P relative to a fixed origin O is $3\mathbf{i}$ m.

(*b*) Find the distance of P from O when $t = 2$.

7. A particle P moves in a straight line so that, at time t seconds, its acceleration a m s^{-2} is given by

$$a = \begin{cases} 4t - t^2, & 0 \le t \le 3, \\ \dfrac{27}{t^2}, & t > 3. \end{cases}$$

At $t = 0$, P is at rest. Find the speed of P when
(*a*) $t = 3$,
(*b*) $t = 6$.

8. **Figure 1**

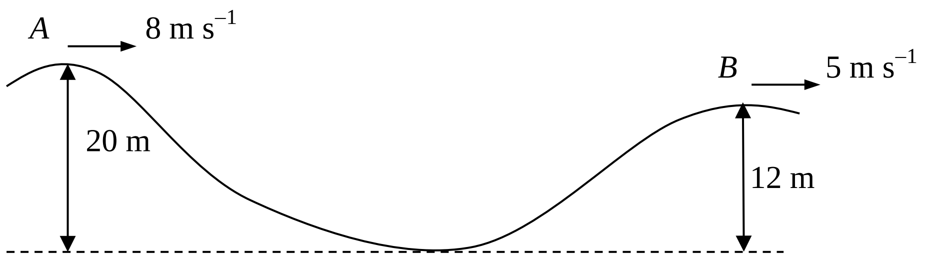


Figure 1 shows the path taken by a cyclist in travelling on a section of a road. When the cyclist comes to the point A on the top of a hill, she is travelling at 8 m s^{-1}. She descends a vertical distance of 20 m to the bottom of the hill. The road then rises to the point B through a vertical distance of 12 m. When she reaches B, her speed is 5 m s^{-1}. The total mass of the cyclist and the cycle is 80 kg and the total distance along the road from A to B is 500 m. By modelling the resistance to the motion of the cyclist as of constant magnitude 20 N,

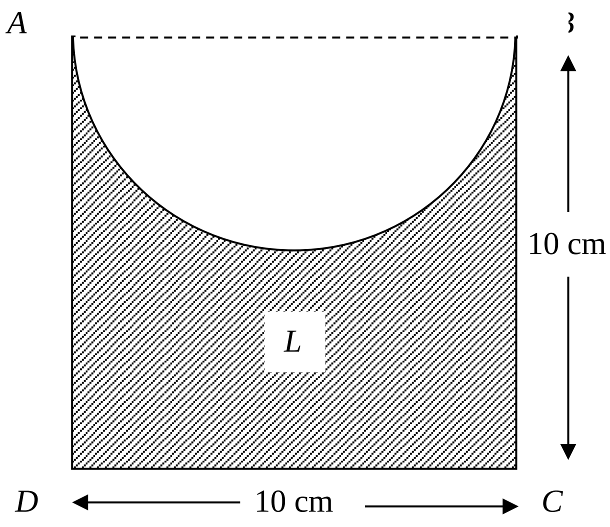
(*a*) find the work done by the cyclist in moving from A to B.

At B the road is horizontal. Given that at B the cyclist is accelerating at 0.5 m s^{-2},

(*b*) find the power generated by the cyclist at B.

9.

Figure 1



A uniform lamina L is formed by taking a uniform square sheet of material $ABCD$, of side 10 cm, and removing the semi-circle with diameter AB from the square, as shown in Fig. 2.

(*a*) Find, in cm to 2 decimal places, the distance of the centre of mass of the lamina L from the mid-point of AB.

[The centre of mass of a uniform semi-circular lamina, radius a, is at a distance $\frac{4a}{3\pi}$ from the centre of the bounding diameter.]

The lamina is freely suspended from D and hangs at rest.

(*b*) Find, in degrees to one decimal place, the angle between CD and the vertical.

10. A particle is projected from a point with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When it has moved a horizontal distance x, its height above the point of projection is y.

(*a*) Show that

$$y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2 \alpha).$$

A shot-putter puts a shot from a point A at a height of 2 m above horizontal ground. The shot is projected at an angle of elevation of 45° with a speed of 14 m s^{-1}. By modelling the shot as a particle moving freely under gravity,

(*b*) find, to 3 significant figures, the horizontal distance of the shot from A when the shot hits the ground,

(*c*) find, to 2 significant figures, the time taken by the shot in moving from A to reach the ground.

12. Liquid is pouring into a container at a constant rate of 20cm^{3}s^{-1} and is leaking out at a rate proportional to the volume of the liquid already in the container.

a) Write a differential equation to model this situation

b) The container is initially empty. Solve the differential equation, giving your answer in terms of k

c) Given that $\frac{dV}{dt} = 10 \; when \; t = 5$, find the volume in the container at 10 seconds after the start

13. The volume of a spherical balloon of radius r cm is Vcm^3, where $V = \frac{4\pi r^3}{3}$

a) find $\frac{dV}{dr}$

b) The volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}$$

Find an expression in terms of r and t for $\frac{dr}{dt}$

c) Given that V = 0 when t = 0, solve the differential equation in b)

d) Hence at t = 5

i) find the radius of the balloon to 3.s.f

ii) show the rate of increase of the radius of the balloon is approximately 2.90 x 10^{-2}cm^{-1}