A2 Assignment Phi Cover Sheet **Name:**

Question		Done	BP	Ready	Topic	Comment
Drill	Aa				C4 Integration	$\frac{1}{4}\tan(4x+1)+c$
	Ab				C4 Integration	$\frac{1}{2}\ln\left\|x^2+2x+5\right\|+c$
	Ac				C4 Integration	$\frac{1}{12}\sin^3(4x-1)+c$
	Ba				C4 Parametric – differentiation	$\frac{dy}{dx}=t-\frac{3}{2}$
	Bb				C4 Parametric – differentiation	$\frac{dy}{dx}=\frac{2t}{t^2-1}$
	Bc				C4 Parametric – differentiation	$\frac{dy}{dx}=4\tan t$
	Ca				C4 Integration – partial fractions	$\ln\|x-1\|+\ln\|x+3\|+c$
	Cb				C4 Integration – partial fractions	$x+\frac{5}{4}\ln\|x-2\|-\frac{5}{4}\ln\|x+2\|+c$
	Cc				C4 Integration – partial fractions	$\frac{1}{2}x^2+\frac{1}{2}\ln\|x+1\|+\frac{1}{2}\ln\|x-1\|+c$
	Da				C3 Functions – MOD solves	$\frac{-5}{8},\frac{5}{2}$
	Db				C3 Functions – MOD solves	$\frac{1}{4},3$
	Dc				C3 Functions – MOD solves	$\pm1,\pm4$
Mechanics	1a				M2 Kinematics – find max v	16
	1b				M2 Kinematics – find time return to O	12
	2a				M2 Projectiles – proof given hori & vert distance	
	2b				M2 Projectiles – find speed at point	9.13 m s^{-1}
	3a				M2 COM – folded over, double density, dist AD	$\frac{13a}{9}$
	3b				M2 COM – folded over, double density, dist AB	$\frac{4a}{9}$
	3c				M2 COM – suspended, angle to DE	45 degrees
	3d				M2 COM – mass added, held horizontal, find m	$m=\frac{5M}{9}$
	4					b) 7.21, 0.588 c) 2.12
C3 Consol.	5a				C3 Trig – proof	
	5b				C3 Trig – simultaneous equations	
	5c				C3 Trig – R method	$5\cos\left(2x-36.87\right)$
	5d				C3 Trig – solve	$x=51.6°,165.2°$
C4 Consolidation	6				C4 Integration using trig identities	$\frac{1}{3}\sin 3x-\frac{1}{7}\sin 7x+c$
	7a				C4 Differential Equations – solve	In partial fractions A and B should be 1/3 and -1/3 c is ln 2 (use the fact that when t = 0, x = 0)

	7b				C4 Differential Equations – show	as $x \to 3, t \to \infty$ so cannot make 3g
	8a				C4 Integration – trapezium rule	2.82843
	8b				C4 Integration – trapezium rule	7.56048
	8c				C4 Integration – Integration	128/15
	9a				C4 Implicit Differentiation	$\frac{dy}{dx} = \frac{2+2y e^{-2x}}{e^{-2x}-2y}$
	9b				C4 Implicit Differentiation – find normal	$x-4y+4=0$
	10a				C4 Vectors – vector equation of line	
	10b				C4 Vectors – magnitude	$\sqrt{(126)}$
	10c				C4 Vectors – angle between	36.7°
	10d				C4 Vectors – shortest distance	$d = 3\sqrt{5} (\approx 6.7)$
	10e				C4 Vectors – area of triangle	30.1 or 30.2
	11				C4 Vectors – shortest distance	13.6

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	**φ**	χ	ψ	ω

"The imaginary number is a fine and wonderful recourse of the divine spirit, almost an amphibian between being and not being" G. W. Leibnitz

A2 Maths with Mechanics Assignment φ (phi)

Due in week beginning March 12th

Drill

Part A Integrate the following:

(a) $\int \sec^2(4x+1)dx$ (b) $\int \left(\frac{x+1}{x^2+2x+5}\right)dx$ (c) $\int \cos(4x-1)\sin^2(4x-1)dx$

Part B Find dy/dx for each of the following, leaving your answer in terms of the parameter t:

(a) $x = 2t, \quad y = t^2 - 3t + 2$ (b) $x = \frac{2t}{1+t^2}, \quad y = \frac{(1-t^2)}{(1+t^2)}$ (c) $x = 2 + \sin t\,,\ y = 3 - 4\cos t$

Part C Integrate the following functions with respect to *x*:

(a) $\frac{2x+2}{(x-1)(x+3)}$ (b) $\frac{x^2+1}{x^2-4}$ (c) $\frac{x^3}{x^2-1}$

Part D Solve the following equations:

(a) $|3x+5| = 5|x|$ (b) $|6x-7| - |2x+5| = 0$ (c) $|x^2-4| = 3|x|$

Mechanics consolidation

1. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2 .$$

(*a*) Find the maximum value of v.
(*b*) Find the time taken for P to return to O.

2.

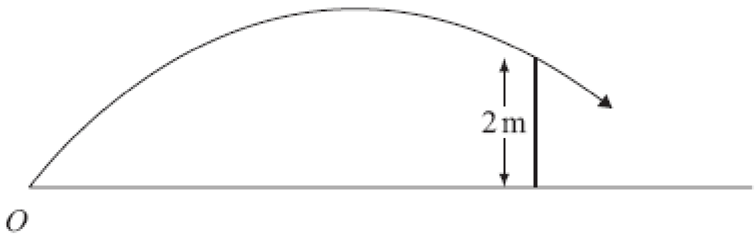


Figure 3

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

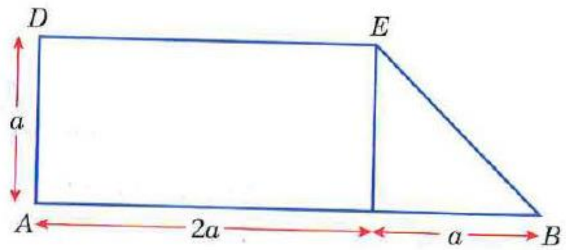
(*a*) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10 \tan \alpha - \frac{50g}{u^2 \cos^2 \alpha}.$$

Given that $\alpha = 45°$,

(*b*) find the speed of the ball as it passes over the fence.

3 A uniform rectangular piece of card *ABCD* has *AB* = 3*a* and *BC* = *a*. One corner of the rectangle is folded over to form a trapezium *ABED* as shown in the diagram:



Find the distance of the centre of mass of the trapezium from

(*a*) *AD*,

(*b*) *AB*.

The lamina *ABED* is freely suspended from *E* and hangs at rest.

(*c*) Find the angle between *DE* and the horizontal.

The mass of the lamina is *M*. A particle of mass *m* is attached to the lamina at the point *B*. The lamina is freely suspended from *E* and it hangs at rest with *AB* horizontal.

(*d*) Find *m* in terms of *M*.

C3 consolidation

4. (a) Show that $2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv \sin \theta(4 \cos \theta + 6 \sin \theta - 3)$.

(*b*) Express $4 \cos \theta + 6 \sin \theta$ in the form $R \sin (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(*c*) Hence, for $0 \leq \theta < \pi$, solve $2 \sin 2\theta = 3(\cos 2\theta + \sin \theta - 1)$, giving your answers in radians to 3 significant figures, where appropriate.

5. (*a*) Use the identity $\cos (A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2 \sin^2 A$$

The curves C_1 and C_2 have equations

$$C_1: y = 3 \sin 2x$$

$$C_2: y = 4 \sin^2 x - 2 \cos 2x$$

(*b*) Show that the *x*-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4 \cos 2x + 3 \sin 2x = 2$$

(*c*) Express $4\cos 2x + 3 \sin 2x$ in the form $R \cos (2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(*d*) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4 \cos 2x + 3 \sin 2x = 2,$$

giving your answers to 1 decimal place.

C4 consolidation

6. Find $\int 2\sin 5x \sin 2x \; dx$.

7. During a chemical reaction, a compound is being made from two other substances. At time t hours after the start of the reaction, x g of the compound has been produced. Assuming that $x = 0$ initially, and that

$$\frac{dx}{dt} = 2(x-6)(x-3)$$

(a) Show that it takes approximately 7 minutes to produce 2 g of the compound.

(b) Explain why it is not possible to produce 3 g of the compound.

8.

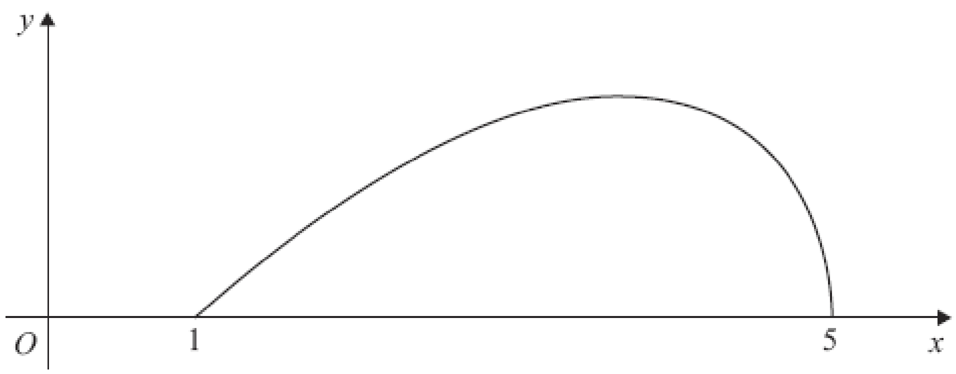


Figure 1

Figure 1 shows the finite region R bounded by the x-axis and the curve with equation

$$y = (x-1)\sqrt{(5-x)}, \quad 1 \le x \le 5$$

The table shows corresponding values of x and y where $y = (x-1)\sqrt{(5-x)}$.

x	1	2	3	4	5
y	0	1.73205		3	0

(*a*) Copy and complete the table above giving the missing value of y to 5 decimal places.

(*b*) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.

(*c*) Use integration to find the exact area of R.

9. The curve C has the equation $ye^{-2x} = 2x + y^2$.

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

The point P on C has coordinates (0, 1).

(*b*) Find the equation of the normal to C at P,
giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

10. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find a vector equation for the line l.

(*b*) Find $\left|\overrightarrow{CB}\right|$.

(*c*) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.

(*d*) Find the shortest distance from the point C to the line l.

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(*e*) find the area of the triangle CXB, giving your answer to 3 significant figures

11. There is a line with equation $r = (4\boldsymbol{i} - 3\boldsymbol{j} - 7\boldsymbol{k}) + \lambda(3\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k})$. A has position vector $(2\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k})$, find the shortest distance from the line to A.