A2 Assignment Upsilon Cover Sheet **Name:**

Question		Done	BP	Ready	Topic	Comment
Drill	Aa				C4 Integration	$-\frac{1}{2}\ln\left\|3-x^2\right\|+c$
	Ab				C4 Integration	$\frac{1}{2}x^2+x+\ln k\left\|x-1\right\|$
	Ac				C4 Integration	$-\frac{1}{8}(4x-1)^{-2}+c$
	Ba				C3 Functions – MOD sketch	Check using desmos
	Bb				C3 Functions – MOD sketch	Check using desmos
	Bc				C3 Functions – MOD sketch	Check using desmos
	Ca				C4 Parametric – axes crossing points	$(0,2)$, $(-1,0)$, (0,–2)
	Cb				C4 Parametric – axes crossing points	(0, 0), (2, 0)
	Cc				C4 Parametric – axes crossing points	(0, 1), (1, 0)
	Da				C4 Differential equations – separate variables	$\int \frac{1}{y}\,dy=\int \frac{1+x}{x}\,dx$
	Db				C4 Differential equations – separate variables	$\int \frac{1}{p-2}\,dp=\int 1\,ds$
	Dc				C4 Differential equations – separate variables	$\int e^{-s}ds=2\int e^{t}dt$
C4	1a				C4 Forming differential equations	$\boldsymbol{\frac{dA}{dt}=kA}$
	1b				C4 Forming differential equations	$\boldsymbol{\frac{dV}{dt}=-kV}$
	1c				C4 Forming differential equations	$\boldsymbol{\frac{dx}{dt}=-kx}$
C3 Consolidation	2a				C3 Numerical methods – find approximations	$\boldsymbol{x_1=2.32}$, $\boldsymbol{x_2=2.37158145}\ldots\approx\boldsymbol{2.372}$, $\boldsymbol{x_3=2.3555935}\ldots\approx\boldsymbol{2.356}$ $\boldsymbol{x_4=2.3604369}\ldots\approx\boldsymbol{2.360}$
	2b				C3 Numerical methods – show root is correct	Test upper and lower bounds. Show change of sign
	3a				C3 e & ln word problems – show initial number	80
	3b				C3 e & ln word problems – solve for t	12.6286…
	3c				C3 e & ln word problems - differentiate	$\boldsymbol{\frac{dP}{dt}=16e^{\frac{t}{5}}}$
	3d				C3 e & ln word problems – find P via a value for t.	250
	4a				C3 Functions – Sketch modulus	Check desmos
	4b				C3 Functions – sketch inverse	Check desmos $\boldsymbol{f(x)}\in\mathbb{R}, \mathrm{f}(x)>-k$ or $y>-k$ or [–k, ∞]
	4c				C3 Functions – state range of f	$\boldsymbol{f(x)}\in\mathbb{R}, \mathrm{f}(x)>-k$ or $y>-k$ or [–k, ∞]
	4d				C3 Functions – find f^{-1}	$\mathrm{f}^{-1}(x)=\frac{1}{2}\ln\|x+k\|$
	4e				C3 Functions – state domain f^{-1}	$\boldsymbol{x}\in\mathbb{R}, x>-k$ or [–k, ∞]
	5a				C3 Algebraic Fractions – simplify	Proof
	5b				C3 Differentiation – quotient rule	Proof
	5c				C3 Differentiation – set derivative = 1	$x=\ln 4$ or $x=0$

M2 Consolidation	6				M2 Vectors Collisions – Impulse Momentum, find speed	23.5
	7a				M2 Work Energy Power – find Power flat road	1200 W
	7b				M2 Work Energy Power – find speed up slope given power	3.6 m s^{-1}
	8a				M2 COM – One shape take away another	50 cm
	8b				M2 COM – angle of suspension to vertical	50.2 degrees
	9a				M2 WEP – find distance moved up slope	
	9b				M2 WEP – find speed after moving back down slope	8.9 m s^{-1}
	10				M2 COMEP – projectiles, find final speed	53.5 m/s
C4 Consolidation	11a				C4 Integration – by parts	$xe^x - e^x$ $(+ c)$
	11b				C4 Integration – by parts	$x^2e^x - 2(xe^x - e^x)(+ c)$
	12a				C4 Implicit differentiation - given gradient	$y - 2x = 0$
	12b				C4 Implicit differentiation - find coordinate points	(2, 4) and (–2, –4)
	13a				C4 Vectors - Collinear	Show AC parallel to AB (multiple of the same vector), sharing a common point therefore single straight line.
	13b				C4 Vectors - Ratio of length of lines	3:2
	13c				C4 Vectors - Perpendicular vectors	Find AD and BD. Show AD**.**BD = 0
	13d				C4 Vectors - Area of triangle	$\frac{9}{2}\sqrt{5}$

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	$\boxed{\upsilon}$	φ	χ	ψ	ω

"The truth of the matter is that, though mathematics may contain beauty, it can only be glimpsed after much hard thinking"

M. Holt

A2 Maths with Mechanics Sheet υ (upsilon)

Due in week beginning March 5th

Drill

Part A Integrate the following functions:

(a) $\int \frac{x}{3-x^2}\,dx$ (b) $\int \frac{x^2}{x-1}\,dx$ (c) $\int \frac{1}{(4x-1)^3}\,dx$

Part B Sketch the following functions:

(a) $y=1+\left|x^2-4\right|$ (b) $y=\sin|x|$ (c) $y = e^{|x|}+3$

Part C Find where these parametric curves cross the x and y axes:

(a) $x=t^2-1$, $y=2t$ (b) $x=1+\cos t$, $y=\sin t$ (c) $x=\sin t$, $y=\cos^2 t$

Part D Separate the variables of these following first order differential equations:

(a) $x\frac{dy}{dx}=y+xy$ (b) $\frac{dp}{ds}=p-2$ (c) $\frac{ds}{dt}=2e^{s+t}$

C4: Forming Differential Equations

1. (a) The area of weed on the surface of the pond is increasing at a rate proportional to its area at that instant. Express this statement as a differential equation.

(b) A simple model suggests that the rate at which a car is depreciating is proportional to the value of the car at that instant. Express this statement as a differential equation.

(c) In a chemical reaction, hydrogen peroxide is converted into water and oxygen. At time t after the start of the reaction, the quantity of hydrogen peroxide that has not been converted is x and the rate at which x is decreasing is proportional to x. Write down a differential equation in x and t.

C3 consolidation

2.

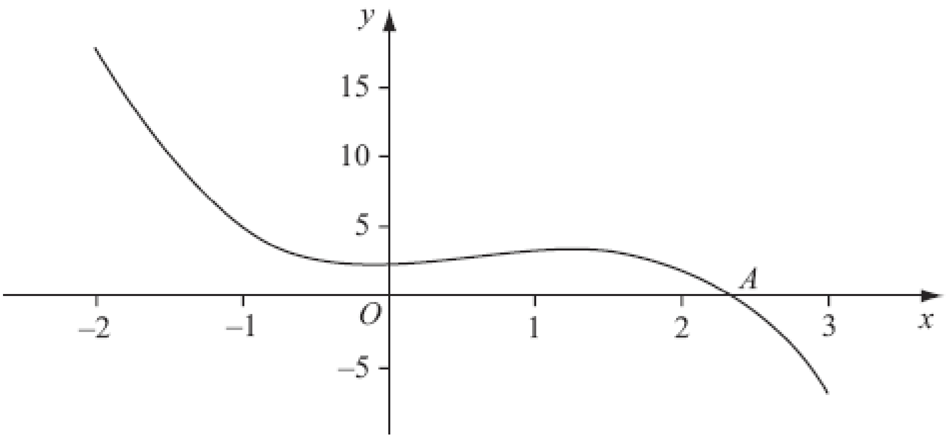

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2 \qquad \text{is used.}$$

(*a*) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(*b*) Show that $\alpha = 2.359$ correct to 3 decimal places.

3. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \quad t \geq 0.$$

(*a*) Write down the number of rabbits that were first introduced to the island.

(*b*) Find the number of years it would take for the number of rabbits to first exceed 1000.

(*c*) Find $\frac{dP}{dt}$.

(*d*) Find P when $\frac{dP}{dt} = 50$.

4.

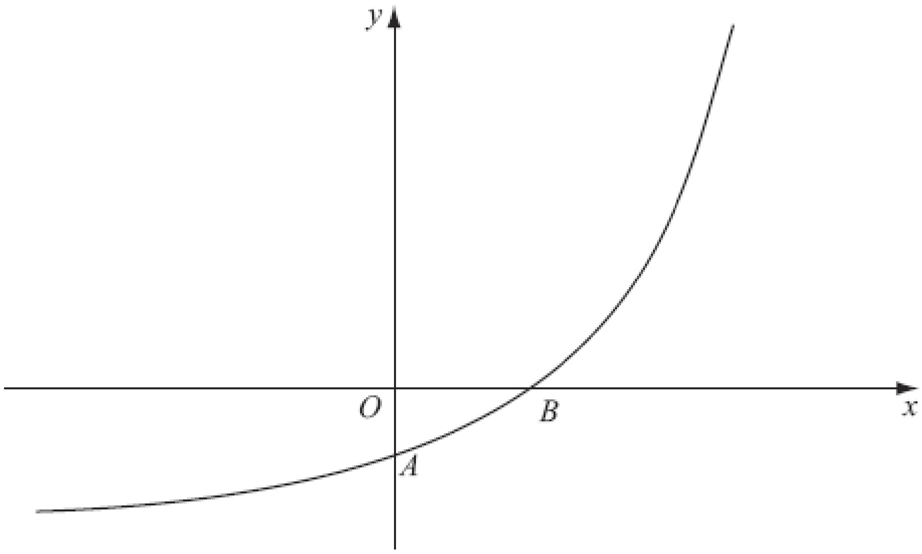

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0,\ 1 - k)$ and $B(\frac{1}{2}\ln k,\ 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(*a*) $y = \mathrm{f}(|x|)$,

(*b*) $y = \mathrm{f}^{-1}(x)$.

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(*c*) state the range of f,

(*d*) find $\mathrm{f}^{-1}(x)$,

(*e*) write down the domain of f^{-1}.

5. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$$

(*a*) Show that $f(x) = \dfrac{x-3}{x-2}$.

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2.$$

(*b*) Differentiate g(*x*) to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$.

(*c*) Find the exact values of *x* for which $g'(x) = 1$

M2 consolidation

6. A particle of mass 0.25 kg is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ m s^{-1} when it receives the impulse $(5\mathbf{i} - 3\mathbf{j})$ N s.

Find the speed of the particle immediately after the impulse. *Hint: Use the M1 Impulse Momentum principle, but with vectors now instead. Remember speed means the magnitude!*

7. A truck of mass of 300 kg moves along a straight horizontal road with a constant speed of 10 m s^{-1}. The resistance to motion of the truck has magnitude 120 N.

(*a*) Find the rate at which the engine of the truck is working.

On another occasion the truck moves at a constant speed up a hill inclined at θ to the horizontal, where $\sin\theta = \frac{1}{14}$. The resistance to motion of the truck from non-gravitational forces remains of magnitude 120 N. The rate at which the engine works is the same as in part (*a*).

(*b*) Find the speed of the truck.

8.

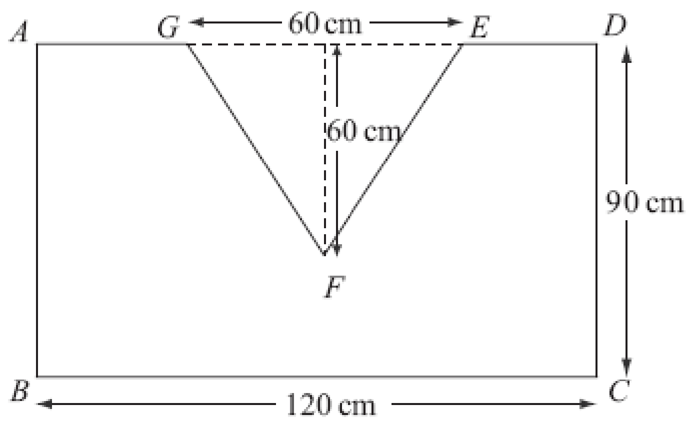

Figure 2

A shop sign *ABCDEFG* is modelled as a uniform lamina, as illustrated in Figure 2. *ABCD* is a rectangle with *BC* = 120 cm and *DC* = 90 cm. The shape *EFG* is an isosceles triangle with *EG* = 60 cm and height 60 cm. The mid-point of *AD* and the mid-point of *EG* coincide.

(*a*) Find the distance of the centre of mass of the sign from the side *AD*.

The sign is freely suspended from *A* and hangs at rest.

(*b*) Find the size of the angle between *AB* and the vertical.

9.

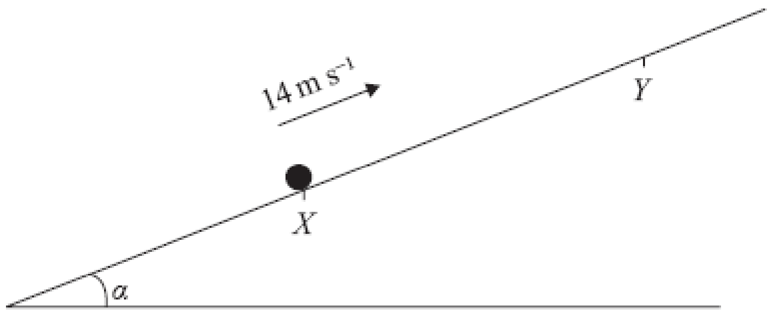

Figure 4

A particle P of mass 2 kg is projected up a rough plane with initial speed 14 m s^{-1}, from a point X on the plane, as shown in Figure 4. The particle moves up the plane along the line of greatest slope through X and comes to instantaneous rest at the point Y. The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{7}{24}$. The coefficient of friction between the particle and the plane is $\frac{1}{8}$.

(*a*) Use the work-energy principle to show that $XY = 25$ m.

After reaching Y, the particle P slides back down the plane.

(*b*) Find the speed of P as it passes through X.

10. A particle is projected at an angle θ from the horizontal at a height of 100m from the ground with speed 30m/s. Find the speed it hits the ground. *think about the quickest way of doing this!*

C4 consolidation

11. (*a*) Use integration by parts to find $\int x e^x \, dx$.

(*b*) Hence find $\int x^2 e^x \, dx$.

12. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(*a*) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(*b*) Find the coordinates of P and Q.

13. Relative to a fixed origin, the points A, B, and C have position vectors $(2\mathbf{i} - \mathbf{j} + 6\mathbf{k})$, $(5\mathbf{i} - 4\mathbf{j})$ and $(7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k})$ respectively.

(a) Show that A, B and C all lie on a single straight line. **(3)**

(b) Write down the ratio $AB : BC$ **(1)**

The point D has position vector $(3\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

(c) Show that AD is perpendicular to BD. **(4)**

(d) Find the exact area of triangle ABD. **(3)**