A2 Assignment Tau Cover Sheet **Name:**

Question		Done	BP	Ready	Topic	Comment
Drill	Aa				C4 Implicit Diff – xy = sin^2	$\frac{y}{\sin 2y - x}$
	Ab				C4 Implicit Diff – tan(2x + y) = x	$\cos^2(2x+y)-2$
	Ac				C4 Implicit Diff – e^(xy) = 4	$-\frac{y}{x}$
	Ba				C3 Functions – MOD sketch	Check using autograph/desmos
	Bb				C3 Functions – MOD sketch	Check using autograph/desmos
	Bc				C3 Functions – MOD sketch	Check using autograph/desmos
	Ca				C4 Parametric – dy/dx chain rule	$\frac{1}{t}$
	Cb				C4 Parametric – dy/dx chain rule	$-\frac{2}{5}t^2$
	Cc				C4 Parametric – dy/dx chain rule	$\cot t$
	Da				C4 Vectors – Find cosine acute angle	$\frac{13}{\sqrt{14}\sqrt{26}}$
	Db				C4 Vectors – Find cosine acute angle	$\frac{3}{7\sqrt{2}}$
	Dc				C4 Vectors – Find cosine acute angle	$\frac{1}{6}$ *remember it's the acute angle! So mod signs! *
Applied consolidation	C4 solomon				C4 SOLOMON PAST PAPER	Mark Scheme Below
	1a				M2 Projectiles – cliff 50m time of flight	5 s
	1b				M2 Projectiles – horizontal landing dist	130m
	1c				M2 Projectiles – speed & direction impact	$43ms^{-1}$, 53^o to the horizontal
	1d				M2 Projectiles – show after T travels 45 deg	Proof
	2a				M2 Kinematics – given F, vectors, find acc	$14\mathbf{i}+18\mathbf{j}$ ms^{-2}
	2b				M2 Kinematics – given F, vectors, find sp	37.4 ms^{-1}
	3				M2 Power – find max acc given power	450N, $\frac{9}{16}$ ms^{-2}
	4a				M2 Power – find power given up slope	50.4kW
	4b				M2 Power – find max speed up slope	95 kmh^{-1}
Core consolidation	5a				C3 Trig – Rmethod, involving 2x	e.g. if using Rcos $R = 13$, $\alpha = 1.176$
	5b				C3 Trig – solve using R method	1.13, 0.0425
	5ci)				C3 Trig – R method write f(x)max	f(x) = 13
	5cii)				C3 Trig – R method smallest +ve value x	$x = 0.588$
	6a				C3 Functions – show f(x) rearrange	show
	6b				C3 Functions – find range f	0 < f(x) < 1/4
	6c				C3 Functions – find f^{-1} and state its domain	$f^{-1}(x) = \frac{1}{x} - 1$, $0 < x < \frac{1}{4}$
	6d				C3 Functions – solve fg(x) = 1/8	$x = \pm\sqrt{5}$

	7ai)				C3 Diff – product rule	$e^{3x}(\sin x + 7\cos x)$
	7aii)				C3 Diff – product rule involving ln	$3x^2 \ln(5x+2) + \frac{5x^3}{5x+2}$
	7b				C3 Diff – quotient rule	
	7c				C3 Diff – second derivative & solve	$x = 1, -3$
	8a				C4 Integral – Trapezium rule complete table	$e^{0.32}$, $e^{1.28}$
	8b				C4 Integral – use trapezium rule	4.922
	9				C4 Connected rates – balloon inflating	$\frac{5k}{3}$
	10a				C4 Diff Eq – partial fractions	$\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}$
	10b				C4 Diff Eq – find particular solution	$\sec^2 x = \frac{8+4y}{2-y}$
	11				C4 Vectors –shortest dist from point to L	$2\sqrt{46}$ * should get $\lambda = 0$ when dotting line with AX line segment *
	12				C4 Connected Rates – Area between circles	$\frac{dA}{dt} = -8\pi$ cm^2s^{-1}

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	**τ**	υ	φ	χ	ψ	ω

"It is not knowledge but the act of learning which grants the greatest enjoyment" K. F. Gauss

A2 Maths with Mechanics Assignment τ (tau)

11 questions including drill, exam style questions and challenge
Due after half term w/b 20/2

Drill

Part A Find $\frac{dy}{dx}$ for each of the following functions:

(a) $xy = \sin^2 y$ (b) $\tan(2x + y) = x$ (c) $e^{xy} = 4$

Part B Sketch the following functions:

(a) $y = 1 - |x+2|$ (b) $y = 3|\sin x|$ (c) $y = 2\ln|x| + 2$

Part C Find $\frac{dy}{dx}$ using the chain rule, for each of the following functions:

(a) $x = t^2 + 1$, $y = 2t$ (b) $x = \frac{5}{t}$, $y = 2t$ (c) $x = 1 - \cos t$, $y = \sin t$

Part D Find the cosine of the acute angle between the following pairs of vectors:

(a) **a** = 2**i** – **j** + 3**k**, **b** = 5**i** + **k**

(b) **a** = 6**i** – 3**j** + 2**k**, **b** = **i** + **j**

(c) **a** = **i** + **j** – 2**k**, **b** = 2**i** – **j** + **k**

C4 SOLOMON L (C4 TOPICS COVERED SO FAR)

1. The number of people, n, in a queue at a Post Office t minutes after it opens is modelled by the differential equation

$$\frac{dn}{dt} = e^{0.5t} - 5, \quad t \geq 0.$$

(a) Find, to the nearest second, the time when the model predicts that there will be the least number of people in the queue. **(3)**

(b) Given that there are 20 people in the queue when the Post Office opens, solve the differential equation. **(4)**

(c) Explain why this model would not be appropriate for large values of t. **(1)**

2. A curve has the equation

$$3x^2 + xy - 2y^2 + 25 = 0.$$

Find an equation for the normal to the curve at the point with coordinates (1, 4), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(8)**

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3. *(a)* Use the substitution $u = 2 - x^2$ to find

$$\int \frac{x}{2 - x^2} \, dx.$$ **(4)**

(b) Evaluate

$$\int_0^{\frac{\pi}{4}} \sin 3x \cos x \, dx.$$ **(6)**

4.

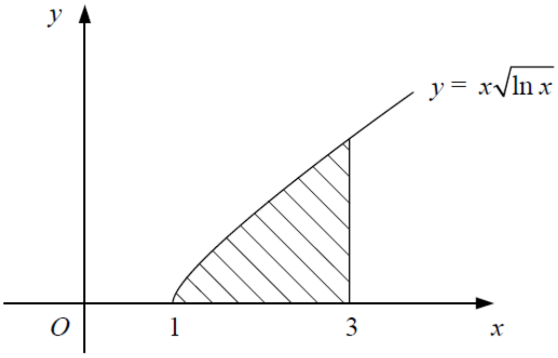

Figure 1

Figure 1 shows the curve with equation $y = x\sqrt{\ln x}$, $x \geq 1$.

The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(a) Using the trapezium rule with two intervals of equal width, estimate the area of the shaded region. **(4)**

5.

$$f(x) = \frac{5 - 8x}{(1 + 2x)(1 - x)^2}.$$

(a) Express f(x) in partial fractions. **(5)**

(b) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(6)**

(c) State the set of values of x for which your expansion is valid. **(1)**

7. The line l_1 passes through the points A and B with position vectors $(3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k})$ and $(8\mathbf{j} - 6\mathbf{k})$ respectively, relative to a fixed origin.

(a) Find a vector equation for l_1. **(2)**

The line l_2 has vector equation

$$\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) + \mu(7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}),$$

where μ is a scalar parameter.

(b) Show that lines l_1 and l_2 intersect. **(4)**

(c) Find the coordinates of the point where l_1 and l_2 intersect. **(2)**

The point C lies on l_2 and is such that AC is perpendicular to AB.

(d) Find the position vector of C. **(6)**

Mechanics Consolidation

1. A stone is thrown up at an angle of 30° to the horizontal with a speed of 30 ms^{-1} from the edge of a cliff 50 m above sea level. If the stone lands in the sea, calculate:

a) How long it is in the air,
b) How far from the base of the cliff it lands,
c) The speed and direction of the stone as it hits the water.
d) Show that after T secs it will be travelling at an angle 45^0 below the horizontal.

where $T = \dfrac{15(\sqrt{3}+1)}{g}$

2. The resultant force **F** Newtons acting on a particle of mass 0.5kg at time t s is given by: $\mathbf{F} = (t^2 - 2)\mathbf{i} + (2t + 3)\mathbf{j}$

a) Find an expression for the acceleration of the particle after 3 secs.

b) Given that the velocity of the particle at time $t = 0$ is $4\mathbf{i}$ ms^{-1}, find the speed after 3 secs.

3. A car has an engine of maximum power 15kW. Calculate the force resisting the motion of the car when it is travelling at its maximum speed of 120 kmh^{-1} on a level road. Assuming an unchanged resistance and taking the mass of the car to be 800kg, calculate the maximum acceleration of the car when travelling at 60 kmh^{-1} on a level road.
(Note: UNITS!)

4. A car of mass 1 tonne is moving at a constant velocity of 60 km per hour up an inclined road which makes an angle of 6^o with the horizontal.

a) Given that the non-gravitational resistance down the slope is 2000N, find the rate at which the car is working,

b) If the engine has a maximum power output of 80kw, calculate the maximum speed of the car up the same slope.

Core consolidation

5. $f(x) = 5\cos 2x + 12\sin 2x.$

(*a*) Using a suitable R-Method involving Rcos or Rsin, find the value of *R* and the value of α to 3 decimal places.

(*b*) Hence solve the equation

$$5\cos 2x + 12\sin 2x = 6 \qquad \text{for } 0 \le x < \pi.$$

(*c*) (i) Write down the maximum value of f(x)

(ii) Find the smallest positive value of *x* for which this maximum value occurs.

6. The function f is defined by

$$f: x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(*a*) Show that $f(x) = \dfrac{1}{x+1}, \quad x > 3.$

(*b*) Find the range of f.

(*c*) Find $f^{-1}(x)$. State the domain of this inverse function.

The function g is defined by

$$g: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(*d*) Solve $fg(x) = \dfrac{1}{8}$.

7. (*a*) Differentiate with respect to *x*,

(i) $e^{3x}(\sin x + 2\cos x)$,

(ii) $x^3 \ln(5x+2)$.

Given that $y = \dfrac{3x^2+6x-7}{(x+1)^2}, \quad x \neq 1,$

(*b*) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{20}{(x+1)^3}$.

(*c*) Hence find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and the real values of x for which $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{15}{4}$.

8.

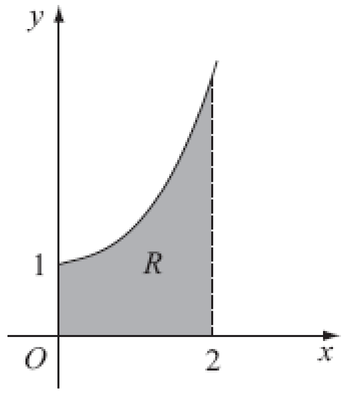

Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(*a*) Copy and complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(*b*) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

9. A spherical balloon is being blown up at a rate proportional to its volume at the time. Given the volume of a sphere is $\frac{4}{3}\pi r^3$, find the rate of change of the radius of the balloon in terms of k at the moment when the radius is 5cm.

10. (*a*) Express $\frac{2}{4-y^2}$ in partial fractions.

(*b*) Hence obtain the solution of

$$2\cot x\,\frac{\mathrm{d}y}{\mathrm{d}x} = (4 - y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = \mathrm{g}(y)$.

11. There is a line with equation $\underset{\sim}{r} = (4\underset{\sim}{i} - 3\underset{\sim}{j} - 7\underset{\sim}{k}) + \lambda(3\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k})$. A has position vector $(2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k})$, find the shortest distance from the line to A.

12 At a given instant, the radii of two concentric circles are 8cm and 12cm. The radius of the outer circle is increasing at a rate of 1cm s^{-1} , and the radius of the inner circle is increasing at a rate of 2cm s^{-1} . Find the rate of change of the area enclosed by the two circles at that instant.

C4 Paper L – Marking Guide

1.	*(a)*	$\frac{dn}{dt} = 0 \Rightarrow e^{0.5t} = 5$	M1	
		$t = 2\ln 5 = 3.219$ mins = 3 mins 13 secs	M1 A1	
	(b)	$\int dn = \int (e^{0.5t} - 5)\ dt$		
		$n = 2e^{0.5t} - 5t + c$	M1 A1	
		$t = 0,\ n = 20 \Rightarrow 20 = 2 + c, \quad c = 18$	M1	
		$n = 2e^{0.5t} - 5t + 18$	A1	
	(c)	as t increases, n rapidly becomes very large $\therefore$ not realistic	B1	**(8)**

2.	$6x + y + x\frac{dy}{dx} - 4y\frac{dy}{dx} = 0$	M1 A2	
	$(1, 4) \Rightarrow 6 + 4 + \frac{dy}{dx} - 16\frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{2}{3}$	M1 A1	
	grad of normal $= -\frac{3}{2}$	M1	
	$\therefore\ y - 4 = -\frac{3}{2}(x - 1)$	M1	
	$2y - 8 = -3x + 3$		
	$3x + 2y - 11 = 0$	A1	**(8)**

3.	*(a)*	$u = 2 - x^2 \Rightarrow \dfrac{du}{dx} = -2x$	M1
		$I = \int \frac{1}{u} \times (-\frac{1}{2})\ du = -\frac{1}{2} \int \frac{1}{u}\ du$	A1
		$= -\frac{1}{2} \ln\lvert u \rvert + c = -\frac{1}{2} \ln\lvert 2 - x^2 \rvert + c$	M1 A1
	(b)	$= \int_0^{\frac{\pi}{4}} (\frac{1}{2} \sin 4x + \frac{1}{2} \sin 2x)\ dx$	M1 A1
		$= [-\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x]_0^{\frac{\pi}{4}}$	M1 A1
		$= (\frac{1}{8} - 0) - (-\frac{1}{8} - \frac{1}{4}) = \frac{1}{2}$	M1 A1 **(10)**

4.	*(a)*	x: 1, 2, 3 y: 0, 1.665, 3.144	B1
		area $\approx \frac{1}{2} \times 1 \times [0 + 3.144 + 2(1.665)] = 3.24$ (3sf)	B1 M1 A1
	(b)	volume $= \pi \int_1^3 x^2 \ln x\ dx$	M1
		$u = \ln x,\ u' = \frac{1}{x},\ v' = x^2,\ v = \frac{1}{3}x^3$	
		$I = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\ dx$	M1 A2
		$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$	A1
		volume $= \pi[\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3]_1^3$	
		$= \pi\{(9 \ln 3 - 3) - (0 - \frac{1}{9})\}$	M1
		$= \pi(9 \ln 3 - \frac{26}{9})$	A1 **(11)**
5.	*(a)*	$\dfrac{5-8x}{(1+2x)(1-x)^2} \equiv \dfrac{A}{1+2x} + \dfrac{B}{1-x} + \dfrac{C}{(1-x)^2}$	
		$5 - 8x \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)(1-x)$	M1
		$x = -\frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$	A1
		$x = 1 \Rightarrow -3 = 3C \Rightarrow C = -1$	A1
		coeffs $x^2 \Rightarrow 0 = A - 2B \Rightarrow B = 2$	M1 A1
		$f(x) = \dfrac{4}{1+2x} + \dfrac{2}{1-x} - \dfrac{1}{(1-x)^2}$	
	(b)	$f(x) = 4(1+2x)^{-1} + 2(1-x)^{-1} - (1-x)^{-2}$	
		$(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(2x)^3 + \ldots$	M1
		$= 1 - 2x + 4x^2 - 8x^3 + \ldots$	A1
		$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$	B1
		$(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{3\times2}(-x)^3 + \ldots$	
		$= 1 + 2x + 3x^2 + 4x^3 + \ldots$	A1
		$f(x) = 4(1 - 2x + 4x^2 - 8x^3) + 2(1 + x + x^2 + x^3) - (1 + 2x + 3x^2 + 4x^3)$	M1
		$= 5 - 8x + 15x^2 - 34x^3 + \ldots$	A1
	(c)	$\lvert x \rvert < \frac{1}{2}$	A1 **(12)**

7.	*(a)*	$\overrightarrow{AB} = (8\mathbf{j} - 6\mathbf{k}) - (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) = (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	M1
		$\therefore\ \mathbf{r} = (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) + \lambda(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	A1
	(b)	$3 - 3\lambda = -2 + 7\mu \quad (1)$	
		$6 + 2\lambda = 10 - 4\mu \quad (2)$	
		$-8 + 2\lambda = 6 + 6\mu \quad (3)$	B1
		$(3) - (2)$: $-14 = -4 + 10\mu,\ \mu = -1,\ \lambda = 4$	M1 A1
		check (1) $3 - 12 = -2 - 7$, true $\therefore$ intersect	B1
	(c)	$\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) - (7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) \quad \therefore (-9, 14, 0)$	M1 A1
	(d)	$\overrightarrow{OC} = [(-2 + 7\mu)\mathbf{i} + (10 - 4\mu)\mathbf{j} + (6 + 6\mu)\mathbf{k}]$	
		$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}]$	M1 A1
		$\therefore\ [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}].(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$	M1
		$15 - 21\mu + 8 - 8\mu + 28 + 12\mu = 0$	A1
		$\mu = 3 \ \therefore\ \overrightarrow{OC} = (19\mathbf{i} - 2\mathbf{j} + 24\mathbf{k})$	M1 A1 **(14)**