A2 Assignment Sigma Cover Sheet **Name:**

Question		Don	BP	Rea	Topic	Comment
Drill	Aa				C3 Functions – Sketch & inverse	$f(x) \in \mathbb{R} : f(x) \neq 0, \quad f^{-1}(x) = \frac{1}{x} - 1, \quad x \in \mathbb{R} : x \neq 0,$
	Ab				C3 Functions – Sketch & inverse	$f(x) \in \mathbb{R}:$ $f(x) \geq 1, \quad f^{-1}(x) = \sqrt{x-1} - 2, \quad x \,\varepsilon\, \Re : x \geq 1$
	Ac				C3 Functions – Sketch & inverse	$f(x) \in \mathbb{R}: 0 < f(x) < 1, \; f^{-1}(x) = \frac{1}{2}\ln(1-x), 0 < x < 1, x \in \mathbb{R}$
	Ba				C4 integral – trig reverse chain	$-\frac{1}{2}(1+\tan x)^{-2} + c$
	Bb				C4 integral – trig reverse chain	$-\frac{1}{2}\cos^4 x + c$
	Bc				C4 integral – reverse chain	$\frac{1}{8}(1-x^2)^{-4} + c$
	Ca				C4 Parametric – Form Cartesian Equation	$y^2 = 4x^2(1-x^2)$
	Cb				C4 Parametric – Form Cartesian Equation	$y^2 = \frac{1}{x^2} - 1$
	Cc				C4 Parametric – Form Cartesian Equation	$y = \frac{1-x^2}{1+x^2}$
	Da				C4 Vectors – Use perpendicular fact to find p	–1
	Db				C4 Vectors – Use perpendicular fact to find p	10
	Dc				C4 Vectors – Use perpendicular fact to find p	0
Current Work	1				M2 Work/Power – Find work done by cyclist	4650J
	2a				M2 Work/Power – Find power on horizontal	14.4 kW
	2b				M2 Work/Power – up hill, find initial acc	0.4m s^{-2}
	3a				M2 Work/Power – connected part, find acc	0.693ms^{-1}
	3b				M2 Work/Power – connected part, find T	7.43kN
	3c				M2 Work/Power – slope, find driving force	27.7kN
	3d				M2 Work/Power – slope, find power	277kW
C4 Consolidation	4a				C4 Integral – expand brackets	$x - 2\ln x - \frac{1}{x} + c$
	4b				C4 Integral – trig reverse chain	$-\frac{2}{5}\cos^5 \frac{x}{2} + c$
	4c				C4 Integral – trig cos^2 and sin^2 convers	$\frac{5x}{2} + \frac{3}{4}\sin 2x + 2\sin^2 x + c$
	4d				C4 Integral – 1/(4-5x)	$-\frac{1}{5}\ln\|4-5x\| + c$
	4e				C4 Integral – ln	$3x\ln x - 3x + c$
	4f				C4 Integral – tan	$\frac{1}{3}\ln\|\sec 3x\| + c$
	4g				C4 Integral – x^2 sinx	$-x^2\cos x + 2x\sin x + 2\cos x + c$
	4h				C4 Integral – substitution show	Proof

					that	
	4i				C4 Integral – cot5x	$\frac{1}{5}\ln\lvert\sin 5x\rvert + c$
	4j				C4 Integral – (x+2) / (x-1)	$x + 3\ln\lvert x-1\rvert + c$
	4k				C4 Integral – reverse chain	$\frac{1}{3}(2-3x)^{-1} + c$
	4l				C4 Integral – standard trig	$-\frac{1}{2}\text{cosec}2x + c$
	4m				C4 Integral – improper partial fractions	$x - \ln\lvert x+2\rvert + \ln\lvert x-2\rvert + c$
	4n				C4 Integral – e^f(x)	$e^{\tan x} + c$
	4o				C4 Integral - x / (9x^2 + 1)	$\frac{1}{18}\left(\ln\lvert 9x^2+1\rvert\right) + c$
	4p				C4 Integral – sec3x	$\frac{1}{3}\ln\lvert\sec 3x + \tan 3x\rvert + c$
	5a				C4 Vectors – show lines intersect & where	**r** = -3**i** +3**j** +7**k**
	5b				C4 Vectors – show lines perpendicular	show scalar product is zero
	5c				C4 Vectors – show A lies on L1	show $\lambda=7$
	5d				C4 Vectors – B reflection of A in L2. Find B	**r** = -11**i** - **j**+11**k**
	6a				C4 Differentiation – prove y=2^x differential	proof *hint ln both sides of $y = 2^x$ *
	6b				C4 Integral – 2^x	$2^x / \ln 2$
	6c				C4 Integral – integrate e^x sinx	$\frac{1}{2}e^x cosx + \frac{1}{2}e^x sinx + c$
	6d				C4 Integral – substitution with partial fraction	$\sqrt{x^2+9} + \frac{3}{2}\ln\left\lvert\sqrt{x^2+9} - 3\right\rvert - \frac{3}{2}\ln\lvert\sqrt{x^2+9} + 3\rvert$
C3 Consolidation	7a				C3 ln & e – given y coord, find x coord	$x = \frac{1}{2}(\ln 2 - 1)$
	7b				C3 ln & e – find equation of curve	$y = 16x + 16 - 8\ln 2$
	8a				C3 Trig – prove cosec^2 identity	Proof
	8b				C3 Trig – Solve cot^2 quadratic	$\theta = 11.5°, 168.5°$
	9a				C3 MOD – sketch |f(x)|	Sketch
	9b				C3 MOD – sketch y = f(-x)	Sketch
	9c				C3 MOD – find coordinates of intersection	$P(-1, 2)$, $Q\ (0, 1)$, $R(1, 0)$
	9d				C3 MOD – Mod solve	x = 2/3, x = - 6
	10a				C3 Numerical methods – show root	Show
	10b				C3 Numerical methods – show iteration form	Show
	10c				C3 Numerical methods – find x1, x2, x3, x4	$x_1 = 1.4371\quad x_2 = 1.4347\quad x_3 = 1.4355$
	10d				C3 Numerical methods – prove root correct	choose interval [1.4345, 1.4355] use change in sign method

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"If you ask a mathematician what they do you always get the same answer; they think. They think about difficult and unusual problems. They do not think about ordinary problems, they just write down the answers." M. Egrafov

A2 Maths with Mechanics Assignment σ (sigma)

Due w/b 6/2

Drill

Part A For each of the following, with their restricted domains

(i) Sketch the (restricted) graph and hence state the range

(ii) Find the inverse function

and
(iii) State the domain of the inverse function:

(a) $f(x) = \dfrac{1}{x+1}, \quad x \, \varepsilon \, \Re : x \neq -1$ (b) $f(x) = x^2 + 4x + 5, \quad x \, \varepsilon \, \Re : x \geq -2$

(c) $f(x) = 1 - e^{2x}, \quad x \varepsilon \, \Re : x < 0$

Part B Integrate the following with respect to *x*:

(a) $\displaystyle\int \frac{\sec^2 x}{(1+\tan x)^3} dx$ (b) $\displaystyle\int 2\sin x \cos^3 x \, dx$ (c) $\displaystyle\int \frac{x}{(1-x^2)^5} dx$

Part C Eliminate θ from the following pairs of equations

a) $y = \sin 2\theta, x = \cos\theta$
b) $y = \cot\theta, x = \sin\theta$
c) $y = \cos 2\theta, x = \tan\theta$

Part D Given that the following vectors are perpendicular, find the value of *p*:

(a) **a** = 2**i** + **j** + *p***k**, **b** = **i** – **j** + **k**

(b) **a** = 4**i** + **j** – 2**k**, **b** = –2**i** + *p***j** + **k**

(c) **a** = **i** + *p***j** – **k**, **b** = **i** – **j** + **k**

Current Mechanics: Work, Energy and Power

1. The total mass of a cyclist and his machine is 140kg. The cyclist rides along a horizontal road against a constant total resistance of magnitude 50N. Find, in Joules, the total work done by the cyclist in increasing his speed from 6ms^{-1} to 9ms^{-1} whilst travelling a distance of 30m.

2. A car of mass 1200 kg moves along a straight horizontal road with a constant speed of 24 m s^{-1}. The resistance to motion of the car has magnitude 600 N.

(*a*) Find, in kW, the rate at which the engine of the car is working.

The car now moves up a hill inclined at α to the horizontal, where $\sin\alpha = \frac{1}{28}$.
The resistance to motion of the car from non-gravitational forces remains of magnitude 600 N. The engine of the car now works at a rate of 30 kW.

(*b*) Find the acceleration of the car when its speed is 20 m s^{-1}.

3. An engine of mass 25 tonnes, pulls a carriage of mass 10 tonnes along a railway line (1 tonne = 1000kg).
The frictional resistances to the motion of the engine and carriage are constant and of magnitude 50N per tonne mass. When the train travels horizontally the **tractive** force exerted by the engine is 26kN. Calculate:

a) The acceleration in ms^{-2} of the engine and carriage.

b) The tension in kN in the coupling between the engine and the carriage.

The engine and carriage now start to climb a slope whose inclination to the horizontal is $\arcsin \frac{1}{70}$, and the frictional resistances are unaltered. At a certain instant the engine and carriage are moving up the slope with speed $10ms^{-1}$ and acceleration $0.6ms^{-2}$. Calculate, at that instant:

c) The tractive force in kN exerted by the engine.

d) The power in kW developed by the engine.

C4 consolidation

Integrate the following using standard integrals, recognition, partial fractions, substitution or parts

4. Integrate with respect to *x*:

(a) $\int\left(1-\frac{1}{x}\right)^2 \mathrm{d}x$ (b) $\int \sin\frac{x}{2}\cos^4\frac{x}{2}\ \mathrm{d}x$ (c) $\int(\sin x + 2\cos x)^2\,\mathrm{d}x$

(d) $\int(4-5x)^{-1}\,\mathrm{d}x$ (e) $\int 3\ln x\,\mathrm{d}x$ (f) $\int \tan 3x\,\mathrm{d}x$

(g) $\int x^2 \sin x\,\mathrm{dx}$

(h) By substitution, show that $\int_0^3 15x\sqrt{x+1}\,dx = 116$ let u = x+1

(i) $\int \cot 5x\,\mathrm{dx}$ (j) $\int \frac{x+2}{(x-1)}\mathrm{dx}$ (k) $\int(2-3x)^{-2}\,\mathrm{dx}$

(l) $\int \operatorname{cosec} 2x \cot 2x \, dx$ (m) $\int \frac{x^2}{x^2 - 4} dx$ (n) $\int \sec^2 x \, e^{\tan x} dx$

(o) $\int \frac{x}{9x^2 + 1} dx$ (p) $\int \sec 3x \, dx$

5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(*a*) Show that l_1 and l_2 meet and find the position vector of their point of intersection.
(*b*) Show that l_1 and l_2 are perpendicular to each other.

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(*c*) Show that A lies on l_1.

The point B is the image of A after reflection in the line l_2.

(*d*) Find the position vector of B.

6 a) Show that the derivative of 2^x is $2^x \ln 2$.

b) What is the integral of 2^x ?

c) Integrate $\int e^x \cos x \, dx$

d) Integrate

$$\int \frac{\sqrt{x^2+9}}{x} dx, letting\ u^2 = x^2 + 9$$

C3 Consolidation

7. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(*a*) Find, in terms of ln 2, the x-coordinate of P.

(*b*) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

8. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$.

(*b*) Solve, for $0 \leq \theta < 180°$, the equation

$$2 \cot^2 \theta - 9 \operatorname{cosec} \theta = 3,$$

giving your answers to 1 decimal place.

9.

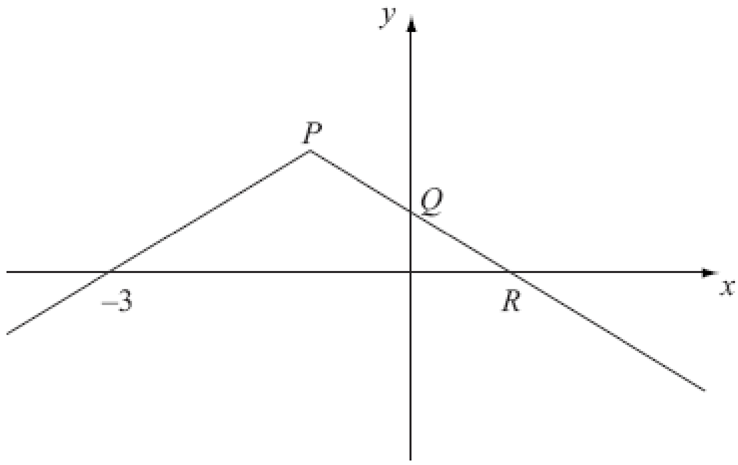

Figure 1

Figure 1 shows the graph of $y = \text{f}(x)$, $x \in \mathbb{R}$,

The graph consists of two line segments that meet at the point P.

The graph cuts the y-axis at the point Q and the x-axis at the points (–3, 0) and R.

Sketch, on separate diagrams, the graphs of

(*a*) $y = |\text{f}(x)|$,
(*b*) $y = \text{f}(-x)$.
Given that $\text{f}(x) = 2 - |x + 1|$,

(*c*) find the coordinates of the points P, Q and R,
(*d*) solve $\text{f}(x) = \frac{1}{2}x$.

10. $$\text{f}(x) = 3x^3 - 2x - 6.$$

(*a*) Show that f $(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$

(*b*) Show that the equation f $(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(*d*) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.