A2 Assignment rho Cover Sheet **Name:**

Question		Done	BP	Ready	Topic	Comment
Drill	Aa				C4 Integration	$\frac{1}{2}\left(x+\frac{1}{4}\sin 4x\right)+c$
	Ab				C4 Integration	$\frac{1}{3}\tan 3x - x + c$
	Ac				C4 Integration	$\frac{2}{3}\ln\lvert 3x-1\rvert + c$
	Ba				C3 e and ln solves	$\frac{1}{2}\ln\frac{3}{2}$
	Bb				C3 e and ln solves	4 or $^1/_4$
	Bc				C3 e and ln solves	$\frac{3}{7}$
	Ca				C3 Modulus solves	x = 6, x = -2
	Cb				C3 Modulus solves	$\frac{1}{6}$ or $\frac{1}{2}$
	Cc				C3 Modulus solves	$\frac{-5}{2}$ or $\frac{-1}{4}$
	Da				C4 Integration	$x-2\ln x-\frac{1}{x}+c$
	Db				C4 Integration	$\frac{5x}{2}+\frac{3}{4}\sin 2x+2\sin^2 x+c$
	Dc				C4 Integration	$\frac{1}{3}\ln\lvert\sec 3x\rvert+c$
Current work	1a				C4 Vectors – distance between	√29
	1b				C4 Vectors – distance between	√34
	1c				C4 Vectors – distance between	p=3
	2a				C4 Vectors – perpendicular	2
	2b				C4 Vectors – perpendicular	-11
	2c				C4 Vectors – perpendicular	$\frac{7}{2}$
	3a				C4 Vectors – direction vector	$\underline{AB}$ = 5$\underline{j}$ + 5$\underline{k}$
	3b				C4 Vectors – equation of a line	Position vector + λ(5**j** + 5**k**) or equivalent
	3c				C4 Vectors – point on line	Yes
	4a				M2 COM – Area of triangle given centroid	$\sqrt{3}\,d^2/3$ (remember, centroid of a triangle is always 2/3 of the way down from each vertex!!)
	4b				M2 COM – COM lamina triangle removed	Proof
	4c				M2 COM – angle of suspension with vertical	22.4 degrees
	5a				C3 e & ln equations	x = 2
	5b				C3 e & ln equations	x = ln 3, x = 0
Consolidation	6a				C3 differentiation	$\frac{dy}{dx}=x^2e^x+2xe^x$
	6b				C3 turning points	x = 0, y = 0 and x = –2, y = 4e^{-2}
	6c				C3 differentiation	$\frac{d^2y}{dx^2}=x^2e^x+2xe^x+2xe^x+2e^x$
	6d				C3 nature of turning points	x = 0 is a minimum, x = –2 is a maximum
	7a				C3 rewrite to iterative formula	

	7b				C3 iteration	$x_2 = 0.6455$, $x_3 = 0.6517$, $x_4 = 0.6526$
	7c				C3 show root is correct	choose interval [0.6525, 0.6535], use change in sign method
	8a				C3 composite function	ln 3
	8b				C3 inverse function	$f^{-1}(x) = \frac{1}{2}(e^x + 1)$, Domain $x \in \mathbb{R}$
	8c				C3 modulus	check using calculator, desmos or autograph
	8d				C3 modulus solve	$x = 11/3$, $x = 7/3$
	9a				M2 Kinematics – given a, find when v is zero	t = 1/3, t = 3
	9b				M2 Kinematics – given a, find distance travel	proof
	10a				M2 Kinematics – vectors, velocity, find acc	$6t\mathbf{i}-4\mathbf{j}$
	10b				M2 Kinematics – vectors, velocity, find force	6.32 N
	11a				M2 COM – Square removed from square	5a/6
	11b				M2 COM – Suspended, find angle with vert	35.5º
	12				C4 connected rates of change	dC/dt = 2/3
	13				C4 vectors	B
	Ch				Challenge	$6\pi\left(2-\sqrt{3}\right)$

"Logic, like whiskey, loses its beneficial effects when taken in too large quantities."

Lord Dunsany

A2 Maths with Mechanics Assignment ρ (rho)

Due in w/b 30/1

Drill

Part A Integrate the following functions with respect to *x*:

(a) $\cos^2 2x$ (b) $\tan^2 3x$ (c) $\dfrac{2}{3x-1}$

Part B Solve the following equations giving *x* exactly:

(a) $2e^x = 3e^{-x}$ (b) $\log_2 x = 4\log_x 2$ (c) $\log_2(1-3x) - \log_2(2x-1) = 1$

Part C Solve the following equations:

(a) $|x-2| = 4$ (b) $2|3x-1| - 1 = 0$ (c) $|x-2| = 3|x+1|$

Part D Integrate the following with respect to *x*:

(a) $\int \left(1-\dfrac{1}{x}\right)^2 \mathrm{d}x$ (b) $\int (\sin x + 2\cos x)^2 \mathrm{d}x$ (c) $\int \tan 3x \,\mathrm{d}x$

Current work: C4 Vectors

1. Find the distance between the points with the following position vectors:

(a) $\mathbf{a} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$

(b) $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$

(c) Given the distance between the points with position vectors

$\mathbf{a} = p\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ is 5, find *p*

2. Given that the following vectors are perpendicular, find the value of *p*:

(a) $\mathbf{a} = p\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$

(b) $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 4\mathbf{i} + p\mathbf{j} - \mathbf{k}$

(c) $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$

3. Point A has position vector $3\mathbf{i}+5\mathbf{j}-2\mathbf{k}$ and point B has position vector $3\mathbf{i}+10\mathbf{j}+3\mathbf{k}$

(a) Find $\overrightarrow{\mathbf{AB}}$

(b) Give the vector equation of the line passing through A and B in its simplest form

(c) Does the point (3,-5,-12) lie on this line?

Current work: M2

4.

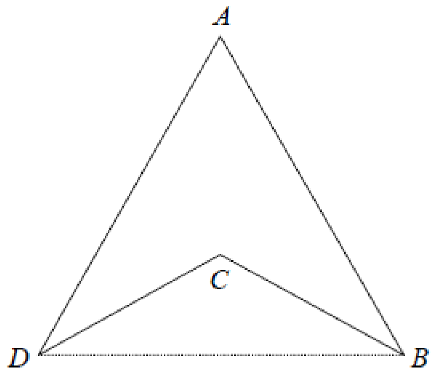


Fig. 2

Figure 2 shows a uniform lamina $ABCD$ formed by removing an isosceles triangle BCD from an equilateral triangle ABD of side $2d$. The point C is the centroid of triangle ABD.

(a) Find the area of triangle BCD in terms of d. **(3 marks)**

(b) Show that the distance of the centre of mass of the lamina from BD is $\frac{4}{9}\sqrt{3}\,d$.

(8 marks)

The lamina is freely suspended from the point B and hangs at rest.

(c) Find in degrees, correct to 1 decimal place, the acute angle that the side AB makes with the vertical.

(4 marks)

C3 Consolidation

5. Find the exact solutions to the equations

(*a*) $\ln x + \ln(x-1) = \ln 6$,

(*b*) $e^x + 3e^{-x} = 4$

6. A curve C has equation $y = x^2e^x$.

(*a*) Find $\frac{dy}{dx}$.

(*b*) Hence find the coordinates of the turning points of C.

(*c*) Find $\frac{d^2y}{dx^2}$.

(*d*) Determine the nature of each turning point of the curve C.

7. $f(x) = -x^3 + 3x^2 - 1$.

(*a*) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}.$$

(*b*) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of x_2, x_3 and x_4, giving all your answers to 4 decimal places.

(*c*) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places.

8. The functions f and g are defined by

$$f : x \mapsto \ln(2x - 1), \quad x \in \mathbb{R}, \quad x > \tfrac{1}{2},$$

$$g : x \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R}, \quad x \neq 3.$$

(*a*) Find the exact value of fg(4).

(*b*) Find the inverse function $f^{-1}(x)$, stating its domain.

(*c*) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the *y*-axis.

(*d*) Find the exact values of *x* for which $\left|\frac{2}{x-3}\right| = 3$.

M2 consolidation

9.

A particle *P* moves in a straight line with an acceleration of $(6t - 10)$ m s^{-2} at time *t* seconds. Initially *P* is at *O*, a fixed point on the line, and has velocity 3 m s^{-1}.

(a) Find the values of *t* for which the velocity of *P* is zero. **(6 marks)**

(b) Show that, during the first two seconds, *P* travels a distance of $6\frac{26}{27}$ m. **(7 marks)**

10. A particle *P* of mass 0.5 kg moves under the action of a single force **F** Newtons. At time *t* seconds, the velocity **v** m s^{-1} of *P* is given by

$$\mathbf{v} = 3t^2\mathbf{i} + (1 - 4t)\mathbf{j}.$$

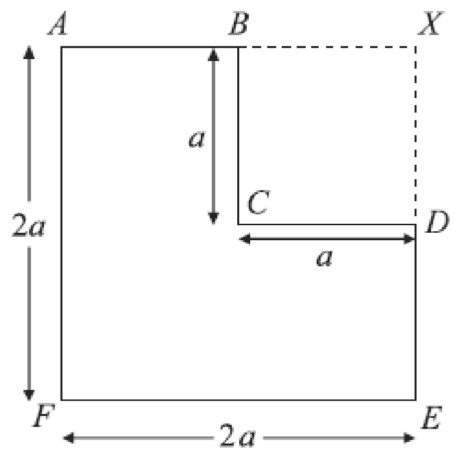
Find

(*a*) the acceleration of *P* at time *t* seconds,

(*b*) the magnitude of **F** when $t = 2$.

11

. **Figure 1**



A uniform lamina *ABCDEF* is formed by taking a uniform sheet of card in the form of a square *AXEF*, of side 2*a*, and removing the square *BXDC* of side *a*, where *B* and *D* are the mid-points of *AX* and *XE* respectively, as shown in Figure 1.

(*a*) Find the distance of the centre of mass of the lamina from *AF*.

The lamina is freely suspended from *A* and hangs in equilibrium.

(*b*) Find, in degrees to one decimal place, the angle which *AF* makes with the vertical.

C4 Consolidation

12. A circular ink blot is spreading at a rate of $1/3\text{cm}^2\text{s}^{-1}$. Find the rate of increase in the circumference of the ink blot when its radius is 1/2cm

13. ***For this question decide which of the responses given is (are) correct then choose***

A if 1, 2 and 3 are correct
B if only 1 and 2 are correct
C if only 2 and 3 are correct
D if only 1 is correct
E if only 3 is correct

$$\overrightarrow{OP} = -2\underline{i} + 3\underline{j} + \underline{k}$$

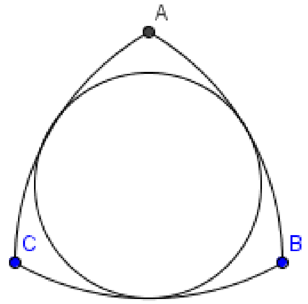
$$\overrightarrow{OQ} = 3\underline{i} - 2\underline{j} + \underline{k}$$

1. $\underset{\boldsymbol{PQ}}{\rightarrow} = +5\boldsymbol{i} - 5\boldsymbol{j}$

2. $\overrightarrow{OP}.\overrightarrow{OQ} = -11$

3. $\angle POQ = \arccos\left(-\dfrac{11}{\sqrt{14}}\right)$

Challenge



The curvy shape ABD shown here is called a Reuleaux triangle (after French engineer Franz Reuleaux (1829-1905)). Its perimeter consists of three equal arcs AB, BC, CA; each with the same radius and centered at the opposite vertex. In the Reuleaux triangle shows, each arc has a radius 3cm. What is the area (in cm^2) of the inscribed circle?