A2 Assignment pi Cover Sheet **Name:**

Question		Done	BP	Read	Topic	Comment
Drill	Aa				C4 Integral – sin^2	$\frac{\pi}{8}-\frac{1}{4}$
	Ab				C4 Integral – cos^2	$\frac{\pi}{8}+\frac{1}{2\sqrt{2}}$
	Ac				C4 Integral – tan^2	2/3
	Ba				C3 Trig Solve – Double angles	210°, 330°, 270°
	Bb				C3 Trig Solve – sec^2 conversion	292.5, 202.5, 112.5, 22.5 degrees
	Bc				C3 Trig Solve – cosec^2 conversion	60, 180 degrees
	Ca				C3 Functions – sketch y = 1 - e^2x	asymptote y = 1, crosses (ln0.5, 0) and (0, -1)
	Cb				C3 Functions – sketch y = 2 + ln(x +1)	asymptote x = -1, crosses (0, 2) and (-1 + e^-2, 0)
	Cc				C3 Functions – sketch y = 10e^2x	asymptote y = 0, crosses (0, 10)
	Da				C4 Parametrics – eliminate t	$y^2 = x^2 + 1$
	Db				C4 Parametrics – eliminate t	$\frac{x^2}{9} + y^2 = 1$
	Dc				C4 Parametrics – eliminate t	
Consolidation	1a				M2 COM – find AG	$AG = 25\ cm$
	1b				M2 COM – Find angle of AB v Vertical	87.6 degrees
	1c				M2 COM – mass added, AB horizontal	$\frac{64\pi}{3} kg$
	2a				M2 Kinematics – given force find a	$\mathbf{a} = \left(3t^2 - 6\right)i + 4t\mathbf{j}$
	2b				M2 Kinematics – given force find v	proof
	3				C4 Integral – Trapezium Rule 4 strips	1.329
	4a				C4 Integral – Integrate secx exactly	ln(2+√3)
	4b				C4 Integral – Trapezium Rule 6 strips	1.326 (4sf)
	4c				C4 Integral – % error	0.687% (using 4.s.f answer)
	5a				C3 Numerical Methods – show root in [2,3]	let f(x)= x^3 -14 ,show change of sign
	5b				C3 Numerical Methods – rearrange eq	7
	5c				C3 Numerical Methods – find X_6 to 3s.f.	2.41
	5d				C3 Numerical Methods – prove root correct	use upper/lower bound, change in sign method
	6a				C3 Functions – Mod solve	x = 1/7, x = 7/3
	6b				C3 Functions – Mod solve	x = –3, x = 2
	7a				C3 Trig – prove sin3θ ≡ 3sin θ - 4sin^3 θ	Proof
	7b				C3 Trig – use proof to find sin3θ	$\frac{9\sqrt{3}}{16}$
	8a				C3 Algebra – make single fraction	Proof
	8b				C3 Algebra – show numerator > 0	Proof
	8c				C3 Algebra – show f(x) > 0	Proof
	9a				C3 Diff – show P lies on curve	Proof
	9b				C3 Diff – show dy/dx = 1/√2 @ P	Proof

	9c				C3 Diff – Find normal equation @ P	$y = -\sqrt{2}x + 2 + \frac{\pi}{4}$
	10a				C4 Binomial – expand (2-x)/√4-2x	$1 - \frac{x}{4} - \frac{x^2}{32} - \frac{x^3}{128}$
	10b				C4 Binomial – estimate 1.9 / √3.8	0.97468
	11a				C4 Integral – Pick your own substitution	$-2\ln\left\|1-\sqrt{x}\right) + 2 - 2\sqrt{x} + c$
	11b				C4 Integral – by parts	$4e^9 - \frac{3}{2}e^4$
	11c				C4 Integral – substitution x = 2sinu	$\frac{\pi}{3}$
	11d				C4 Integral – involving ln	$x^2(ln3x)^2 - x^2 ln3x + \frac{1}{2}x^2 + c$
challenge					Given y is imaginary, find values of x	$-4 < x < 0$

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"I don't believe in mathematics."

Albert Einstein

A2 Maths with Mechanics Assignment π(pi)

Due in w/b 29/1

Drill

Part A Evaluate, giving exact answers

(a) $\int_0^{\frac{\pi}{4}} \sin^2 x dx$ (b) $\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \cos^2 x dx$ (c) $\int_{-\frac{\pi}{12}}^{\frac{\pi}{12}} (\tan^2 3x + 1)\, dx$

***note these two lower limits have a negative sign, not very clear in print**

Part B Solve the following equations in the range $0 \le x \le 360^o$

(a) $\cos 2x = 3\sin x + 2$ (b) $\sec^2 2x = 2tan2x$

(c) $cosec^2\left(\frac{x}{2}\right) = \sqrt{3}\cot\left(\frac{x}{2}\right) + 1$

Part C Sketch the following functions: show clearly any asymptotes, vertical and horizontal, and any crossings with the coordinate axes.

(a) $y = 1 - 2e^x$ (b) $y = 2 + \ln(x+1)$ (c) $y = 10e^{2x}$

Part D Eliminate *t* from the following pairs of equations:

(a) $x = \tan t, \quad y = \frac{1}{\cos t}$ (b) $x = 3\sin t, \quad y = \cos t$

Mechanics

1. **Figure 1**

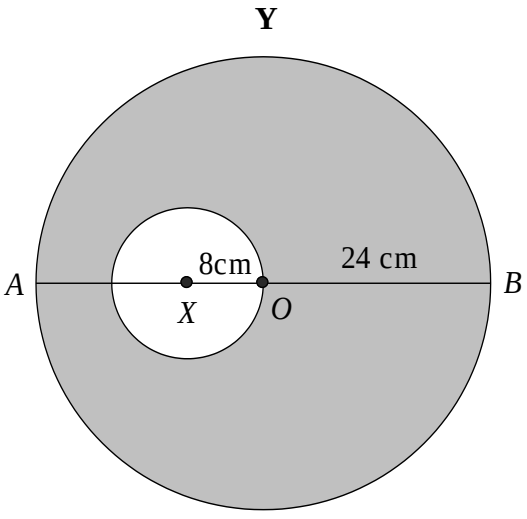

Figure 1 shows a template *T* made by removing a circular disc, of centre *X* and radius 8 cm, from a uniform circular lamina, of centre *O* and radius 24 cm. The point *X* lies on the diameter *AOB* of the lamina and *AX* = 16 cm. The centre of mass of *T* is at the point *G*.

a) Find *AG*
b) An axel is inserted through Y such that the axel is perpendicular to AB, and the lamina is left to rotate freely about Y. What is the acute angle between AB and the downward vertical?
c) With the axel still in place at Y, a mass is attached at the point A such that AB hangs horizontally to the downward vertical. What is the mass required?

2. A particle P of mass 0.5 kg is moving under the action of a single force **F** newtons. At time
t seconds, $\mathbf{F} = (1.5t^2 - 3)\mathbf{i} + 2t\mathbf{j}$. When $t = 2$, the velocity of P is $(-4\mathbf{i} + 5\mathbf{j})\,\text{m s}^{-1}$.

(a) Find the acceleration of P at time t seconds.

(b) Show that, when $t = 3$, the velocity of P is $(9\mathbf{i} + 15\mathbf{j})\,\text{m s}^{-1}$.

Pure

3. Find an approximate value to 3 decimal places for $I = \int_0^1 e^x \tan x\,dx$ using four strips.

4. For the integral $I = \int_0^{\frac{\pi}{3}} \sec x\,dx$

a) Find the exact value of I.
b) Use the trapezium rule to find an approximation of I using six strips to 4s.f.
c) Find the percentage error of this approximation.

5. (a) Show that $x^3 = 14$ has a root lying between 2 and 3.

(b) Show that $x^3 = 14$ can be rearranged into the form $x = \frac{p}{x^2} + \frac{x}{2}$ where p is a constant to be found.

(c) Using the iteration formula $x_{n+1} = \frac{p}{x_n^2} + \frac{x_n}{2}$, starting with $x_0 = 2.5$, find x_1, x_2, x_3, x_4, x_5, x_6.

Using your answer for x_6, give a root to 3 significant figures of $x^3 = 14$.

d) Prove that your answer is correct to 3.s.f.

6. Solve the following equations:

(a) $|5x - 4| = |2x + 3|$ (b) $|x^2 + x| = 6$

7. (a) Show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(b) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

8. $f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}$, $x \neq -2$.

(a) Show that $f(x) = \frac{x^2 + x + 1}{(x+2)^2}$, $x \neq -2$.

(*b*) Show that $x^2 + x + 1 > 0$ for all values of x.
(*c*) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

9. The curve C has equation $x = 2 \sin y$.

(*a*) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(*b*) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(*c*) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

10. a) Expand $\frac{2-x}{\sqrt{4-2x}}$ in ascending powers of x up to x^3

b) using your expansion, estimate $\frac{1.9}{\sqrt{3.8}}$ $correct\ to\ 5.d.p.$

11. a) using a suitable substitution of your choosing $\int \frac{1}{1-x^{\frac{1}{2}}} dx$

b) $using\ integration\ by\ parts, find\ the\ exact\ integral$: $\int_2^3 x^3 e^{x^2} dx$

c) using the substitution $x = 2sinu$, $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx$

d) $\int 2x(ln3x)^2 dx$

Challenge Question

Given that $y = \frac{x}{x+\left(\frac{x}{x+y}\right)}$, find the range of x values if y is not a real number.

(Hint on the VLE)