A2 Assignment pi Cover Sheet **Name:**

Question		Done	BP	Ready	Topic	Comment
Drill	Aa				C4 Integration – standard results & T.Ids	$1-\frac{1}{\sqrt{3}}$
	Ab				C4 Integration – standard results & T.Ids	$\frac{\pi}{16}-\frac{1}{8}$
	Ac				C4 Integration – standard results & T.Ids	$\frac{27-e^3}{3}$
	Ba				C4 Integration – partial fractions	$\frac{2}{3}\ln k\left\|\frac{x-2}{x+1}\right\|$
	Bb				C4 Integration – partial fractions	$\frac{1}{10}\ln\|2x-1\|+\frac{7}{5}\ln\|x+2\|+c$
	Bc				C4 Integration – partial fractions	$\frac{1}{4}\ln\frac{3}{2}$
	Ca				C3 Sketching e and ln	Check graph using Autograph/Desmos
	Cb				C3 Sketching e and ln	Check graph using Autograph/Desmos
	Cc				C3 Sketching e and ln	Check graph using Autograph/Desmos
	Da				C4 Integration - parts	$-x\cos x+\sin x+c$
	Db				C4 Integration - parts	$\frac{x^3}{3}\ln x-\frac{x^3}{9}+c$
	Dc				C4 Integration - parts	$2\ln 2-\frac{3}{4}$
Current work	1ai				M2 COM – Lamina Find distance AD	1.7a
	1aii				M2 COM – lamina find distance from AB	1.1 a
	1b				M2 COM – Maximum tilt point	32.5 degrees
	2ai				M2 COM – Frame find distance from AB	4a/5
	2aii				M2 COM – Frame find distance from BC	a/2
	2b				M2 COM – Frame mass added s.t. horiz	m = a kg
	2c				M2 COM – Frame mass added s.t. 80 deg	M = 1.44a kg
	3				C4 Connected rates of change	6/25
	4a				C4 Connected rates of change	0.00255 cm s^{-1} (3sf)
	4b				C4 Connected rates of change	0.48 cm^3 s^{-1}
	5				C4 Implicit differentiation	tangent 3x+2y-6=0; normal 2x-3y+9=0
	6ai				C3 Composite functions	
	6aii				C3 Composite functions – solve	$x=-4$
	6bi				C3 Inverse function	$\frac{3x}{x-1}$
Consolidation	6bii				C3 Inverse function - domain	$x\in\mathbb{R}\quad x\neq 1$
	7a				C3 Modulus function - sketch	Check on Desmos/Autograph
	7b				C3 Modulus function - solve	$1-\sqrt{6},\ 1$
	8a				C3 Rcos	$\sqrt{13}\sin(x+0.588)$
	8b				C3 Rcos – max value	169
	8c				C3 Rcos – solve	$x=2.273$ or $x=5.976$
	9a				C3 Trig proof	Proof
	9b				C3 Trig sketch	Check on Desmos/Autograph
	9c				C3 Trig solve	$\theta=20.9°, 69.1°, 200.9°, 249.1°$
	10a				C3 e & ln problem	5.353
	10b				C3 e & ln problem	Show

	10c				C3 e & ln problem	T = 13.06…
	11a				M2 Projectiles – Find Cartesian eq of proj	$y = xtan70 - x^2 \frac{49}{9000} \sec^2 70$
	11b				M2 Projectiles – Find Cartesian eq slope	$y = xtan5$
	11c				M2 Projectiles – Find intersection of eqs	x = 57.2 m, y = 5.00m
	11d				M2 Projectiles – Find distance to origin	57.4 m
	12				C4 Integration by substitution	Show that

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"It is easier to square the circle than to get round a mathematician."

Augustus De Morgan

A2 Maths with Mechanics Assignment π (pi).

Due in w/b 23/1

Drill

Part A Integrate the following functions with respect to *x*, giving an EXACT answer

(a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \text{cosec}^2 x\,dx$ (b) $\int_0^{\frac{\pi}{8}} \sin^2 2x\,dx$ (c) $\int_1^{\ln 3} e^{3x}\,dx$

Part B Integrate the following with respect to *x* using partial fractions:

(a) $\dfrac{2}{(x+1)(x-2)}$ (b) $\dfrac{3x-1}{(2x-1)(x+2)}$

(c) Evaluate giving an exact answer $\int_4^6 \dfrac{1}{x^2-4}\,dx$

Part C Sketch the following functions: show all asymptotes clearly

(a) $y=1-e^{-x}$ (b) $y=1-\ln 2x$ (c) $y=2e^{-2x}$

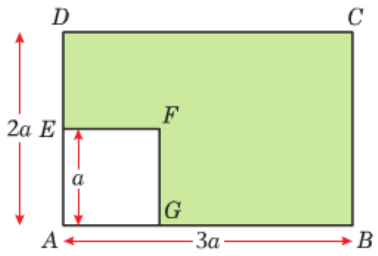
Part D Integrate the following functions with respect to x,

(a) $\int x \sin x\,dx$ (b) $\int x^2 \ln x\,dx$ (c) $\int_0^{\ln 2} xe^{2x}dx$

Current work: M2

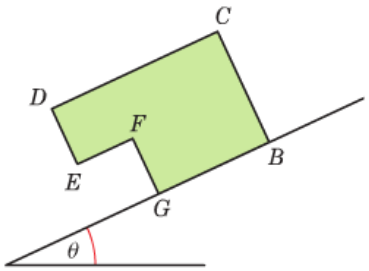
1.

A uniform lamina consists of a rectangle *ABCD*, where $AB = 3a$ and $AD = 2a$, with a square hole *EFGA*, where $EF = a$, as shown in the diagram:



a Find the distance of the centre of mass of the lamina from
i *AD*, **ii** *AB*.

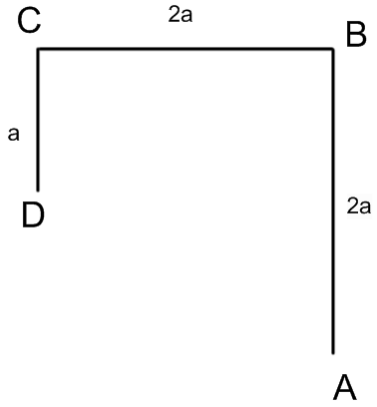
The lamina is balanced on a rough plane inclined to the horizontal at an angle θ. The plane of the lamina is vertical and the inclined plane is sufficiently rough to prevent the lamina from slipping. The side *GB* is in contact with the plane with *G* lower than *B*, as shown in the diagram:



b Find, in degrees to 1 decimal place, the greatest value of θ for which the lamina can rest in equilibrium without toppling.

2.

A thin uniform wire of length $5a$ is bent to form the shape $ABCD$, where $AB = 2a$, $BC = 2a$, $CD = a$ and BC is perpendicular to both AB and CD, as shown in the diagram:



a Find the distance of the centre of mass of the wire from
i AB, **ii** BC.

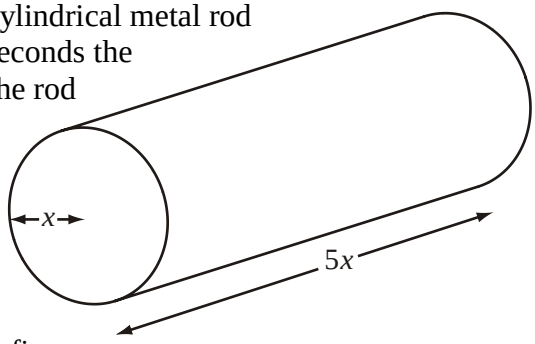
b) A mass m is attached at the point C such that when the wire is suspended from the midpoint of BC, BC hangs horizontally. Find m.

c) The mass is replaced by another mass M, attached again at C. The wire is suspended from the midpoint of BC, such that BC hangs at an angle of 80 degrees to the vertical, with B above C. Find the mass M needed to 3s.f.

Current work: C4 Connected rates of change

3. The volume of a cube is increasing at a rate of $18\text{cm}^3\text{s}^{-1}$. Find the rate of increase of a side when the volume is 125cm^3.

4. The diagram above shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm. The cross-sectional area of the rod is increasing at the constant rate of $0.032\ \text{cm}^2\ \text{s}^{-1}$.



(a) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(b) Find the rate of increase of the volume of the rod when $x = 2$.

Consolidation: C3

5. Find the tangent and normal to $y^2e^x + x^2 = 9$ at the point (0,3)

6. Functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{x}{x-3}, x \in \mathbb{R}, x \neq 3 \qquad \mathrm{g}: x \mapsto \frac{1}{2x-1}, x \in \mathbb{R}, x \neq \tfrac{1}{2}$$

(a) (i) Show that $\mathrm{gf}(x) = 1 - \frac{6}{x+3}$. (ii) Solve $\mathrm{gf}(x) = 7$.

(b) (i) Find an expression for $\mathrm{f}^{-1}(x)$. (ii) Find the domain of f^{-1}.

7. (a) Sketch, on the same diagram, the graphs of

$$y = |2x+1| \text{ and } y = 4 - x^2$$

indicating the coordinates of any points where the graphs meet the axes.

(b) Solve the equation $|2x+1| = 4 - x^2$, giving the exact value of each root.

8. (*a*) Express 3 sin x + 2 cos x in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(*b*) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$.

(*c*) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places.

9. (*a*) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n°.$$

(*b*) Sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.

(*c*) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3$$

giving your answers to 1 decimal place.

10. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(*a*) Find the amount of the drug in the bloodstream 5 hours after the dose is given. Give your answer in mg to 3 decimal places.

A second dose of 10 mg is given after 5 hours.

(*b*) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(*c*) Find the value of T.

Consolidation: M2

11. A golfer hits a golf ball at a speed of 30ms^{-1} at 70° to the horizontal up a slope which is angled at 5° to the horizontal.

a) Find the equation of the path of the ball.

b) Find the equation of the slope.

c) By eliminating y from the equations found above, find where the ball lands.

d) How far from O, the point of projection, does the ball land?

Consolidation: C4

14. Use the substitution $u = 1 + \sin x$ and integration to show that

$$\int \sin x \cos x (1 + \sin x)^5 \, dx = \frac{1}{42} (1 + \sin x)^6 [6 \sin x - 1] + \text{constant}.$$