

Question	Done	BP	Ready	Topic	Comment
Drill	Aa			C4 Integration	$\frac{1}{4} \sin 2x + \frac{1}{2}x + c$
	Ab			C4 Integration	$\frac{1}{3}e^{3x-2} + c$
	Ac			C4 Integration	$\frac{1}{2} \ln 2x-5  + c$
	Ba			C3 Show root	<i>change of sign</i>
	Bb			C3 Show root	<i>change of sign</i>
	Bc			C3 Show root	<i>f(x) is not continuous on the interval, and f(0) and f(2) will both be positive i.e. there will be no change in sign. Have a look at it in your graphics or on Autograph</i>
	Ca			C3 Log equations	4, 12
	Cb			C3 Log equations	$\frac{1}{e-1}$
	Da			C4 Implicit Differentiation	$\frac{dy}{dx} = \frac{2x+3y}{2y-3x}$
	Db			C4 Implicit Differentiation	$\frac{dy}{dx} = \frac{4x-y}{x-3y}$
	Dc			C4 Implicit Differentiation	$\frac{dy}{dx} = \frac{2}{3} \tan 2x \cot 3y$
	Dd			C4 Implicit Differentiation	$\frac{dy}{dx} = \frac{e^y}{1-xe^y}$
	De			C4 Implicit Differentiation	$\frac{dy}{dx} = -\frac{y \ln y}{2y^2+x}$
	Df			C4 Implicit Differentiation	$\frac{dy}{dx} = \frac{\sin y + 2x \cos y}{x^2 \sin y - x \cos y}$
Current work	1a			C4 Trig integration	$\tan x - x + c$
	1b			C4 Trig integration	$-\frac{1}{3} \cot 3x - x + c$
	1c			C4 Trig integration	$\frac{1}{2}x - \frac{1}{4} \sin 2x + c$
	1d			C4 Trig integration	$-\frac{1}{2} \operatorname{cosec} 2x + c$
	1e			C4 Trig integration	$3x + 4 \cos x - \sin 2x + c$
	1f			C4 Trig integration	$-\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x + c$
	2			C3 natural log knowledge	Think about what values x can take in for ln x to exist, and what the modulus does
	3a			C4 Integration using partial fractions	$\frac{1}{4} \ln \left  \frac{x-2}{x+2} \right  + c$
	3b			C4 Integration using partial fractions	$\ln x-3  - 2 \ln x-2  + c$
	3c			C4 Integration using partial fractions	$\ln x-1  - 2 \ln 2x+1  + c$
	4a			C4 Trapezium rule	3.983 (3dp)
	4b			C4 Integration	4.047 (3dp)
4c			C4 Percentage error	1.58%	

	5			M2 Projectiles – perpendicular to original	$t = 0.638$
	6			M2 Projectiles – horizontal, find height	31m
	7			M2 Projectiles – find needed angle of projection	24 degrees
	8a			C3 Trig proof	Proof
	8b			C3 Trig solve	$0.333^c, 1.24^c, 3.47^c, 4.38^c$
	9a			C3 Binomial expansion –compare coefficients	$a = 2, b = -3$
	9b			C3 Binomial expansion – find coefficient	-80
Consolidation	10a			C4 Implicit Differentiation & coordinate geom.	$y = 18 - 4x$
	10b			C4 Coordinate geom.	Q (-4, -2)
	11a			C3 Modulus Sketching	Sketch
	11b			C3 Graph Sketching	Sketch
	11c			C3 Solutions vs sketch	one point of intersection
	11d			C3 Modulus solve	$\frac{1}{2}$
	12ai			C3 Inverse functions	$f^{-1}: x \rightarrow \frac{x-2}{5}, x \in \mathbb{R}$
	12aai			C3 Composite functions	$fg: x \rightarrow \frac{5}{x} + 2, x \in \mathbb{R}, x \neq 0$
	12aai			C3 Inverse and composite functions	$(fg)^{-1}: x \rightarrow \frac{5}{x-2}, x \in \mathbb{R}, x \neq 2$
	12b			C3 function solve	$x = -1.81, 13.81$
	13			Challenge	$\sqrt{6} / 3$
Mock Exam	Ai			Trig	$\cos x \equiv 2 \cos^2 \frac{x}{2} - 1$ $\Rightarrow 2 \cos^2 \frac{x}{2} \equiv 1 + \cos x \Rightarrow \cos^2 \frac{x}{2} \equiv \frac{1 + \cos x}{2}$
	Aii			Trig	$\cos x \equiv 1 - 2 \sin^2 \frac{x}{2}$ $\Rightarrow 2 \sin^2 \frac{x}{2} \equiv 1 - \cos x \Rightarrow \sin^2 \frac{x}{2} \equiv \frac{1 - \cos x}{2}$
	Bi			Trig	$\frac{2\sqrt{5}}{5}$
	Bii			Trig	$\frac{\sqrt{5}}{5}$
	Biii			Trig	$\frac{1}{2}$
	C			Trig	$\cos^4 \frac{A}{2} \equiv \left( \frac{1 + \cos A}{2} \right)^2 \equiv \frac{1 + 2 \cos A + \cos^2 A}{4}$ $\equiv \frac{1 + 2 \cos A + \left( \frac{1 + \cos 2A}{2} \right)}{4}$ $\equiv \frac{2 + 4 \cos A + 1 + \cos 2A}{8}$ $\equiv \frac{3 + 4 \cos A + \cos 2A}{8}$

$\alpha$	$\beta$	$\gamma$	$\delta$	$\varepsilon$	$\zeta$	$\eta$	$\theta$	$\iota$	$\kappa$	$\lambda$	$\mu$	$\nu$	$\xi$	$\omicron$	$\pi$	$\rho$	$\sigma$	$\tau$	$\upsilon$	$\phi$	$\chi$	$\psi$	$\omega$
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*“The mathematician is fascinated with the marvellous beauty of the forms he constructs, and in their beauty he finds everlasting truth”*

J B Shaw

**A2 Maths with Mechanics Assignment  $\mu$  (mu)**  
**INCLUDING a past paper: C3 past Jan 2009**  
**due in w/b 01/01/18**

*Happy Christmas*  
*And a successful New Year!*



**Drill**

**Part A:** Integrate with respect to  $x$  (use the correct notation  $\int(\dots) dx = \text{etc}$ )

- (a)  $\cos^2 x$  (*hint write in terms of  $\cos 2x$  first*)      (b)  $e^{3x-2}$       (c)  $\frac{1}{2x-5}$

**Part B:** Show that each of the following functions has a root on the interval given:

- (a)  $x^3 - x + 3 = 0$        $(-3, 3)$       (b)  $3 + 4x - x^4 = 0$        $(1, 2)$

(c) Explain why we cannot use a change of sign to show there is a root in the following equation:

$$\tan 2x + 1 = 0 \text{ on the interval } (0^\circ, 2^\circ).$$

You may want to look at the graph to answer this.

**Part C:** Solve the following equations give an exact answer

- (a)  $2 \ln 2x - 6 \ln 2 = \ln(x - 3)$       (b)  $\ln(x + 1) - \ln x = 1$

**Part D:** Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

- (a)  $x^2 + 3xy - y^2 = 0$       (b)  $4x^2 - 2xy + 3y^2 = 8$       (c)  $\cos 2x \sec 3y + 1 = 0$   
 (d)  $xe^y - y = 5$       (e)  $y^2 + x \ln y = 3$       (f)  $x \sin y = 1 - x^2 \cos y$

1. Integrate the following functions with respect to  $x$ :

- (a)  $\int \tan^2 x dx$  (*hint write in terms of  $\sec^2 x$* )      (d)  $\int \frac{\cos 2x}{\sin^2 2x} dx$

(b)  $\int \cot^2 3x \, dx$  (e)  $\int (1 - 2 \sin x)^2 \, dx$

(c)  $\int \sin^2 x \, dx$  (f)  $\int \sin 3x \cos x \, dx$

2. Question: Why do we put modulus signs around  $\ln$  when integrating?

3. Integrate the following functions using partial fractions:

(a)  $\int \frac{1}{x^2 - 4} \, dx$  (b)  $\int \frac{4 - x}{(x - 2)(x - 3)} \, dx$  (c)  $\int \frac{5 - 2x}{(x - 1)(2x + 1)} \, dx$

4. The area under the curve  $y = \ln x$ , is bounded by the  $x$  - axis and the line  $x = 5$ .

(a) Estimate the area of the shaded region to 3 decimal places using the trapezium rule with 4 strips.

(b) Given that  $\int \ln x \, dx = x \ln x - x + c$ , find the true value of the area correct to 3 decimal places. (extension – find out why this is the integral!)

(c) Calculate the percentage error of the trapezium rule approximation.

5. A particle is projected from a height of 30m above the ground, with initial velocity  $3\mathbf{i} + 4\mathbf{j}$ . Find the time it takes for the particle to be travelling perpendicular to its original projection

6. A particle is projected horizontally with speed 40m/s from a point A. It hits the ground 100m horizontally from A. Find the height of A

7. A field 100m in length has two barriers of height 2m at a distance of 5m from both ends. A ball is kicked with speed 25m/s. What is the minimum angle the ball would need to be kicked at to the horizontal to clear both walls.

8. (a) Prove the following identity: set out proof correctly  
 $\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x$

(b) Solve the following equation on the interval  $0 \leq \theta \leq 2\pi$ . Give answers to 3sf.  
 $\cos 2\theta = \tan 2\theta$

9. The first three terms in the expansion of  $(1 + ax)^b$ , in ascending powers of  $x$ , for  $|ax| < 1$ , are

$$1 - 6x + 24x^2.$$

(a) Find the values of the constants  $a$  and  $b$ .

(b) Find the coefficient of  $x^3$  in the expansion.

10. A curve has the equation  $x^2 + 4xy - 3y^2 = 36$ .

(a) Find an equation for the tangent to the curve at the point P (4, 2).

Given that the tangent to the curve at the point Q on the curve is parallel to the tangent at P,

(b) find the coordinates of Q.

11. (a) Sketch the graph of  $y = |2x + a|$ ,  $a > 0$ , showing the coordinates of the points where the graph meets the coordinate axes.

- (b) On the same axes, sketch the graph of  $y = \frac{1}{x}$ .
- (c) Explain how your graphs show that there is only one solution of the equation

$$x|2x + a| - 1 = 0.$$

- (d) Find, using algebra, the value of  $x$  for which  $x|2x + 1| - 1 = 0$ .

12. The functions  $f$  and  $g$  are defined by

$$f: x \rightarrow 5x + 2, \quad x \in \mathbb{R} \qquad g: x \rightarrow \frac{1}{x}, \quad x \in \mathbb{R}, x \neq 0$$

- (a) Find the following functions stating the domain in each case.
- (i)  $f^{-1}(x)$                       (ii)  $fg(x)$                       (iii)  $(fg)^{-1}(x)$
- (b) Solve the equation  $f^{-1}(x) = fg(x)$ , giving your answers to 2 decimal places.

### Challenge – have a go at this!

A cube ABCDEFGH has the square ABCD as its base with EFGH above ABCD respectively. What is the cosine of the angle CAG?

### Optional extra questions for you if you are catching up on work from the C3 mock exam

**MEA)** Using  $\cos 2A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$ , show that:

$$(i) \cos^2 \frac{x}{2} \equiv \frac{1 + \cos x}{2} \qquad (ii) \sin^2 \frac{x}{2} \equiv \frac{1 - \cos x}{2}$$

**MEB)** Given that  $\cos \theta = 0.6$  and that  $\theta$  is acute, write down the values of:

$$(i) \cos \frac{\theta}{2} \qquad (ii) \sin \frac{\theta}{2} \qquad (iii) \tan \frac{\theta}{2}$$

**MEC)** Show that  $\cos^4 \frac{A}{2} \equiv \frac{1}{8}(3 + 4\cos A + \cos 2A)$

### Past paper work

Do **C3 January 2009** available on the VLE then mark it using the mark scheme only after you have completed it in timed conditions. You may want to try doing in reverse order. Don't forget to redo the **C3 paper in the mock exam** if you did not achieve your AS grade