

A2 Assignment Mu Cover Sheet

Name:

Question	Done	BP	Ready	Topic	Comment
Drill	Aa			C4 Integration – Repeated linear factors	$-\frac{3}{(x-2)} - \frac{3}{(x-2)^2} + c$
	Ab			C4 Integration – cos <sup>2</sup> conversion	$x + \sin x + c$
	Ac			C4 Integration – sin <sup>2</sup> conversion	$\frac{3}{2}x - \frac{3}{16}\sin 8x + c$
	Ba			C4 Integration – R method x solve	1.17 <sup>c</sup> , 3.26 <sup>c</sup>
	Bb			C4 Integration – R method 2x solve	1.13 <sup>c</sup> , 3.18 <sup>c</sup> , 4.28 <sup>c</sup>
	Bc			C4 Integration – R method x/2 solve	0 <sup>c</sup> , π <sup>c</sup>
	Ca			C3 Num Meth – confirm root in interval	Use a change of sign argument. Remember to say that $f(x)$ is continuous on the interval!
	Cb			C3 Num Meth – confirm root in interval	Use a change of sign argument. Remember to say that $f(x)$ is continuous on the interval!
	Cc			C3 Num Meth – confirm root in interval	Use a change of sign argument. Remember to say that $f(x)$ is continuous on the interval!
	Da			C3 MOD – sketch & define range	$f(x) \geq 0$
	Db			C3 MOD – sketch & define range	$f(x) \geq -2$
	Dc			C3 MOD – sketch & define range	$f(x) \neq 2$
Tracking Test 3	TT3Ai			R, α transformations	$R = 25$
	TT3Aii			R, α transformations	-25
	TT3Aiii			R, α transformations	1.85
	TT3Aiv			R, α transformations	3.84, 6.16
	TT3B			Implicit differentiation	$9y + 7x = 23$
Current work	1a			C4 Integration – sin(3x+1)	$\frac{1}{3}\cos(3x + 1) + c$
	1b			C4 Integration – cos(x/2)	$8\sin\left(\frac{x}{2}\right) + c$
	1c			C4 Integration – tanx	$-\ln(\cos x) + c$
	1d			C4 Integration – cot4x	$\frac{1}{4}\ln(\sin 4x) + c$
	1e			C4 Integration – trig reverse chain	$\frac{1}{10}\sec^5 2x + c$
	1f			C4 Integration – tan5x	$-\frac{1}{5}\ln(\cos 5x) + c$
	1g			C4 Integration – sin <sup>2</sup> conversion	$\frac{1}{2}x - \frac{1}{24}\sin 12x + c$

	1h			C4 Integration – cos <sup>2</sup> conversion	$\frac{3}{2}x + \frac{3}{8}\sin 4x + c$
	1i			C4 Integration – tan <sup>2</sup> conversion	$\frac{3}{4}\tan 4x - 3x + c$
	1j			C4 Integration – trig reverse chain	$\frac{1}{6}\tan^6 y + c$
	1k			C4 Integration – cosec <sup>3</sup> u cot <sup>3</sup> u	$-\frac{1}{3}\operatorname{cosec} 3u + c$
	1l			C4 Integration – reverse chain	$\frac{2}{21}(3x^2 + 1)^7 + c$
	1m			C4 Integration – trig fraction	$\frac{1}{3}\ln(2 + \tan 3x) + c$
	1n			C4 Integration – partial fractions	$\ln x-3  - 2\ln x-2  + c$
	2			C4 Integration – definite case 3 partial fract	Proof
	3a			C3 Trig Proof	Proof
	3b			Hence C4 Integral	$\frac{\pi}{3} + \frac{1}{4}$
	4a			C3 Trig Proof	Proof
	4b			Hence C4 Integral	0
	5			C4 Integral – definite trig	$\frac{\sqrt{3}}{4} + \frac{1}{3}$
	6a			C4 integral – definite sin <sup>2</sup> conversion	$\frac{\pi}{2} - 1$
	6b			C4 Integral – definite cos <sup>2</sup> conversion	$\frac{3\pi}{2} + \frac{9\sqrt{3}}{4}$
	7a			C4 Binomial – expand 2/root(2-3x)	$\sqrt{2} + \left(\frac{3\sqrt{2}x}{4}\right) + \left(\frac{27\sqrt{2}x^2}{32}\right) + \left(\frac{135\sqrt{2}x^3}{128}\right) + \dots$
	7b			C4 Binomial – state validity	Valid $ x  < 2/3$
	7c			C4 Binomial – estimate 2/root170	0.1534
Consolidation	8a			M2 Kinematics – find velocity given displace	56ms <sup>-1</sup>
	8b			M2 Kinematics – find acc given displacement	28 ms <sup>-2</sup>
	9a			M2 Kinematics – find position given velocity	29m
	9b			M2 Kinematics – find accel given velocity	-6 ms <sup>-2</sup>



$\alpha$	$\beta$	$\gamma$	$\delta$	$\varepsilon$	$\zeta$	$\eta$	$\theta$	$\iota$	$\kappa$	$\lambda$	$\mu$	$\nu$	$\xi$	$\omicron$	$\pi$	$\rho$	$\sigma$	$\tau$	$\upsilon$	$\varphi$	$\chi$	$\psi$	$\omega$
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“One of the most endearing things about mathematicians is the extent to which they will go to avoid doing any work.”  
M Pardage

## A2 Maths with Mechanics Assignment $\mu$ (mu)

### 12/12

#### Drill

**Part A** Integrate the following functions

a)  $\int \frac{3x}{(x-2)^3} dx$

(b)  $\int 2 \cos^2 \frac{x}{2} dx$

(c)  $\int 3 \sin^2 4x dx$

**Part B** Write in form indicated and hence solve for  $x$  (in radians to 2dp) in the interval  $[0, 2\pi]$ :

(a)  $4 \sin x - 3 \cos x = 2.5$       (b)  $5 \cos 2x + 12 \sin 2x = 6$       (c)  $\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) = 1$   
 $R \sin(x - \alpha)$                        $R \cos(2x - \alpha)$                        $R \cos\left(\frac{x}{2} - \alpha\right)$

**Part C** By first defining  $f(x)$  show that each of the following functions has a root on the interval given:

(a)  $x^3 + 6x^2 = 9x - 2$  on  $(0, 0.5)$       (b)  $1 + x^2 = x^3$  on  $(1, 2)$       (c)  $x^2 = \sin x$  on  $(0.5, 1)$

**Part D** For each of the following functions, with domain  $x \in \mathbb{R}$ , sketch its graph and state its range: mark any asymptotes

(a)  $f(x) = |3x - 2|$                       (b)  $f(x) = |x| - 2$                       (c)  $f(x) = 2 + \frac{3}{x}$

#### Focus from Tracking Test 3

**TT3A**  $f(x) \equiv 7 \cos x - 24 \sin x$

Given that  $f(x) \equiv R \cos(x + \alpha)$ , where  $R \geq 0, 0 \leq \alpha \leq \frac{\pi}{2}$ , and  $x$  and  $\alpha$  are measured in radians,

i) find  $R$  and show that  $\alpha = 1.29$  to 2 decimal places.

Hence write down

ii) the minimum value of  $f(x)$ ,

iii) the value of  $x$  in the interval  $0 \leq x \leq 2\pi$  which gives this minimum value.

iv) Find the smallest two positive values of  $x$  for which  $7 \cos x - 24 \sin x = 10$

**TT3B** Find the equation of the tangent to the curve with implicit equation  $x^2 + 3xy^2 - y^3 = 9$  at the point  $(2, 1)$ .

## Current work

Learning formulae \* you will need these in C4 integration

Complete the following:

$\sin(A+B) \equiv$
$\sin(A-B) \equiv$
$\cos(A+B) \equiv$
$\cos(A-B) \equiv$
$\tan(A+B) \equiv$
$\tan(A-B) \equiv$
$\sin 2A \equiv$
$\cos 2A \equiv$
$\text{or} \equiv$
$\text{or} \equiv$
$\tan 2A \equiv$
$\sin^2 x + \cos^2 x =$
$1 + \cot^2 x \equiv$
$\tan^2 x + 1 \equiv$
$*2 \sin A \cos B \equiv$
$*2 \cos A \sin B \equiv$
$*2 \cos A \cos B \equiv$
$*2 \sin A \sin B \equiv$

\*factor formulae express as sum or difference of sines and cosines

Complete the table: you will feel more confident about integration if you learn these

<b>f(x)</b>	<b><math>\int f(x)</math></b>	<b>f(x)</b>	<b><math>\int f(x)</math></b>
<b><math>x^n</math></b>		<b>cosecx</b>	
<b><math>e^x</math></b>		<b>secx</b>	
<b>1/x</b>		<b>cotx</b>	
<b>sinx</b>		<b><math>\sec^2 x</math></b>	
<b>cosx</b>		<b><math>-\text{cosecxcotx}</math></b>	
<b>tanx</b>		<b>secxtanx</b>	
		<b><math>-\text{cosec}^2 x</math></b>	

1. Integrate the following functions using an appropriate method when required

- (a)  $\int -\sin(3x+1) dx$       (b)  $\int 4 \cos\left(\frac{x}{2}\right) dx$       (c)  $\int \tan x dx$
- (d)  $\int \cot 4x dx$       (e)  $\int \sec^5 2x \tan 2x dx$       (f)  $\int \tan 5x dx$
- (g)  $\int \sin^2 6x dx$       (h)  $\int 3 \cos^2 2x dx$       (i)  $\int 3 \tan^2 4x dx$
- (j)  $\int \sec^2 y \tan^5 y dy$       (k)  $\int \text{cosec} 3u \cot 3u du$       (l)  $\int 4x(3x^2+1)^6 dt$

$$(m) \int \frac{\sec^2 3x}{2 + \tan 3x} dy \qquad (n) \int \frac{4-x}{(x-2)(x-3)} dx$$

2. Show that  $\int_0^2 \frac{x^2 + 6x + 7}{(x+2)(x+3)} dx = 2 + \ln\left(\frac{25}{18}\right)$

**For the following definite integrations with limits give an exact answer in terms of  $\pi$ .**

3. a) Show that  $\sin^2 x + 3\cos^2 x \equiv 2 + \cos 2x$ .

b) Hence evaluate \* check using your calculator to see if you're right !!! \*

$$\int_{\pi/12}^{\pi/4} (\sin^2 x + 3\cos^2 x) dx$$

4. a) Show that  $\frac{4\cos 2x}{\sin^2 2x} \equiv \sec^2 x - \csc^2 x$ .

b) Hence evaluate  $\int_{\pi/6}^{\pi/3} \frac{4\cos 2x}{\sin^2 2x} dx$  \* check using your calculator to see if you're right !!! \*

5. Evaluate  $\int_0^{\pi/6} (\sin 3x + \cos 2x) dx$  \* check using your calculator to see if you're right !!! \*

6. Evaluate and check using your calculator to see if you're right !!!

$$(a) \int_0^{\pi} \sin^2 \frac{1}{4} x dx \qquad (b) \int_0^{\pi} 3 \cos^2 \left(\frac{x}{6}\right) dx$$

7. a) Expand, in ascending powers of  $x$  up to  $x^3$ ,  $\frac{2}{\sqrt{2-3x}}$

b) State the validity of your expansion

c) Use your expansion to estimate  $\frac{2}{\sqrt{170}}$  correct to 4.d.p (check this on your calculator !!)

## Mechanics

Use differentiation and integration for questions where the acceleration is not constant

8 A particle P moves in a straight line such that at time  $t$  its displacement from a fixed point O is given by  $x = 4t^3 + 2t^2$ .

(a) Find the velocity when  $t = 2$

(b) Find the acceleration when  $t = 1$

9 A particle P is moving along the  $x$ -axis with velocity  $v = (18t - 6t^2) \text{ ms}^{-1}$ .

When  $t = 0$ , P is at  $x = 2\text{m}$ .

(a) Find the position of P when  $t = 3$ .

(b) Find the acceleration when  $t = 2$ .

(c) Find the maximum velocity.

(d) Find the distance OP when the velocity is a maximum.

10 Given that the acceleration vector of an object is  $\mathbf{a} = -0.2\mathbf{j} \text{ ms}^{-2}$ , with initial velocity  $0.5\mathbf{i} \text{ ms}^{-1}$ , and that the object starts at the origin, find:



- (a) The velocity at time  $t = 10$
- (b) The speed when  $t = 6$
- (c) The position vector at  $t = 2$
- (d) The displacement from the origin at  $t = 4$

11 A particle is projected with a speed of  $21 \text{ m s}^{-1}$  at an angle of elevation  $60^\circ$ . Find its

- i) speed
- ii) and direction of motion after

- a) 1 second
- b) 2 seconds

12. *For these questions decide which of the responses given is (are) correct then choose*

- A if 1, 2 and 3 are correct*
- B if only 1 and 2 are correct*
- C if only 2 and 3 are correct*
- D if only 1 is correct*
- E if only 3 is correct*

a)

$$x = 1 + \cos^2 \theta - \cos^4 \theta$$

$$y = 1 + \sin^2 \theta - \sin^4 \theta$$

1.  $x - y = 0$

2.  $x + y = 2 \cos^2 \theta \sin^2 \theta$

3.  $x = 1 + \frac{1}{2} \sin^2 2\theta$

b)

$$f(x) = e^{\sin x}$$

1.  $f(0) = 1$

2.  $f'(0) = 1$

3.  $f''(0) = 1$