

A2 Assignment lambda Cover Sheet

Name:

Question	Done	BP	Ready	Topic	Comment
Drill	Aa			C4 Integration	$\frac{2}{3} \sin \frac{3}{2}x + c$
	Ab			C4 Integration	$\tan x + c$
	Ac			C4 Integration	$-\frac{1}{3} \cot 3x + c$
	Ba			C3 Modulus solves	$\frac{1}{6}$ and $\frac{7}{4}$
	Bb			C3 Modulus solves	$-3, 2$
	Bc			C3 Modulus solves	$-\frac{5}{8}, \frac{5}{2}$
	Bd			C3 Modulus solves	$\frac{1}{4}, 3$
	Be			C3 Modulus solves	$\pm 1, \pm 4$
	Ca			C4 Binomial Expansion (simple)	$\frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \dots,  x  < 2$
	Cb			C4 Binomial Expansion (simple)	$\frac{1}{27} + \frac{1}{27}x + \frac{2}{81}x^2 + \frac{10}{729}x^3 + \dots  x  < 3$
	Cc			C4 Binomial Expansion (simple)	$2 - 2x - 2x^2 - \frac{10}{3}x^3 + \dots,  x  < \frac{1}{3}$
	Cd			C4 Binomial Expansion (simple)	$\frac{1}{2} - \frac{3}{8}x + \frac{27}{64}x^2 - \frac{135}{256}x^3 + \dots,  x  < \frac{2}{3}$
	Da			C4 Partial Fractions	$\frac{1}{x+2} + \frac{1}{x+3}$
	Db			C4 Partial Fractions	$\frac{4}{x} - \frac{2}{x^2} + \frac{3}{x+1}$
	Dc			C4 Partial Fractions	$\frac{5}{x+2} - \frac{4}{x+3}$
Current work	1a			M2 Projectiles – find height given horizontal dist	4.4m
	1b			M2 Projectiles – find speed after 2 sec	48 ms <sup>-1</sup>
	1c			M2 Projectiles – times when moving tan <sup>-1</sup> (1/4)	0.20s and 2.7s
	2a			M2 Projectiles – time ball above 2.5m	0.22 < t < 1.8
	2b			M2 Projectiles – closest distance to catch at 2.5m	44m
	3a			M2 Projectiles – Greatest height	54m
	3b			M2 Projectiles – Furthest distance	43m
Consolidation	4a			C4 Integration	$\frac{3}{4}e^{4x+2} + c$
	4b			C4 Integration	$-4e^{4-x} + 2x + c$
	4c			C4 Integration	
	5a			C3 Trig exact values given known values	tan A = 4/3
	5b			C3 Trig exact values given known values	sin B = $\frac{\sqrt{5}}{3}$
	5c			C3 Trig exact values given known values	cos(A+B) = $\frac{2}{15}(3 - 2\sqrt{5})$

	5d			C3 Trig exact values given known values	$\sin(A+B) = \frac{1}{15}(8 + 3\sqrt{5})$
	6			C3 Differentiation - tangent	$y = 9x \ln 3 - 18 \ln 3 + 9$
	7			C3 Differentiation – max value	Proof
	8a			C3 Numerical methods	Proof
	8b			C3 Numerical methods	1.58, 1.68, 1.70
	8c			C3 Show root correct	change of sign on $f(1.695)$ and $f(1.705)$
	9a			C3 Rcos	$5 \cos(\theta - 0.927)$
	9bi			C3 Rcos range	$-4 \leq f(\theta) \leq 6$
	9bii			C3 Rcos solve	$\theta = 1.15, 2.92$ (2dp)
	9c			C3 Rcos turning point	$(0.93, 2/5)$ and $(4.07, -2/5)$
	10a			C4 partial fractions	$\frac{1}{1-x} - \frac{3}{1-2x}$
	10b			C4 Binomial expansion	$-2 - 5x - 11x^2 - 23x^3$ $+ \dots, \text{ valid for }  x  < \frac{1}{2}$
Mock Exam	A(i)			Trig	$\frac{2p}{1+p^2}$
	A(ii)			Trig	$\frac{1-p^2}{1+p^2}$
	A(iii)			Trig	$\frac{2p}{1-p^2}$
	B			Trig	R.H.S. $= \frac{1 - \tan^2\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)} = \frac{1 - \tan^2\left(\frac{\theta}{2}\right)}{\sec^2\left(\frac{\theta}{2}\right)}$ $= \cos^2\left(\frac{\theta}{2}\right) \left\{1 - \tan^2\left(\frac{\theta}{2}\right)\right\}$ $= \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) = \cos \theta = \text{L.H.S}$
	C			Trig	R.H.S $= \frac{2 \tan\left(\frac{\theta}{2}\right)}{\sec^2\left(\frac{\theta}{2}\right)} = 2 \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} \times \frac{\cos^2\left(\frac{\theta}{2}\right)}{1}$ $= 2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) = \sin \theta$

$\alpha$	$\beta$	$\gamma$	$\delta$	$\varepsilon$	$\zeta$	$\eta$	$\theta$	$\iota$	$\kappa$	$\lambda$	$\mu$	$\nu$	$\xi$	$\omicron$	$\pi$	$\rho$	$\sigma$	$\tau$	$\upsilon$	$\phi$	$\chi$	$\psi$	$\omega$
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“The mathematician has reached the highest rung on the ladder of human thought”

H Ellis

## A2 Maths with Mechanics Assignment $\lambda$ (lambda)

### Drill

**Part A** Integrate with respect to  $x$ : use the correct notation

(a)  $\cos \frac{3x}{2}$                                       (b)  $\sec^2 x$                                       (c)  $\operatorname{cosec}^2 3x$

**Part B** Solve the following equations: graphically or otherwise

(a)  $|5x - 4| = |x + 3|$                               (b)  $|x^2 + x| = 6$                                       (c)  $|3x + 5| = |5x|$   
 (d)  $|6x - 7| = |2x + 5|$                               (e)  $|x^2 - 4| = 3|x|$

**Part C** Expand each of the following in ascending powers of  $x$  up to and including the term in  $x^3$  and state the set of values of  $x$  for which each expression is valid.

(a)  $(2 + x)^{-1}$                               (b)  $(3 - x)^{-3}$                                       (c)  $(8 - 24x)^{\frac{1}{2}}$                                       (d)  $(4 + 6x)^{-\frac{1}{2}}$

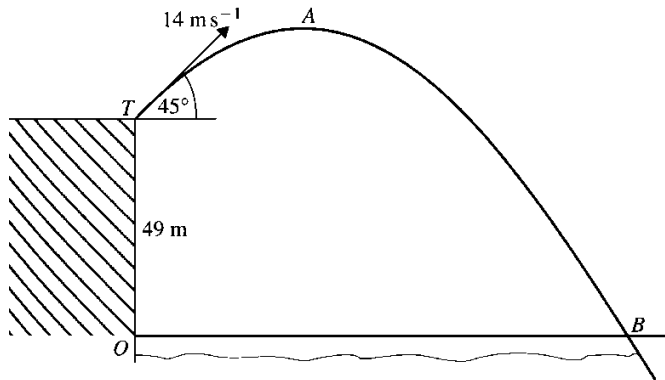
**Part D** Express as partial fractions:

(a)  $\frac{2x+5}{(x+2)(x+3)}$                               (b)  $\frac{7x^2+2x-2}{x^2(x+1)}$                                       (c)  $\frac{x+7}{x^2+5x+6}$

### Current work M2

- A particle P is projected from a point O on level ground with speed  $50 \text{ ms}^{-1}$  at an angle  $\theta$  where  $\sin \theta = \left(\frac{7}{25}\right)$  above the horizontal. Find
  - the height of P at the point where its horizontal displacement from O is 120 m,
  - the speed of P two seconds after projection,
  - the times after projection at which P is moving at an angle of  $\tan^{-1}\left(\frac{1}{4}\right)$  to the ground
- A cricket ball, which may be modelled as a particle moving freely under gravity, is struck from a height of 0.5m above a horizontal field with a velocity of  $26 \text{ ms}^{-1}$  at an angle  $\alpha$  above the horizontal, where  $\tan \alpha = \frac{5}{12}$ .
  - The fielders can reach up to a height of 2.5m. Between what times is the ball out of reach of the fielders?
  - The captain wishes his fielders to catch the ball as soon as it is within reach. How far from the bat should the fielders be placed in order to do this?

3.



A golf ball is struck from the point  $T$ , at the top of a cliff 49 m above sea level, with a speed of  $14 \text{ m s}^{-1}$  at an angle of  $45^\circ$  to the horizontal, as shown in the diagram. The point  $O$  is at sea level and vertically below  $T$ . The point  $A$  is the highest point reached by the ball in its motion. The ball strikes the sea at the point  $B$ .

- (a) Find the height  $A$  above sea level.  
 (b) Find the distance  $OB$ .

4. Integrate the following w.r.t.  $x$ :

(a)  $\int 3e^{4x+2} dx$       (b)  $\int (4e^{4-x} + 2) dx$

(c)  $\int \frac{e^{2x} + 1}{4e^{-x}} dx$  hint: split into  $\frac{e^{2x}}{4e^{-x}} + \frac{1}{4e^{-x}}$

5. Given that  $\sin A = \frac{4}{5}$ ,  $0 < A < 90^\circ$  and that  $\cos B = \frac{2}{3}$ ,  $0 < B < 90^\circ$ , find without using a calculator the value of

- (a)  $\tan A$       (b)  $\sin B$       (c)  $\cos(A+B)$       (d)  $\sin(A+B)$

6. A curve has the equation  $y = 3^x$ .

Find an equation for the tangent to the curve at the point  $(2, 9)$

7. Show that the curve with equation  $y = \frac{\ln x}{x}$  has a maximum value of  $\frac{1}{e}$  at  $x = e$ .8.  $f(x) = x^3 + x^2 - 4x - 1$ . The equation  $f(x) = 0$  has only one positive root,  $\alpha$ .

(a) Show that  $f(x) = 0$  can be rearranged as  $x = \sqrt{\left(\frac{4x+1}{x+1}\right)}$ ,  $x \neq -1$ .

The iterative formula  $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$  is used to find an approximation to  $\alpha$ .

- (b) Taking  $x_1 = 1$ , find, to 2 decimal places, the values of  $x_2$ ,  $x_3$  and  $x_4$ .  
 (c) Prove that  $\alpha = 1.70$ , is correct to 2dp.

9. (a) Express  $3 \cos \theta + 4 \sin \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$
- (b) Given that the function  $f$  is defined by  
 $f(\theta) \equiv 1 - 3 \cos 2\theta - 4 \sin 2\theta$ ,  $\theta \in \mathbb{R}$ ,  $0 \leq \theta \leq \pi$   
 i) state the range of  $f$ ,  
 ii) solve the equation  $f(\theta) = 0$
- (c) Find the coordinates of the turning points of the curve with equation  
 $y = \frac{2}{3 \cos x + 4 \sin x}$  in the interval  $[0, 2\pi]$ .
10. (a) Express  $\frac{x-2}{(1-x)(1-2x)}$  in partial fractions.
- (b) Hence find the series expansion of  $\frac{x-2}{(1-x)(1-2x)}$  in ascending powers of  $x$  up to and including the term in  $x^3$  and state the set of values of  $x$  for which the expression is valid.

### Mechanics challenge question – give it a try!

Karen is standing 4 m away from a wall which is 2.5 m high. She throws a ball at  $10\text{ms}^{-1}$  at an angle of  $40^\circ$  to the horizontal, releasing the ball from a height of 1 m above the ground. Will the ball pass over the wall? Fully justify your answer.

### Optional extra questions for you if you are catching up on work from the C3 mock exam

A) Given  $\tan x = p$ , find in terms of  $p$ :

- i)  $\sin 2x$
- ii)  $\cos 2x$
- iii)  $\tan 2x$

B) Show that:  $\cos \theta \equiv \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$

C) Show that  $\sin \theta \equiv \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$