

Question	Done	BP	Ready?	Topic	Answers
Drill	Aa			C3 Differentiation all methods	$e^x(\sin x + \cos x)$
	Ab			C3 Differentiation all methods	$\frac{2e^{2x}(\cos x + \sin x)}{\cos^3 x}$
	Ac			C3 Differentiation all methods	$\sin 2x \ln x + \frac{\sin^2 x}{x}$
	Ad			C3 Differentiation all methods	$\frac{\cos x}{2\sqrt{\sin x}}$
	Ae			C3 Differentiation all methods	$\cos 2x$
	Af			C3 Differentiation all methods	$\frac{1 + \sin x + \cos x}{(1 + \cos x)^2}$
	Ba			C3 Trig solves	π
	Bb			C3 Trig solves	$\frac{5\pi}{6}, \frac{11\pi}{6}$
	Bc			C3 Trig solves	$\frac{2\pi}{3}, \frac{4\pi}{3}$
	Ca			C3 Rcos	17, 5.20
	Cb			C3 Rcos	13, 0.395
	Cc			C3 Rcos	$\sqrt{10}, 1.89$
	Da			C3 Integration by inspection	$-\frac{1}{6}(2x+1)^{-3} + c$
	Db			C3 Integration by inspection	$-\frac{1}{2}(1-x)^6 + c$
	Dc			C3 Integration by inspection	$2e^{\frac{1}{2}x} + c$
	Dd			C3 Integration by inspection	$-\cos(x+1) + c$
	De			C3 Integration by inspection	$\frac{1}{2}\ln 2x-3 + c$
	Df			C3 Integration by inspection	$\frac{1}{3}\tan 3x + c$
Mock Exam	MEA			Trig	$\cos(\theta - \theta) \equiv \cos \theta \cos \theta + \sin \theta \sin \theta$ $\Rightarrow \sin^2 \theta + \cos^2 \theta \equiv 1$ as $\cos \theta = 0$
	MEBi			Trig	$\sin 35^\circ$
	MEBii			Trig	$\cos 7\theta$
	MEBiii			Trig	$\tan 5\theta$

	MECi			Differentiation	$e^{2x} (2 \cos x - \sin x)$
	MECii			Differentiation	$e^x \sec 3x (1 + 3 \tan 3x)$
	MECiii			Differentiation	$\frac{e^{\sin x} (\cos^2 x + \sin x)}{\cos^2 x}$
	MEDi			Trig	$\frac{1}{\sqrt{1+q^2}}$
	MEDii			Trig	$(1-q)/(1+q)$
	MEDiii			Trig	$(q^2-1)/(2q)$
	1			C3 Inverse trig functions	Check on google inc asymptotes
	2			C3 Inverse trig functions	$2 - \sqrt{3}$
	3a			C3 Creating an iterative formula	$a = 5, b = -1$
	3b			C3 Creating an iterative formula	$a = 5, b = -1$
	3c			C3 Creating an iterative formula	$a = \frac{1}{4}$
	3d			C3 Creating an iterative formula	$a = 3, b = -\frac{1}{3}$
	3e			C3 Creating an iterative formula	$a = 5$
	3f			C3 Creating an iterative formula	$a = \frac{1}{2}, b = -\frac{3}{2}$
	4			C3 Numerical methods – root	1.12
	5a			C3 Numerical methods	2.422
	5b			C3 Numerical methods – justify	Discuss in class
	6a			C3 Functions- domain and range	(sketches, make sure they are one to one in order to have an inverse) $0 < f(x) < 1; f^{-1}(x) = \frac{1-x}{x}$
	6b			C3 Functions- domain and range	$f(x) \geq 0; f^{-1}(x) = (x+1)^{\frac{1}{2}} - 1$
	6c			C3 Functions- domain and range	$f(x) \geq 5; f^{-1}(x) = (x-1)^{\frac{1}{2}} - 2$
	7			C3 Using dx/dy + trig ids to find dy/dx	PROOF
	8a			C3 Trig proof	PROOF
	8b			C3 Trig solve	$0, \frac{\pi}{7}, \frac{3\pi}{7}, \frac{5\pi}{7}, \pi$
	9a			C3 Trig solve	π
	9b			C3 Trig solve	$0, \pi, 2\pi$

M1 Practice	10				M1 Force diagram with friction	a) 10.8N b) 0.725
						c) $F_{\max} = 3.94\text{N}$, $6\sin 25 = 2.54\text{N}$, $2.54 < 3.94$ so the weight remains in equilibrium
Challenge					Challenge!	1/3
					C3 Past Paper	C3 Past Paper

α	β	γ	δ	ε	ζ	η	θ	i	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω
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“A linguist would be shocked to learn that if a set is not closed this does not mean that it is open, or that ‘E is dense in E’ does not mean the same thing as ‘E is dense in itself’.”

J E Littlewood

A2 Maths with Mechanics Assignment θ theta

C3 Past Paper at the end of this assignment to be completed during reading week.

Drill

Part A Differentiate with respect to x :

- (a) $e^x \sin x$ (b) $\frac{e^{2x}}{\cos^2 x}$ (c) $\sin^2 x \ln x$
 (d) $\sqrt{\sin x}$ (e) $\sin x \cos x$ (f) $\frac{1 + \sin x}{1 + \cos x}$

Part B Solve these equations for $0 \leq \theta \leq 2\pi^c$

- (a) $\sec \theta = -1$ (b) $\cot \theta = -\sqrt{3}$ (c) $\operatorname{cosec} \frac{1}{2} \theta = \frac{2\sqrt{3}}{3}$

Part C By writing each of these functions in the form given, state the greatest value of each function and the smallest positive value of x (in radians to 2dp) at which this occurs.

- (a) $8 \cos x - 15 \sin x, R \cos(x + \alpha)$ (b) $5 \sin x + 12 \cos x, R \sin(x + \alpha)$
 (c) $3 \sin x - \cos x, R \sin(x - \alpha)$

Part D Integrate the following with respect to x : by considering the reverse of differentiation

- (a) $\int (2x+1)^{-4} dx$ (b) $\int 3(1-x)^5 dx$ (c) $\int e^{\frac{x}{2}} dx$
 (d) $\int \sin(x+1) dx$ (e) $\int \frac{1}{2x-3} dx$ (f) $\int \sec^2 3x dx$

Practising your trig for C3 Mock exam w/b 20th November 17

Learning trig formula is vital unless you know the trig formula you will not recognise them in questions. Here are examples.

A) Using the expansion of $\cos(A - B)$ with $A = B = \theta$, show that $\sin^2 \theta + \cos^2 \theta \equiv 1$.

B) Express the following as a single sine, cosine or tangent:

- i) $\sin 15^\circ \cos 20^\circ + \cos 15^\circ \sin 20^\circ$
 ii) $\cos 4\theta \cos 3\theta - \sin 4\theta \sin 3\theta$

$$\text{iii) } \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta}$$

Write these out as many times as you need to remember them

Trig Identity table

$$\begin{aligned} \sin 2x &= \\ \cos 2x &= \\ \cos 2x &= \\ \cos 2x &= \\ \tan 2x &= \\ \sec^2 x &= \\ \operatorname{cosec}^2 x &= \\ \sin^2 x &= \\ \cos^2 x &= \\ \tan(A - B) &= \\ \sin(A - B) &= \\ \cos(A - B) &= \\ \sin P + \sin Q &= \\ \sin P - \sin Q &= \\ \cos P + \cos Q &= \\ \cos P - \cos Q &= \end{aligned}$$

Trig Identity table

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x \\ \cos 2x &= 2 \cos^2 x - 1 \\ \cos 2x &= 1 - 2 \sin^2 x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} \\ \sec^2 x &= 1 + \tan^2 x \\ \operatorname{cosec}^2 x &= 1 + \cot^2 x \\ \sin^2 x &= \frac{1}{2} - \frac{1}{2} \cos 2x \\ \cos^2 x &= \frac{1}{2} + \frac{1}{2} \cos 2x \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ \sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \sin P + \sin Q &= 2 \sin \left(\frac{P + Q}{2} \right) \cos \left(\frac{P - Q}{2} \right) \\ \sin P - \sin Q &= 2 \cos \left(\frac{P + Q}{2} \right) \sin \left(\frac{P - Q}{2} \right) \\ \cos P + \cos Q &= 2 \cos \left(\frac{P + Q}{2} \right) \cos \left(\frac{P - Q}{2} \right) \\ \cos P - \cos Q &= -2 \sin \left(\frac{P + Q}{2} \right) \sin \left(\frac{P - Q}{2} \right) \end{aligned}$$

Practising the product and quotient rules for differentiation

C) Find the function $f'(x)$ where $f(x)$ is

i) $e^{2x} \cos x$

ii) $e^x \sec 3x$

iii) $\frac{e^{\sin x}}{\cos x}$

D) A tricky question!

Given $\cot x = q$ find in terms of q i) $\sin x$ ii) $\tan(x - 45)$ iii) $\cot 2x$

hint: Make a triangle add sides q and 1 using $\cot x = q$ and hence find $\sin x$, $\cos x$ and $\tan x$ in terms of q first.

- Sketch the graph of $y = \arcsin x$, $y = \arccos x$, $y = \arctan x$ labelling your axes and axes crossing points clearly

2. Given that $\arctan(x-2) = -\frac{\pi}{3}$, find the value of x
3. Show the steps by which the following iterative formulae can be derived from the given equations. State the values of the constants a and b in each case:

<u>Equation to be solved</u>	<u>Iterative formula</u>
(a) $x^2 - 5x + 1 = 0$	$x_{n+1} = \sqrt{ax_n + b}$
(b) $x^2 - 5x + 1 = 0$	$x_{n+1} = a + \frac{b}{x_n}$
(c) $4 \ln x = x$	$x_{n+1} = e^{ax_n}$
(d) $3e^x + x - 9 = 0$	$x_{n+1} = \ln(a + bx_n)$
(e) $x \ln x = 5$	$x_{n+1} = e^{\left(\frac{a}{x_n}\right)}$
(f) $2 \cos x + 3x - 1 = 0$	$x_{n+1} = \arccos(a + bx_n)$

4. Given that $f(x) = 5x - 4 \sin x - 2$ where x is measured in radians, show that $f(x) = 0$ has a root in the interval $(1.1, 1.15)$. Use the iterative formula $x_{n+1} = p \sin x_n + q$ (where p and q are constants to be found) and $x_0 = 1.1$ to find x_4 to 3sf.

5. The root of the equation $f(x) = 0$, where $f(x) = x + \ln 2x - 4$ is to be estimated using the iterative formula $x_{n+1} = 4 - \ln 2x_n$, with $x_0 = 2.4$.

- (a) Showing your values of x_1, x_2, x_3, \dots , obtain the value, to 3 decimal places, of the root.
- (b) By considering the change of sign of $f(x)$ in a suitable interval, justify the accuracy of your answer to part (a).

6. For each of the following functions, whose domain is the set of **positive** real numbers, sketch the function and hence state the range. For each function find its inverse

(a) $f(x) = \frac{1}{x+1}$ (b) $f(x) = (x+1)^2 - 1$ (c) $f(x) = x^2 + 4x + 5$

7. Prove that if $x = \sec y$, then $\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$

8. Using the result that $\sin P - \sin Q = 2 \cos \frac{(P+Q)}{2} \sin \frac{(P-Q)}{2}$

(a) Show that $\sin 105^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$

(b) Solve, for $0 \leq \theta \leq \pi$, $\sin 4\theta - \sin 3\theta = 0$

9. Solve the following equations on the interval $0 \leq x \leq 2\pi$. Give **exact** answers:

(a) $\cos x(\sin^2 x - 1) = 1$

(b) $\cot\left(x + \frac{\pi}{6}\right) = \sqrt{3}$

M1 Practice (Preparation for M2)

10. A weight of 6N rests on a rough 25° incline. The perpendicular reaction is measured to be 10N. A horizontal force H pushes the weight so that it is just on the point of slipping up the plane.

- a) Complete a force diagram and find force H
- b) Find μ , the coefficient of friction.

Force H is now removed

- c) Showing all your calculations clearly, justify whether the 6N weight will slide down the plane, or remain in equilibrium.

Challenge Question - have a go!

Triangle ABC has $\hat{B}C = 90$ and $\hat{C}B = 30$. If a point inside the triangle is chosen at random, what is the probability it is nearer AB than it is to AC?

PAST PAPER NEXT PAGE

Complete this past paper (C3 June 2006) in 1 hour 30mins

If it takes longer draw a line under where you got to in the time allowed and continue.

- Do it under exam conditions.
- Do not use the mark scheme until you have done the whole paper
- Mark it yourself using the mark scheme on the VLE and write down your % on the paper.
- Do your corrections in another colour.
- Hand it to your teacher with your assignment

1. (a) Simplify $\frac{3x^2 - x - 2}{x^2 - 1}$. (3)

(b) Hence, or otherwise, express $\frac{3x^2 - x - 2}{x^2 - 1} - \frac{1}{x(x+1)}$ as a single fraction in its simplest form. (3)

2. Differentiate, with respect to x ,

(a) $e^{3x} + \ln 2x$, (3)

(b) $(5 + x^2)^{\frac{3}{2}}$. (3)

3.

Figure 1

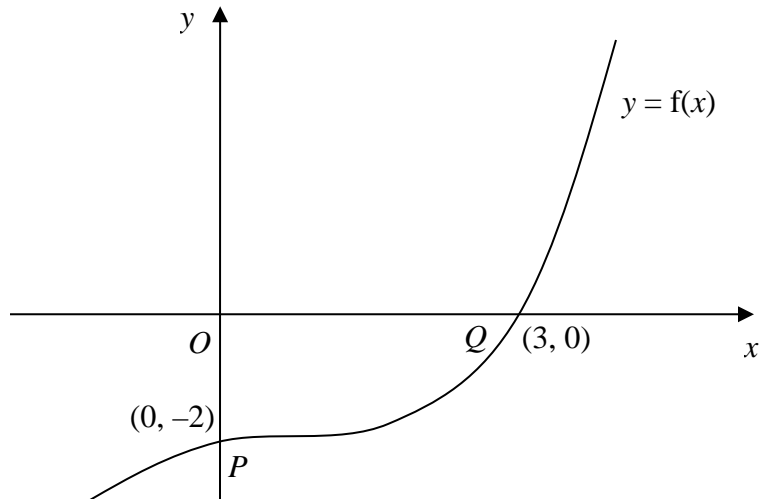


Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x . The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$, (3)

(c) $y = \frac{1}{2}f(3x)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400e^{-0.05t} + 25, \quad t \geq 0.$$

- (a) Find the temperature of the ball as it enters the liquid. (1)
- (b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures. (4)
- (c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures. (3)
- (d) From the equation for temperature T in terms of t , given above, explain why the temperature of the ball can never fall to 20 °C. (1)
-

5.

Figure 2

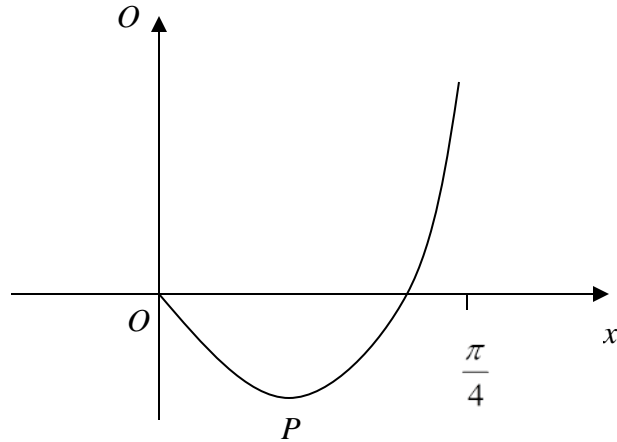


Figure 2 shows part of the curve with equation

$$y = (2x - 1) \tan 2x, \quad 0 \leq x < \frac{\pi}{4}.$$

The curve has a minimum at the point P . The x -coordinate of P is k .

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k .

(b) Calculate the values of x_1 , x_2 , x_3 and x_4 , giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)

6. (a) Using $\sin^2 \theta + \cos^2 \theta \equiv 1$, show that the $\operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$. (2)

(b) Hence, or otherwise, prove that

$$\operatorname{cosec}^4 \theta - \cot^4 \theta \equiv \operatorname{cosec}^2 \theta + \cot^2 \theta. \quad (2)$$

(c) Solve, for $90^\circ < \theta < 180^\circ$,

$$\operatorname{cosec}^4 \theta - \cot^4 \theta = 2 - \cot \theta. \quad (6)$$

7. For the constant k , where $k > 1$, the functions f and g are defined by

$$f: x \mapsto \ln(x+k), \quad x > -k,$$

$$g: x \mapsto |2x-k|, \quad x \in \mathbb{R}.$$

(a) On separate axes, sketch the graph of f and the graph of g .

On each sketch state, in terms of k , the coordinates of points where the graph meets the coordinate axes. (5)

(b) Write down the range of f . (1)

(c) Find $fg\left(\frac{k}{4}\right)$ in terms of k , giving your answer in its simplest form. (2)

The curve C has equation $y = f(x)$. The tangent to C at the point with x -coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k . (4)

8. (a) Given that $\cos A = \frac{3}{4}$, where $270^\circ < A < 360^\circ$, find the exact value of $\sin 2A$. (5)

(b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$. (3)

Given that

$$y = 3 \sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

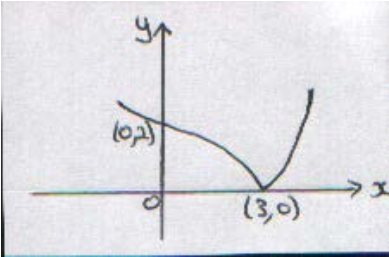
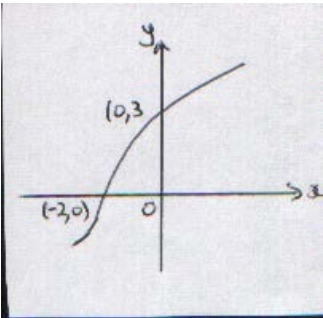
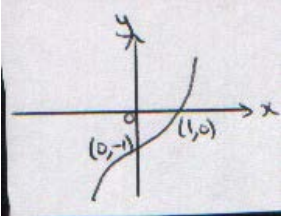
(ii) show that $\frac{dy}{dx} = \sin 2x$. (4)

TOTAL FOR PAPER: 75 MARKS

END

Finished and done all the questions?

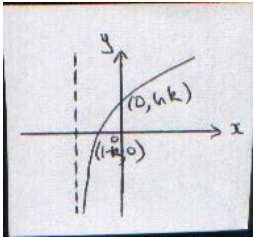
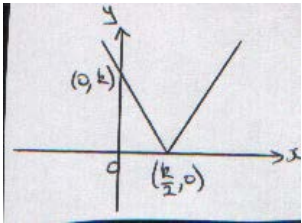
Now mark it using the **MARK SCHEME** on the next page. **ONLY** look at this once you have completed the full 1 hour 30 mins exam

Question Number	Scheme	Marks
3. (a)	 <p data-bbox="841 279 1198 317">Mod graph, reflect for $y < 0$</p> <p data-bbox="841 373 1175 443">(0, 2), (3, 0) or marked on axes</p> <p data-bbox="841 499 1221 537">Correct shape, including cusp</p>	<p data-bbox="1252 279 1300 310">M1</p> <p data-bbox="1252 390 1295 422">A1</p> <p data-bbox="1252 499 1446 537">A1 (3)</p>
3. (b)	 <p data-bbox="841 642 1214 680">Attempt at reflection in $y = x$</p> <p data-bbox="841 810 1068 848">Curvature correct</p> <p data-bbox="841 978 1156 1016">-2, 0), (0, 3) or equiv.</p>	<p data-bbox="1252 642 1300 674">M1</p> <p data-bbox="1252 810 1295 842">A1</p> <p data-bbox="1252 978 1446 1016">B1 (3)</p>
3. (c)	 <p data-bbox="841 1146 1117 1184">Attempt at 'stretches'</p> <p data-bbox="841 1325 1052 1362">(0, -1) or equiv.</p> <p data-bbox="841 1503 922 1541">(1, 0)</p>	<p data-bbox="1252 1146 1300 1178">M1</p> <p data-bbox="1252 1304 1295 1335">B1</p> <p data-bbox="1252 1503 1446 1541">B1 (3)</p>
		Total 9 marks

Question Number	Scheme	Marks
<p>4.</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>425 °C</p> <p>$300 = 400e^{-0.05t} + 25 \Rightarrow 400e^{-0.05t} = 275$</p> <p>sub. $T = 300$ and attempt to rearrange to $e^{-0.05t} = a$, where $a \in \mathbb{Q}$</p> <p>$e^{-0.05t} = \frac{275}{400}$</p> <p>M1 correct application of logs</p> <p>$t = 7.49$</p> <p>$\frac{dT}{dt} = -20 e^{-0.05 t}$ (M1 for $ke^{-0.05 t}$)</p> <p>At $t = 50$, rate of decrease = $(\pm) 1.64$ °C/min</p> <p>$T > 25$, (since $e^{-0.05 t} \rightarrow 0$ as $t \rightarrow \infty$)</p>	<p>B1 (1)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>B1 (1)</p> <p>Total 9 marks</p>

Question Number	Scheme	Marks
5. (a)	<p>Using product rule: $\frac{dy}{dx} = 2 \tan 2x + 2(2x - 1) \sec^2 2x$</p> <p>Use of “$\tan 2x = \frac{\sin 2x}{\cos 2x}$” and “$\sec 2x = \frac{1}{\cos 2x}$” [$= 2 \frac{\sin 2x}{\cos 2x} + 2(2x - 1) \frac{1}{\cos^2 2x}$]</p> <p>Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions [$\Rightarrow 2 \sin 2x \cos 2x + 2(2x - 1) = 0$]</p> <p>Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. AG</p>	<p>M1 A1 A1</p> <p>M1</p> <p>M1</p> <p>A1* (6)</p>
(b)	<p>$x_1 = 0.2670, x_2 = 0.2809, x_3 = 0.2746, x_4 = 0.2774,$</p> <p>Note: M1 for first correct application, first A1 for two correct, second A1 for all four correct Max -1 deduction, if ALL correct to > 4 d.p. M1 A0 A1 SC: degree mode: M1 $x_1 = 0.4948$, A1 for $x_2 = 0.4914$, then A0; max 2</p>	<p>M1 A1 A1 (3)</p>
(c)	<p>Choose suitable interval for k: e.g. [0.2765, 0.2775] and evaluate $f(x)$ at these values</p> <p>Show that $4k + \sin 4k - 2$ changes sign and deduction [$f(0.2765) = -0.000087\dots, f(0.2775) = +0.0057$]</p> <p>Note: Continued iteration: (no marks in degree mode) Some evidence of further iterations leading to 0.2765 or better M1; Deduction A1</p>	<p>M1</p> <p>A1 (2)</p> <p>(11 marks)</p>

Question Number	Scheme	Marks
6. (a)	Dividing $\sin^2 \theta + \cos^2 \theta \equiv 1$ by $\sin^2 \theta$ to give $\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} \equiv \frac{1}{\sin^2 \theta}$ Completion: $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta \Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$ AG	M1 A1* (2)
(b)	$\operatorname{cosec}^4 \theta - \cot^4 \theta \equiv (\operatorname{cosec}^2 \theta - \cot^2 \theta)(\operatorname{cosec}^2 \theta + \cot^2 \theta)$ $\equiv (\operatorname{cosec}^2 \theta + \cot^2 \theta) \quad \text{using (a)} \quad \text{AG}$ <p>Notes: (i) Using LHS = $(1 + \cot^2 \theta)^2 - \cot^4 \theta$, using (a) & elim. $\cot^4 \theta$ M1, conclusion {using (a) again} A1*</p> (ii) Conversion to sines and cosines: needs $\frac{(1 - \cos^2 \theta)(1 + \cos^2 \theta)}{\sin^4 \theta}$ for M1	M1 A1* (2)
(c)	Using (b) to form $\operatorname{cosec}^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta$ Forming quadratic in $\cot \theta$ $\Rightarrow 1 + \cot^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta \quad \{\text{using (a)}\}$ $2\cot^2 \theta + \cot \theta - 1 = 0$ Solving: $(2\cot \theta - 1)(\cot \theta + 1) = 0$ to $\cot \theta =$ $\left(\cot \theta = \frac{1}{2} \right) \quad \text{or} \quad \cot \theta = -1$ $\theta = 135^\circ \quad (\text{or correct value(s) for candidate dep. on 3Ms})$ <p>Note: Ignore solutions outside range Extra “solutions” in range loses A1√, but candidate may possibly have more than one “correct” solution.</p>	M1 M1 A1 M1 A1 A1√ (6) (10 marks)

Question Number	Scheme	Marks
7. (a)	 <p>Log graph: Shape</p> <p>Intersection with -ve x-axis</p> <p>$(0, \ln k), (1 - k, 0)$</p>	B1 dB1 B1
	 <p>Mod graph :V shape, vertex on +ve x-axis</p> <p>$(0, k)$ and $(\frac{k}{2}, 0)$</p>	B1 B1 (5)
(b)	$f(x) \in \mathbb{R}, -\infty < f(x) < \infty, -\infty < y < \infty$	B1 (1)
(c)	$fg\left(\frac{k}{4}\right) = \ln\left\{k + \left \frac{2k}{4} - k\right \right\} \text{ or } f\left(\left -\frac{k}{2}\right \right)$ $= \ln\left(\frac{3k}{2}\right)$	M1 A1 (2)
(d)	$\frac{dy}{dx} = \frac{1}{x+k}$ <p>Equating (with $x = 3$) to grad. of line;</p> $\frac{1}{3+k} = \frac{2}{9}$ $k = 1\frac{1}{2}$	B1 M1; A1 A1√ (4) (12 marks)

Question Number	Scheme	Marks
<p>8. (a)</p> <p>(b)(i)</p> <p>(b)(ii)</p>	<p>Method for finding $\sin A$</p> $\sin A = -\frac{\sqrt{7}}{4}$ <p>Note: First A1 for $\frac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given)</p> <p>Use of $\sin 2A \equiv 2\sin A \cos A$</p> $\sin 2A = -\frac{3\sqrt{7}}{8} \text{ or equivalent exact}$ <p>Note: \pm f.t. Requires exact value, dependent on 2nd M</p> $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x \cos \frac{\pi}{3} - \sin 2x \sin \frac{\pi}{3} +$ $\cos 2x \cos \frac{\pi}{3} + \sin 2x \sin \frac{\pi}{3}$ $\equiv 2\cos 2x \cos \frac{\pi}{3}$ <p>[This can be just written down (using factor formulae) for M1A1] $\equiv \cos 2x$ AG</p> <p>Note: M1A1 earned, if $\equiv 2\cos 2x \cos \frac{\pi}{3}$ just written down, using factor theorem Final A1* requires some working after first result.</p> $\frac{dy}{dx} = 6\sin x \cos x - 2\sin 2x$ <p>or $6\sin x \cos x - 2\sin\left(2x + \frac{\pi}{3}\right) - 2\sin\left(2x - \frac{\pi}{3}\right)$</p> $= 3\sin 2x - 2\sin 2x$ $= \sin 2x \quad \text{AG}$ <p>Note: First B1 for $6\sin x \cos x$; second B1 for remaining term(s)</p>	<p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1√ (5)</p> <p>M1</p> <p>A1</p> <p>A1* (3)</p> <p>B1 B1</p> <p>M1</p> <p>A1* (4)</p> <p>(12 marks)</p>