A2 Assignment eta Cover Sheet **Name:**

Question		Done	Backpack	Ready?	Topic	Answers
Drill	Aa				C3 Differentiation all methods	$\frac{-1}{(x-1)^2}$
	Ab				C3 Differentiation all methods	$\frac{e^x(2x-3)}{(2x+1)^3}$
	Ac				C3 Differentiation all methods	$e^x\cos(e^x)$
	Ad				C3 Differentiation all methods	$2x\ln x+x$
	Ae				C3 Differentiation all methods	$\frac{4}{1+4x}$
	Af				C3 Differentiation all methods	$3x^2\sec^2 x^3$
	Ag				C3 Differentiation all methods	$-\sin x e^{\cos x}$
	Ah				C3 Differentiation all methods	$-6e^x(3-e^x)^5$
	Ba				C3 Modulus function	Check on google inc asymptotes
	Bb				C3 Modulus function	Check on google inc asymptotes
	Bc				C3 Modulus function	Check on google inc asymptotes
	Ca				C3 Sketch and find range	$\mathrm{f}(x)\geq 5$
	Cb				C3 Sketch and find range	$\mathrm{f}(x)\leq 0$
	Cc				C3 Sketch and find range	$-\infty<\mathrm{f}(x)<\infty$ or f(x) ε $\mathbb{R}$
	Da				C3 Integration by inspection	$-\frac{1}{2}\operatorname{cosec} 2x+c$
	Db				C3 Integration by inspection	$\frac{1}{2}\ln\lvert 2x-1\rvert+c$
	Dc				C3 Integration by inspection	$e^{x^3}+c$
Further Practice	A				C3 Algebraic Division & ln & e solve	a) Check! b) $x=\frac{3-e}{2e-1}$
	B				C3 Factor Formula Trig Proof & solve	a) Proof B) $x=0$, π/4 , π/3, 2π/3 , 3π/4
	C				C3 differentiation & area of triangle	a) (4,0) B) 1 c) $x=4e^{-\frac{2}{5}}$
	1a				C3 Rcos(x-a) max and min	Min f(x) = 7, max f(x) = 17
	1b				C3 Rcos(x-a) max and min	Min f(x) = 5, max f(x) = 9

	1c				C3 Rcos(x-a) max and min	Min f(x) = 1/7, max f(x) = 1/3
	2a				C3 Rcos(x-a) min	$R=\sqrt{2},\ \alpha=\frac{\pi}{4}$ min $-\sqrt{2},\ \theta=\frac{5\pi}{4}$
	2b				C3 Rcos(x-a) min	$R=13, \alpha=1.18$ min $-13, \theta=1.96$
	2c				C3 Rcos(x-a) min	$R=2\sqrt{3},\ \alpha=\frac{\pi}{3}$ min $-2\sqrt{3}, \theta=\frac{7\pi}{6}$
	2d				C3 Rcos(x-a) min	$R=\sqrt{58}, \alpha=\arctan\frac{7}{3}$ min $-\sqrt{58}, \theta=5.88$
	3				C3 Rcos(x-a) with solving	R = 13, α = 0.3948. x = 0.963 or 2.968
	4a				C3 Trig proof	PROOF
	4b				C3 Trig proof	PROOF
	5a				C3 Inverse	$\frac{1}{3}\left(e^{x}+6\right)$
	5b				C3 Domain and range of inverse	sketch f(x) to find range of f equals domain of f^{-1}
	5c				C3 Solve function	8.70
	6				C3 Find normal	$2y+x-2=0$
	7				C3 Minimum points	$\left(\ln 2, 2-\ln 4\right)$ min by showing second derivative is positive
	8a				C3 Trig solve	2.7, 5.8 radians
	8b				C3 Trig solve	0, π, 2π, 2π/3, 4π/3 radians
	9					7
M1 Practice	10				M1 Impulse/Momentum vectors	6.2 ms^{-1}

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	o	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"Perhaps the greatest paradox of all is that there are paradoxes in mathematics"

J Newman

A2 Maths with Mechanics Assignment η (eta)

Due in w/b 13/11

Drill

Part A Differentiate the following functions with respect to *x*:

(a) $\dfrac{x}{x-1}$ (c) $\dfrac{e^x}{(2x+1)^2}$ (c) $\sin\left(e^x\right)$ (d) $x^2 \ln x$

(e) $\ln(1+4x)$ (f) $\tan\left(x^3\right)$ (g) $e^{\cos x}$ (h) $\left(3-e^x\right)^6$

Part B For each of these functions sketch $f\left(|x|\right)$ and $|f(x)|$

(a) $y=\ln(x+1)$ (b) $y=1-e^x$ (c) $y=1-\dfrac{1}{x+2}$

Part C Sketch the following functions where each function is defined on domain, $x \in \mathbb{R}$..
State the range of each function.

(a) $f(x)=(x+1)^2+4, \quad x \geq 0$ (b) $f(x)=1-e^x, \quad x \geq 0$ (c) $f(x)=3\ln x, \quad x>0$

Part D Integrate the following functions by working out what has been differentiated:

(a) $\int \operatorname{cosec} 2x \cot 2x \, dx$ (b) $\int \dfrac{1}{2x-1} \, dx$ (c) $\int 3x^2 e^{x^3} \, dx$

Further Practice:

A)

(a) Simplify

$$\frac{x^2+7x+12}{2x^2+9x+4}.$$

(3)

(b) Solve the equation

$$\ln(x^2+7x+12)-1=\ln(2x^2+9x+4),$$

giving your answer in terms of e. **(4)**

B)

(a) Use the identities for $\sin(A+B)$ and $\sin(A-B)$ to prove that

$$\sin P + \sin Q \equiv 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}.$$ **(4)**

(b) Find, in terms of π, the solutions of the equation

$$\sin 5x + \sin x = 0,$$

for x in the interval $0 \le x < \pi$. **(5)**

C)

The curve with equation $y = x^{\frac{5}{2}} \ln\frac{x}{4}$, $x > 0$ crosses the x-axis at the point P.

(a) Write down the coordinates of P. **(1)**

The normal to the curve at P crosses the y-axis at the point Q.

(b) Find the area of triangle OPQ where O is the origin. **(6)**

The curve has a stationary point at R.

(c) Find the x-coordinate of R in exact form. **(3)**

1. Write down the maximum and minimum values of the following functions

(a) $f(x) = 12 + 5\sin x$ (b) $f(x) = 7 - 2\sin(2x + \pi)$

(c) $f(x) = \dfrac{2}{10 - 4\sin 2x}$ hint: think about max and min values of $\sin x$

2. Express the following in the form $R\sin(\theta \pm \alpha)$ or $R\cos(\theta \pm \alpha)$ as appropriate (with α in radians) and hence find the **minimum** value of the function, and the first positive value of θ for which it occurs: check using your graphic calculator

(a) $\cos\theta + \sin\theta$ [use $R\cos(\theta - \alpha)$]

(b) $5\cos\theta - 12\sin\theta$ [use $R\cos(\theta + \alpha)$]

(c) $\sqrt{3}\sin\theta + 3\cos\theta$ [use $R\sin(\theta + \alpha)$]

(d) $3\sin\theta - 7\cos\theta$ [use $R\sin(\theta - \alpha)$]

3. Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Hence solve the equation $12\sin x - 5\cos x = 7$ for $0 < x < 2\pi$ giving x correct to 3 decimal places.

4. Prove the following identities:

(a) $$\frac{1}{\operatorname{cosec} x - 1} + \frac{1}{\operatorname{cosec} x + 1} \equiv 2\sec x \tan x$$

(b) $$\frac{\sin A}{\sin B} + \frac{\cos A}{\cos B} \equiv \frac{2\sin(A+B)}{\sin 2B}$$

5. The function f is given by f: $x \to \ln(3x - 6)$, $x \in \mathbb{R}$, $x > 2$.

(a) Find $f^{-1}(x)$.
(b) Write down the domain of f^{-1} and the range of f^{-1}.
(c) Find, to 3 significant figures, the value of x for which $f(x) = 3$.

6. Find the equation of the normal to the curve $y = e^x(\cos x + \sin x)$ at the point (0,1)

7 A curve has equation $y = e^x - 2x$. Find the coordinates of the turning point in terms of natural logarithms, and show that it is a minimum point.

8. Solve the following equations in the interval $0 \le x \le 2\pi^c$ giving x in terms of π or to 1dp as appropriate:

(a) $\cos\left(x - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}\sin x$ (b) $\sin 2x + \sin x = 0$

9 If $x^2 - 3x + 1 = 0$, what is the value of $x^2 + \left(\frac{1}{x}\right)^2$

M1 Practice (Preparation for M2)

10 A particle of mass 5 kg is moving with velocity (3i + 4j) ms^{-1} when it is given an impulse of (2i + 6j) Ns. Find the speed of the particle after the impact.