A2 Assignment epsilon Cover Sheet **Name:**

Question		Done	Backpack		Topic	Answers
Drill	1i				C3 Differentiation trig	$-5\cos^4 x\sin x$
	1ii				C3 Differentiation trig	$\sec x\tan x$
	1iii				C3 Differentiation trig	$-\operatorname{cosec}^2 x$
	2i				C4 Integration Reverse chain	$\frac{2}{15}(3x-3)^5+c$
	2ii				C4 Integration Reverse chain	$\frac{3}{2}\tan 2x+c$
	2iii				C4 Integration Reverse chain	$\cot(\pi-x)+c$
	3i				C3 Sketching modulus function	Check on google inc asymptotes
	3ii				C3 Sketching modulus function	Check on google inc asymptotes
	3iii				C3 Sketching modulus function	Check on google inc asymptotes
	4i				C3 Sketch and give range	f $\mathcal{E}\mathbb{R}$: $0\le f(x)\le 16$
	4ii				C3 Sketch and give range	f $\mathcal{E}\mathbb{R}$: $\frac{1}{10}\le f(x)\le\frac{1}{2}$
	4iii				C3 Sketch and give range	f $\mathcal{E}\mathbb{R}$: $\frac{1}{4}\le f(x)\le 16$
	TT1A				C3 Differentiation	$\frac{2}{x}$
	TT1B				C3 Differentiation	$2x\sin 3x+3x^2\cos 3x$
	TT1C				C3 Trig proofs	Proof
	TT1D				C3 Trig proofs	Proof
	1a				C3 Sketching modulus function	Check on google inc asymptotes
	1b				C3 Sketching modulus function	Check on google inc asymptotes
	1c				C3 Sketching modulus function	Check on google inc asymptotes
	1d				C3 Sketching modulus function	Check on google inc asymptotes
	2				C3 Sketch and solve modulus equation with unknown	$x=3a$ or $\frac{3}{2}a$
	3ai				C3 Inverse function and domain	$f^{-1}(x)=5x-6$, $x\in\mathbb{R}$
	3aii				C3 Inverse function and domain	$f^{-1}(x)={}^{5}/_{x}$, $\{x\in\mathbb{R}: x\neq 0\}$
	3aiii				C3 Inverse function and domain	$f^{-1}: x\rightarrow x^2-4$, $\{x\in\mathbb{R}: x\geq 0\}$
	3aiv				C3 Inverse function and domain	$f^{-1}: x\rightarrow {}^{(x+2)}/_{(x-3)}$, $\{x\in\mathbb{R}: x\neq 3\}$

	3bi				C3 Conditions for inverse function	The inverse is a 1 to many function (3bii) $x \in \mathbb{R}, x \geq 3$
	3bii				C3 Changing domain to create inverse function	$X \geq 3$
	4a				C3 Inverse and composite functions	$f^{-1}(x) = {}^1/_3(x-2)$, $g^{-1}(x) = {}^1/_x$, $x \neq 0$, $gf(x) = {}^1/_{(3x+2)}$, $x \neq -{}^2/_3$
	4b				C3 Inverse and composite functions	PROOF
	5a				C4 dx/dy in terms of y	$\frac{dx}{dy} = \sec^2 y$
	5b				C4 using dx/dy to find dy/dx in terms of x	PROOF
	6a				C3 Solving equations using composite functions	3
	6b				C3 Solving equations using composite functions	2
	6c				C3 Solving equations using composite functions	$\frac{109}{4}(27.25)$
	7				C3 Find normal	$\left(\frac{1}{5}\left(\frac{\pi}{4}+8\right), \frac{1}{5}\left(\frac{\pi}{4}+8\right)\right)$
	8a				C2 Solve trig equations	$0.424^c, 1.15^c, 3.57^c, 4.29^c$
	8b				C2 Solve trig equations	$\pi/3, 5\pi/3$
	9a					$\frac{1}{4}\pi$
	9b					$\frac{3}{16}\pi$
M1 Practice	10				M1 Force diagrams	34 N
Challenge					C4 Parametric differentiation	$\frac{1}{3}\cos t$

α	β	γ	δ	**ε**	ζ	η	θ	ι	κ	λ	μ	ν	ξ	ο	π	ρ	σ	τ	υ	φ	χ	ψ	ω

"Logic is the art of going wrong with confidence" M Kline

A2 Maths with Mechanics Assignment ε (epsilon)

C3 past paper* to do in the week's break found at the end of this assignment

due w/b 30/10

Drill

Part A Find dy/dx:

(a) $\cos^5 x$ (b) $\dfrac{1}{\cos x}$ (c) $\dfrac{1}{\tan x}$

Part B Integrate the following with respect to *x*:

(a) $2(3x-3)^4$ (b) $3\sec^2 2x$ (c) $\operatorname{cosec}^2(\pi - x)$

Part C For each of the following function sketch f(*x*), $\mathrm{f}\left(|x|\right)$ and $\left|\mathrm{f}\left(x\right)\right|$

(a) $\mathrm{f}(x)=(x-1)^2+3$ (b) $\mathrm{f}(x)=2^x-4$ (c) $\mathrm{f}(x)=(x-2)^3$

Part D Sketch the following functions where each function is defined $x \in \mathbb{R}$. on its given domain, State the range of each function.

(a) $f(x)=x^2, -4 \le x \le 4$ (b) $f(x)=\dfrac{1}{x}, \quad 2 \le x \le 10$

(c) $f(x)=2^x, \quad -2 \le x \le 4$

TT1 FOCUS:

A) Differentiate $ln\ x^2$

B) Differentiate $x^2 sin\ 3x$

C) Prove that $1 + cos2\theta + cos4\ \theta \equiv (4cos^2\theta - 1)cos2\theta$

D) Prove that $sin(x+y)sin(x-y) \equiv cos^2y - cos^2x$

Current work

1. For each of the following functions, sketch $f(x)$, $f(|x|)$ and $|f(x)|$ on separate axes.

   (a) $f(x)=2x-4$ (b) $f(x)=-x$

   (c) $f(x)=\sin x$ (d) $f(x)=(x-2)^2$

2. Sketch the graph of $y = |x - 2a|$ (where *a* is a positive constant) showing the points of q intersection with the coordinate axes. Solve $|x - 2a| = \frac{1}{3}x$ for *x* in terms of *a*.

3. (a) For each of these functions, find the inverse function, $f^{-1}(x)$ and state its domain.

   (i) $f(x) = \frac{x+6}{5}$, $x \in \mathbb{R}$ (ii) $f(x) = \frac{5}{x}$, $\{x \in \mathbb{R}: x \neq 0\}$

   (iii) $f : x \to \sqrt{x+4}$, $\{x \in \mathbb{R}: x \geq -4\}$ (iv) $f : x \to \frac{3x+2}{x-1}$, $\{x \in \mathbb{R}: x \neq 1\}$

   (b) (i) State why the inverse $f^{-1}(x)$ does not exist for $f : x \to 2(x-3)^2 - 5$, $\{x \in \mathbb{R}\}$

   (ii) Change the domain of the above function so that the inverse does exist.

4. $f(x) = 3x + 2$ and $g(x) = \frac{1}{x}$ with $x \neq 0$.

   (a) Find $f^{-1}(x)$, $g^{-1}(x)$ and $gf(x)$. (b) Show that $(gf)^{-1}(x) = f^{-1}g^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$.

   Note: you will need to show *both* that $f^{-1}g^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$ and that $(gf)^{-1}(x) = \frac{1}{3}\left(\frac{1}{x} - 2\right)$.

Consolidation

5. Given that $x = \tan y$

   (a) find $\frac{dx}{dy}$ in terms of y

   (b) hence show $\frac{dy}{dx} = \frac{1}{1+x^2}$

6. The functions *f*, *g* and *h* each have the set of real numbers as their domain and are defined as follows:

   $f(x) = 7 - 2x$ $g(x) = 4x - 1$ $h(x) = 3(x - 1)$

   Find $fg(x)$, $gh(x)$ and $ff(x)$ and hence find the values of *x* for which:

   (a) $fg(x) = -15$ (b) $gh(x) = 11$ (c) $ff(x) = 102$

7. The normal to the curve $y = \sec^2 x$ at the point $P\left(\frac{\pi}{4}, 2\right)$ meets the line $y = x$ at the point *Q*.

   Find the exact coordinates of *Q*.

8. Solve the following equations on the interval $0 \leq x \leq 2\pi$. Give exact answers where you can, but otherwise give your answers to 3sf:

   (a) $8\sin x \cos x = 3$ (b) $10\cos x = 2(1 + 2\sin^2 x)$

9. Evaluate

(a) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 2dx$

(b) $\int_0^{\frac{\pi}{2}} \cos^4 xdx$

M1 (Preparation of M2)

10. A mass of 10 kg rests in equilibrium on a rough plane inclined at θ to the horizontal.
   a) Draw a force diagram and find the magnitude of the frictional force when θ = 20°.
   b) If R is the magnitude of the normal contact force, show that $F = R\tan\theta$.

Are you up for a challenge? Then try this question:

Use the chain rule $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

to find $\frac{dy}{dx}$ in terms of t for the curve defined by the parametric equations

$x = 3\cot t,\ y = \cos ect$

PAST PAPER NEXT PAGE

Complete this past paper (C3 June 2005) in 1 hour 30mins

If it takes longer draw a line under where you got to in the time allowed and continue.

- **Do it under exam conditions.**
- **Do not use the mark scheme until you have done the whole paper**
- **Mark it yourself using the mark scheme on the VLE and write down your % on the paper.**
- **Do your corrections in another colour.**
- **Hand it to your teacher with your assignment**

1. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)

---

2. (*a*) Differentiate with respect to x

(i) $3\sin^2 x + \sec 2x$,

(3)

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(*b*) show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$.

(6)

---

3. The function f is defined by

$$f: x \mapsto \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(*a*) Show that $f(x) = \dfrac{2}{x-1}$, $x > 1$.

(4)

(*b*) Find $f^{-1}(x)$.

(3)

The function g is defined by

$$g: x \mapsto x^2 + 5, \quad x \in \mathbb{R}.$$

(*c*) Solve $fg(x) = \frac{1}{4}$.

(3)

---

4. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(*a*) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(*b*) Show that $\alpha = \frac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \frac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(*c*) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(*d*) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)

---

5. (*a*) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(*b*) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(*c*) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(*d*) Hence, for $0 \leq \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)

---

6. **Figure 1**

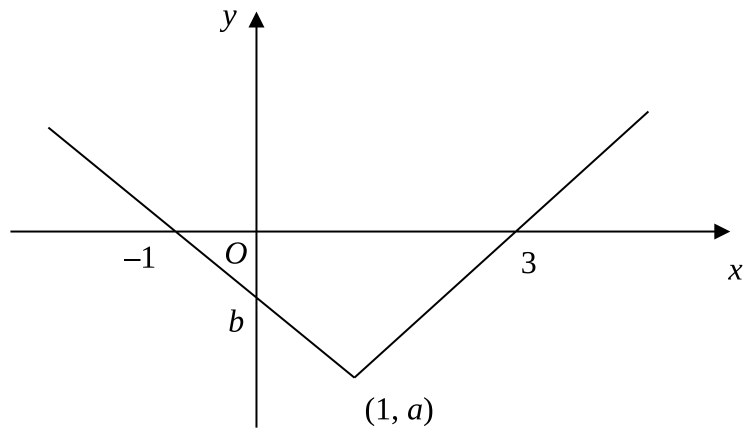


Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(–1, 0)$ and the y-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(*a*) $y = \mathrm{f}(x + 1)$,

(2)

(*b*) $y = \mathrm{f}(|x|)$.

(3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $\mathrm{f}(x) = |x – 1| – 2$, find

(*c*) the value of a and the value of b,

(2)

(*d*) the value of x for which $\mathrm{f}(x) = 5x$.

(4)

7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800ae^{0.2t}}{1 + ae^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(*a*) show that $a = 0.12$,

(3)

(*b*) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(*c*) Show that $p = \dfrac{336}{0.12 + e^{-0.2t}}$.

(1)

(*d*) Hence show that the population cannot exceed 2800.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Finished and done all the questions?
Now mark it using the MARK SCHEME on the next page. ONLY look at this once you have completed the full 1 hour 30 mins exam

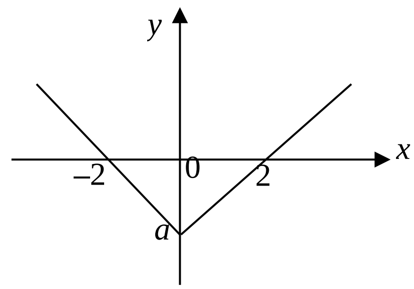
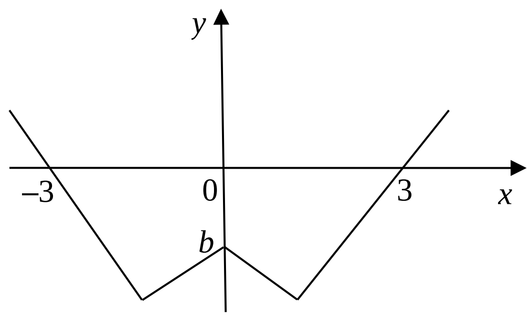
Question Number	Scheme	Marks
1 (a)	Dividing by $\cos^2\theta$: $\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\cos^2\theta}{\cos^2\theta}\equiv\dfrac{1}{\cos^2\theta}$	M1
	Completion **:** $1+\tan^2\theta\equiv\sec^2\theta$ (no errors seen)	A1 (2)
(b)	Use of $1+\tan^2\theta=\sec^2\theta$: $2(\sec^2\theta-1)+\sec\theta=1$	M1
	[$2\sec^2\theta+\sec\theta-3=0$]	
	Factorising or solving: $(2\sec\theta+3)(\sec\theta-1)=0$	M1
	[$\sec\theta=-\frac{3}{2}$ or $\sec\theta=1$]	
	$\theta=0$	B1
	$\cos\theta=-\frac{2}{3}$; $\theta_1=131.8°$	M1 A1
	$\theta_2=228.2°$	A1√ (6)
	[A1ft for $\theta_2=360°-\theta_1$]	
		[8]

Question Number	Scheme	Marks
2 (a)	(i) $6\sin x\cos x + 2\sec 2x\tan 2x$	M1A1A1 (3)
	or $3\sin 2x + 2\sec 2x\tan 2x$ [M1 for 6 sin *x*]	
(ii)	$3(x+\ln 2x)^2(1+\frac{1}{x})$	B1M1A1 (3)
	[B1 for $3(x+\ln 2x)^2$]	
(b)	Differentiating numerator to obtain 10*x* – *10*	
	Differentiating denominator to obtain 2(*x*-1)	
	Using quotient rule formula correctly:	
	To obtain $\frac{dy}{dx} = \frac{(x-1)^2(10x-10)-(5x^2-10x+9)2(x-1)}{(x-1)^4}$	
	Simplifying to form $\frac{2(x-1)[5(x-1)^2-(5x^2-10x+9)}{(x-1)^4}$	
	$= -\frac{8}{(x-1)^3}$ * (c.s.o.)	
3 (a)	$\frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$	B1
	$= \frac{5x+1-3(x-1)}{(x+2)(x-1)}$	M1
	M1 for combining fractions even if the denominator is not lowest common	
	$= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1}$ *	M1 A1 cso (4)
	M1 must have linear numerator	
(b)	$y = \frac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$	M1A1
	$f^{-1}(x) = \frac{2+x}{x}$ o.e.	A1 (3)
	$fg(x) = \frac{2}{x^2+4}$ (attempt) [$\frac{2}{"g"-1}$]	M1
	Setting $\frac{2}{x^2+4} = \frac{1}{4}$ and finding $x^2 = \dots$; $x = \pm 2$	M1; A1 (3)

Question Number	Scheme	Marks
		[10]

Question Number	Scheme	Marks
4 (a)	$f'(x) = 3\,e^{x} - \dfrac{1}{2x}$	M1A1A1 (3)
	$3e^{x} - \dfrac{1}{2x} = 0$	M1
	$\Rightarrow 6\alpha\, e^{\alpha} = 1 \qquad \Rightarrow \alpha = \frac{1}{6}\, e^{-\alpha}$ (*)	A1 cso (2)
(c)	$x_1 = 0.0613...,\; x_2 = 0.1568..,\; x_3 = 0.1425...,\; x_4 = 0.1445....$	M1 A1 (2)
	[M1 at least x_1 correct, A1 all correct to 4 d.p.]	
(d)	Using $f'(x) = 3\,e^{x} - \dfrac{1}{2x}$ with suitable interval	
	e.g. $f'(0.14425) = -0.0007$	M1
	$f'(0.14435) = +0.002(1)$	
	Accuracy (change of sign and correct values)	A1 (2)
		[9]

Question Number	Scheme	Marks
5 (a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1
	$= (1 - \sin^2 A); - \sin^2 A = 1 - 2\sin^2 A$ (*)	A1 (2)
(b)	$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta\cos\theta; -3(1 - 2\sin^2\theta) - 3\sin\theta + 3$	B1; M1
		M1
	$\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$	
	$\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*)	A1 (4)
(c)	$4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
	Complete method for R (may be implied by correct answer)	
	[$R^2 = 4^2 + 6^2$, $R\sin\alpha = 4$, $R\cos\alpha = 6$]	M1
	$R = \sqrt{52}$ or 7.21	A1
	Complete method for α ; $\alpha = 0.588$ (allow 33.7º)	M1 A1 (4)
(d)	$\sin\theta\ (4\cos\theta + 6\sin\theta - 3) = 0$	M1
	$\theta = 0$	B1
	$\sin(\theta + 0.588) = \dfrac{3}{\sqrt{52}} = 0.4160..$ (24.6º)	M1
	$\theta + 0.588 = (0.4291),\ 2.7125$ [or $\theta + 33.7° = (24.6º),\ 155.4°$]	dM1
	$\theta = 2.12$ cao	A1 (5)
		[15]

Question Number	Scheme		Marks
6. (a)		Translation ← by 1	M1
		Intercepts correct	A1 (2)
(b)		$x \geq 0$, correct "shape"	B1
		[provided not just original]	
		Reflection in y-axis	B1√
		Intercepts correct	B1 (3)
(c)	$a = -2$, $b = -1$		B1 B1 (2)
(d)	Intersection of $y = 5x$ with $y = -x - 1$		M1A1
	Solving to give $x = -\frac{1}{6}$		M1A1 (4)
			[11]
	[Notes: (i) If both values found for $5x = -x - 1$ and $5x = x - 3$, or solved algebraically, can score 3 out of 4 for $x = -\frac{1}{6}$ and $x = -\frac{3}{4}$; required to eliminate $x = -\frac{3}{4}$ for final mark. (ii) Squaring approach: M1 correct method, $24x^2 + 22x + 3 = 0$ (correct 3 term quadratic, any form) A1 Solving M1, Final correct answer A1.]		

Question Number	Scheme	Marks
7 (a)	Setting $p = 300$ at $t = 0 \Rightarrow\ 300 = \dfrac{2800a}{1 + a}$	M1
	(300 = 2500a); a = 0.12 (c.s.o) *	dM1A1 (3)
(b)	$1850 = \dfrac{2800(0.12)e^{0.2t}}{1 + 0.12e^{0.2t}}$; $e^{0.2t} = 16.2...$	M1A1
	Correctly taking logs to $0.2\,t = \ln k$	M1
	$t = 14$ (13.9..)	A1 (4)
(c)	Correct derivation:	
	(Showing division of num. and den. by $e^{0.2t}$; using a)	B1 (1)
(d)	Using $t \to \infty$, $e^{-0.2t} \to 0$,	M1
	$p \to \dfrac{336}{0.12} = 2800$	A1 (2)
		[10]