BHASVIC MαTHS

A2 Doubles assignment *summer 2*

Section: *Mech and FP1*

<u>Past</u>

1. Evaluate

$$\int_{1}^{15} \frac{x+2}{(x+1)(x+3)}\,dx$$

2. Evaluate

$$\int_{0}^{\frac{\pi}{3}} x \sin 3x \, dx$$

3. Evaluate

$$\int_{-1}^{2} x^2\sqrt{x^3+1}\,dx$$

4. Solve the following in the interval $0 \le x \le 2\pi$

a) $2\sin\theta\cos\theta + \sin\theta = 0$

Solve the following in the interval $0 \le x \le 360$

b) $4\cos\theta = \cos\theta\,\mathrm{cosec}\,\theta$

5. Without using your calculator find , for each of the following equations, all the solutions in the interval $0 \le x \le 180$

a) $\cos(x+30) = \cos(60-3x)$

b) $\sin(x+20) = \cos 3x$

<u>Present</u>

6. Find the vector equation of the line in the given direction through the given point

a) Direction $\begin{pmatrix}1\\4\end{pmatrix}$, point $(4,-1)$

b) Direction $\begin{pmatrix}1\\3\\-3\end{pmatrix}$, point $(1,0,5)$

c) Direction $2i+3j$, point $(4,0)$

d) Direction $i-3k$, point $(0,2,3)$

e) Direction $\begin{pmatrix}2\\-3\end{pmatrix}$, point $(4,1)$

f) Direction $\begin{pmatrix}3\\-2\\2\end{pmatrix}$, point $(-1,1,5)$

g) Direction $(i-3j)$, point $(-1,1,5)$

h) Direction $(2i+3j-k)$, point $(4,-3,0)$

7. Find the vector equation of the line through two given points

a) $(4,1)$ and $(1,2)$

b) $(-5,-2,3)$ and $(4,-2,3)$

c) $(2,7)$ and $(4,-2)$

d) $(1,1,3)$ and $(10,-5,0)$

8. Write down the Cartesian equation of each line

a) $$r = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -7 \\ 4 \end{pmatrix}$$

b) $$r = \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}$$

c) $$r = \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$$

d) $$r = \begin{pmatrix} -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

e) $$r = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

f) $$r = \begin{pmatrix} 3 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix}$$

9. Find a vector equation of each line

a) $$y = \frac{3}{5}x + 2$$

b) $$3x - 5y = 17$$

c) $$y = -\frac{4}{3}x - 1$$

d) $$2x + 3y + 4 = 0$$

10. Write down a vector equation of each line

a) $$\frac{x-2}{5} = \frac{y-2}{3} = \frac{z+1}{7}$$

b) $$\frac{x+1}{3} = \frac{y}{-7} = \frac{z-1}{-5}$$

c) $$\frac{x-11}{3} = \frac{y+1}{6}, z = -2$$

d) $$\frac{x+1}{4} = \frac{y-6}{-1} = \frac{z-5}{3}$$

e) $$\frac{x-3}{2} = \frac{y+1}{4} = \frac{z}{5}$$

f) $$\frac{x+1}{5} = \frac{3-z}{2}, y = 1$$

11. Determine whether the following pairs of lines intersect and, if they do, find the coordinates of the intersection point

a) $r = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ and $r = \begin{pmatrix} 2 \\ 1 \\ -14 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$

b) $r = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ and $r = \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$

c) $\frac{x+1}{2} = \frac{y-6}{-2} = \frac{z+7}{1}$ and $\frac{x-2}{-1} = \frac{y-5}{3} = \frac{z}{5}$

d) $\frac{x-2}{-3} = \frac{y+1}{2} = \frac{z-1}{2}$ and $\frac{x+1}{1} = \frac{y}{6} = \frac{z-1}{3}$

12. Decide whether or not the given point lies on the given line

a) Line $r = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$, point $(0,5,9)$

c) Line $r = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}$, point $(-1,0,3)$

b) Line $r = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$, point $(0,0,0)$

d) Line $r = \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix}$, point $(-1,3,8)$

13. A line passes through the point $A(3,-1,4)$ and has the direction $5i - j + 2k$

a) Write down the vector equation of the line

b) Point B has coordinates $(-7,1,0)$. Show that B lies on the line

c) Find the exact distance AB

14. Show that the points $A(4,-1,-8)$ and $B(2,1,-4)$ line on the line l with equation $r = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

15.

In a chemical reaction, the rate of change of the mass, m grams, of a substance at time t minutes is inversely proportional to the square root of m. Initially, the mass is 9 grams and is increasing at a rate of 10 grams per minute.

Formulate a differential equation between m and t. **[3 marks]**

b) Solve the differential equation

16.

Forces $\mathbf{F}_1 = (2a\mathbf{i} + 3b\mathbf{j})$ N, $\mathbf{F}_2 = (-b\mathbf{i} + a\mathbf{j})$ N and $\mathbf{F}_3 = (10\mathbf{i} - 2\mathbf{j})$ N are in equilibrium, so that $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$. Find a and b. [4 marks]

The triangle ABC has coordinates A $(1, 2, 0)$, B $(-1, 3, -1)$ and C$(3, 0, -6)$.

(i) Find the column vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$. [2 marks]

(ii) Prove that triangle ABC is right angled. [3 marks]

17.

In Figure 1, ABCD is a parallelogram. D divides CB in the ratio 1:2 and E divides OD in the ratio 3:1.

$\overrightarrow{OA} = \mathbf{u}$ and $\overrightarrow{OC} = \mathbf{v}$.

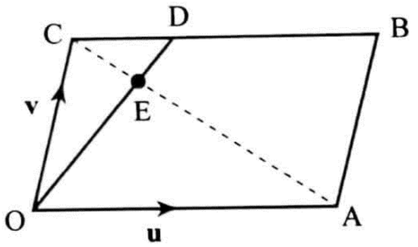

Figure 1

(i) Find the vectors $\overrightarrow{OE}$ and $\overrightarrow{CE}$ in terms of $\mathbf{u}$ and $\mathbf{v}$. [3 marks]

(ii) Hence prove that the point E lies on the diagonal AC. [2 marks]

18.

(i) Use the Newton–Raphson method to find a root of $f(x) = 0$, where $f(x) = 4\sin 2x - x^3$. Use a starting value $x_0 = 1$, and give the results of the first 4 iterations correct to 5 decimal places. Hence suggest the value of the root to 3 significant figures. [4 marks]

(ii) Verify that your root is indeed correct to 3 significant figures. [1 mark]

19.

Ben and Carrie are attempting to solve the equation $x^3 - 2x - 5 = 0$ using fixed point iteration.

Ben rearranges the equation to get an iterative formula of the form $x_{n+1} = ax_n^3 + b$.

(i) Find the constants a and b. [1 mark]

(ii) Using an initial value $x_0 = 1$, find x_1, x_2 and x_3. Comment on your results. [2 marks]

Carrie rearranges the equation to get an iterative formula of the form $x_{n+1} = \sqrt[3]{c\,x_n + d}$.

(iii) Find the constants c and d. [1 mark]

(iv) Use this iterative formula, together with an initial value $x_0 = 1$, to find x_1, x_2 and x_3. [2 marks]

(v) Verify that x_3, when rounded, gives a root of the equation correct to 2 decimal places. [2 marks]

20.

Figure 3 shows a curve with parametric equations $x = 1 + 2t$, $y = 2t + t^2$ for $-2 \leqslant t \leqslant 2$.

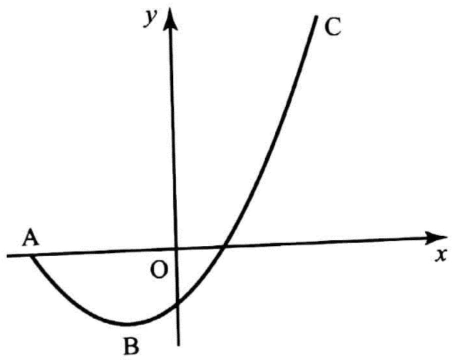

Figure 3

(i) Find the coordinates of A and C. [4 marks]

(ii) Find $\frac{dy}{dx}$ in terms of t. Hence find the coordinates of the turning point B of the curve. [5 marks]

(iii) Find the cartesian equation of the curve in the form $y = ax^2 + bx + c$. [3 marks]

diver is moving

21.

$y = ax^2 + bx + c$.

At the moment he opens his parachute, a sky diver is moving vertically downwards at a speed of 10 metres per second. His speed, $v\,\text{m}\,\text{s}^{-1}$, t seconds after this is modelled by the differential equation $\frac{dv}{dt} = -\frac{1}{3}kv(v-3)$, where k is a positive constant.

[10 marks]

(i) Show that $v = \dfrac{3}{1 - 0.7e^{-kt}}$.

[2 marks]

(ii) Hence find the terminal velocity of the sky diver.