## Paper Reference(s)

## 6679/01 <br> Edexcel GCE

## Mechanics M3

## Advanced Level

# Wednesday 13 May 2015 - Morning <br> Time: 1 hour 30 minutes 

Materials required for examination<br>Items included with question papers<br>Mathematical Formulae (Pink)<br>Nil


#### Abstract

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.


## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle $P$ of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 1.2 m and modulus of elasticity $\lambda$ newtons. The other end of the spring is attached to a fixed point $A$ on a ceiling. The particle is hanging freely in equilibrium at a distance 1.5 m vertically below $A$.
(a) Find the value of $\lambda$.

The particle is now raised to the point $B$, where $B$ is vertically below $A$ and $A B=0.8 \mathrm{~m}$. The spring remains straight. The particle is released from rest and first comes to instantaneous rest at the point $C$.
(b) Find the distance $A C$.
2. The finite region bounded by the $x$-axis, the curve with equation $y=2 \mathrm{e}^{x}$, the $y$-axis and the line $x=1$ is rotated through one complete revolution about the $x$-axis to form a uniform solid.

Use algebraic integration to
(a) show that the volume of the solid is $2 \pi\left(\mathrm{e}^{2}-1\right)$,
(b) find, in terms of e, the $x$-coordinate of the centre of mass of the solid.
3.


Figure 1
A small ball $P$ of mass $m$ is attached to the midpoint of a light inextensible string of length $4 l$. The ends of the string are attached to fixed points $A$ and $B$, where $A$ is vertically above $B$. Both strings are taut and $A P$ makes an angle of $30^{\circ}$ with $A B$, as shown in Figure 1. The ball is moving in a horizontal circle with constant angular speed $\omega$.
(a) Find, in terms of $m, g, l$ and $\omega$,
(i) the tension in $A P$,
(ii) the tension in $B P$.
(b) Show that $\omega^{2} \geq \frac{g \sqrt{ } 3}{3 l}$.
4. A vehicle of mass 900 kg moves along a straight horizontal road. At time $t$ seconds the resultant force acting on the vehicle has magnitude $\frac{63000}{k t^{2}} \mathrm{~N}$, where $k$ is a positive constant. The force acts in the direction of motion of the vehicle. At time $t$ seconds, $t \geq 1$, the speed of the vehicle is $v \mathrm{~m} \mathrm{~s}^{-1}$ and the vehicle is a distance $x$ metres from a fixed point $O$ on the road. When $t=1$ the vehicle is at rest at $O$ and when $t=4$ the speed of the vehicle is $10.5 \mathrm{~m} \mathrm{~s}^{-1}$.
(a) Show that $v=14-\frac{14}{t}$.
(b) Hence deduce that the speed of the vehicle never reaches $14 \mathrm{~m} \mathrm{~s}^{-1}$.
(c) Use the trapezium rule, with 4 equal intervals, to estimate the value of $x$ when $v=7$.
5.


Figure 2
Figure 2 shows a uniform solid spindle which is made by joining together the circular faces of two right circular cones. The common circular face has radius $r$ and centre $O$. The smaller cone has height $h$ and the larger cone has height $k h$. The point $A$ lies on the circumference of the common circular face. The spindle is suspended from $A$ and hangs freely in equilibrium with $A O$ at an angle of $30^{\circ}$ to the vertical.

Show that $k=\frac{4 r}{h \sqrt{ } 3}+1$.
6.


Figure 3
Two points $A$ and $B$ are 6 m apart on a smooth horizontal floor. A particle $P$ of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 2.5 m and modulus of elasticity 20 N . The other end of the spring is attached to $A$. A second light elastic spring, of natural length 1.5 m and modulus of elasticity 18 N , has one end attached to $P$ and the other end attached to $B$, as shown in Figure 3. Initially $P$ rests in equilibrium at the point $O$, where $A O B$ is a straight line.
(a) Find the length of $A O$.

The particle $P$ now receives an impulse of magnitude 6 Ns acting in the direction $O B$ and $P$ starts to move towards $B$.
(b) Show that $P$ moves with simple harmonic motion about $O$.
(c) Find the amplitude of the motion.
(d) Find the time taken by $P$ to travel 1.2 m from $O$.
7. A solid smooth sphere, with centre $O$ and radius $r$, is fixed to a point $A$ on a horizontal floor. A particle $P$ is placed on the surface of the sphere at the point $B$, where $B$ is vertically above $A$. The particle is projected horizontally from $B$ with speed $\frac{\sqrt{ }(g r)}{2}$ and starts to move on the surface of the sphere. When $O P$ makes an angle $\theta$ with the upward vertical and $P$ remains in contact with the sphere, the speed of $P$ is $v$.
(a) Show that $v^{2}=\frac{g r}{4}(9-8 \cos \theta)$.

The particle leaves the surface of the sphere when $\theta=\alpha$.
(b) Find the value of $\cos \alpha$.

After leaving the surface of the sphere, $P$ moves freely under gravity and hits the floor at the point $C$.

Given that $r=0.5 \mathrm{~m}$,
(c) find, to 2 significant figures, the distance $A C$.

## END

