# Edexcel GCE 

## Mechanics M3

# Advanced/Advanced Subsidiary 

# Monday 10 June 2013 - Morning 

Time: 1 hour 30 minutes

Materials required for examination<br>Mathematical Formulae (Pink)

Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

## Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75 .
There are 28 pages in this question paper. Any blank pages are indicated.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1.


Figure 1
A rough disc is rotating in a horizontal plane with constant angular speed 20 revolutions per minute about a fixed vertical axis through its centre $O$. A particle $P$ rests on the disc at a distance 0.4 m from $O$, as shown in Figure 1. The coefficient of friction between $P$ and the disc is $\mu$. The particle $P$ is on the point of slipping.

Find the value of $\mu$.
2. A particle $P$ of mass 0.5 kg is moving along the positive $x$-axis in the positive $x$-direction. The only force on $P$ is a force of magnitude $\left(2 t+\frac{1}{2}\right) \mathrm{N}$ acting in the direction of $x$ increasing, where $t$ seconds is the time after $P$ leaves the origin $O$. When $t=0, P$ is at rest at $O$.
(a) Find an expression, in terms of $t$, for the velocity of $P$ at time $t$ seconds.

The particle passes through the point $A$ with speed $6 \mathrm{~m} \mathrm{~s}^{-1}$.
(b) Find the distance $O A$.
3.


Figure 2
Two particles $P$ and $Q$, of mass $m$ and $2 m$ respectively, are attached to the ends of a light inextensible string of length $6 l$. The string passes through a small smooth fixed ring at the point $A$. The particle $Q$ is hanging freely at a distance $l$ vertically below $A$. The particle $P$ is moving in a horizontal circle with constant angular speed $\omega$. The centre $O$ of the circle is vertically below $A$. The particle $Q$ does not move and $A P$ makes a constant angle $\theta$ with the downward vertical, as shown in Figure 2.

Show that
(i) $\theta=60^{\circ}$,
(ii) $\omega=\sqrt{\left(\frac{2 g}{5 l}\right)}$.
4. A particle $P$ of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m . The other end of the string is attached to a fixed point $O$ on a rough horizontal plane. The coefficient of friction between $P$ and the plane is $\frac{2}{5}$. The particle is held at rest at a point $B$ on the plane, where $O B=1.5 \mathrm{~m}$. When $P$ is at $B$, the tension in the string is 20 N . The particle is released from rest.
(a) Find the speed of $P$ when $O P=1.2 \mathrm{~m}$.

The particle comes to rest at the point $C$.
(b) Find the distance $B C$.
5.


Figure 3
The shaded region $R$ is bounded by the curve with equation $y=(x+1)^{2}$, the $x$-axis, the $y$-axis and the line with equation $x=2$, as shown in Figure 3. The region $R$ is rotated through $2 \pi$ radians about the $x$-axis to form a uniform solid $S$.
(a) Use algebraic integration to find the $x$ coordinate of the centre of mass of $S$.


Figure 4
A uniform solid hemisphere is fixed to $S$ to form a solid $T$. The hemisphere has the same radius as the smaller plane face of $S$ and its plane face coincides with the smaller plane face of $S$, as shown in Figure 4. The mass per unit volume of the hemisphere is 10 times the mass per unit volume of $S$. The centre of the circular plane face of $T$ is $A$. All lengths are measured in centimetres.
(b) Find the distance of the centre of mass of $T$ from $A$.
6.


## Figure 5

The points $A$ and $B$ are 3.75 m apart on a smooth horizontal floor. A particle $P$ has mass 0.8 kg . One end of a light elastic spring, of natural length 1.5 m and modulus of elasticity 24 N , is attached to $P$ and the other end is attached to $A$. The ends of another light elastic spring, of natural length 0.75 m and modulus of elasticity 18 N , are attached to $P$ and $B$. The particle $P$ rests in equilibrium at the point $O$, where $A O B$ is a straight line, as shown in Figure 5.
(a) Show that $A O=2.4 \mathrm{~m}$.

The point $C$ lies on the straight line $A O B$ between $O$ and $B$. The particle $P$ is held at $C$ and released from rest.
(b) Show that $P$ moves with simple harmonic motion.

The maximum speed of $P$ is $\sqrt{2} \mathrm{~m} \mathrm{~s}^{-1}$.
(c) Find the time taken by $P$ to travel 0.3 m from $C$.
7.


Figure 6
A particle $P$ of mass $5 m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The particle is held at the point $A$, where $O A=a$ and $O A$ is horizontal, as shown in Figure 6. The particle is projected vertically downwards with speed $\sqrt{\left(\frac{9 a g}{5}\right)}$. When the string makes an angle $\theta$ with the downward vertical through $O$ and the string is still taut, the tension in the string is $T$.
(a) Show that $T=3 m g(5 \cos \theta+3)$.

At the instant when the particle reaches the point $B$ the string becomes slack.
(b) Find the speed of $P$ at $B$.

At time $t=0, P$ is at $B$.
At time $t$, before the string becomes taut once more, the coordinates of $P$ are $(x, y)$ referred to horizontal and vertical axes with origin $O$. The $x$-axis is directed along $O A$ produced and the $y$-axis is vertically upward.
(c) Find
(i) $x$ in terms of $t, a$ and $g$,
(ii) $y$ in terms of $t, a$ and $g$.

## END

