Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced

Thursday 24 January 2008 – Morning

Time: 1 hour 30 minutes

Materials required for examination

Items included with question papers

Mathematical Formulae (Green)

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 7 questions in this question paper.

The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

- 1. A light elastic string of natural length 0.4 m has one end A attached to a fixed point. The other end of the string is attached to a particle P of mass 2 kg. When P hangs in equilibrium vertically below A, the length of the string is 0.56 m.
 - (a) Find the modulus of elasticity of the string.

(3)

A horizontal force is applied to P so that it is held in equilibrium with the string making an angle θ with the downward vertical. The length of the string is now 0.72 m.

(b) Find the angle θ .

(3)

2. A particle *P* of mass 0.1 kg moves in a straight line on a smooth horizontal table. When *P* is a distance *x* metres from a fixed point *O* on the line, it experiences a force of magnitude $\frac{16}{5x^2}$ N away from *O* in the direction *OP*. Initially *P* is at a point 2m from *O* and is moving towards *O* with speed 8 m s⁻¹.

Find the distance of *P* from *O* when *P* first comes to rest.

(8)

3.

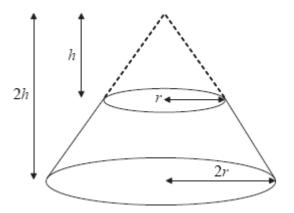


Figure 1

A uniform solid S is formed by taking a uniform solid right circular cone, of base radius 2r and height 2h, and removing the cone, with base radius r and height h, which has the same vertex as the original cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of S from its larger plane face is $\frac{11}{28}h$.

(5)

The solid S lies with its larger plane face on a rough table which is inclined at an angle θ° to the horizontal. The table is sufficiently rough to prevent S from slipping.

Given that h = 2r,

(b) find the greatest value of θ for which S does not topple.

(3)

4. A particle P of mass m lies on a smooth plane inclined at an angle 30° to the horizontal. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity 2mg. The other end of the string is attached to a fixed point O on the plane. The particle P is in equilibrium at the point A on the plane and the extension of the string is $\frac{1}{4}a$. The particle P is now projected from A down a line of greatest slope of the plane with speed V. It comes to instantaneous rest after moving a distance $\frac{1}{2}a$.

By using the principle of conservation of energy,

(a) find V in terms of a and g,

(6)

(b) find, in terms of a and g, the speed of P when the string first becomes slack.

(4)

5. A car of mass m moves in a circular path of radius 75 m round a bend in a road. The maximum speed at which it can move without slipping sideways on the road is 21 m s⁻¹.

Given that this section of the road is horizontal,

(a) show that the coefficient of friction between the car and the road is 0.6.

(3)

The car comes to another bend in the road. The car's path now forms an arc of a horizontal circle of radius 44 m. The road is banked at an angle α to the horizontal, where tan $\alpha = \frac{3}{4}$. The coefficient of friction between the car and the road is again 0.6. The car moves at its maximum speed without slipping sideways.

(b) Find, as a multiple of mg, the normal reaction between the car and road as the car moves round this bend.

(4)

(c) Find the speed of the car as it goes round this bend.

(5)

6.

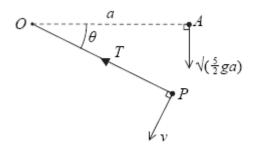


Figure 2

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. At time t = 0, P is projected vertically downwards with speed $\sqrt{(\frac{5}{2}ga)}$ from a point A which is at the same level as O and a distance a from O. When the string has turned through an angle θ and the string is still taut, the speed of P is V and the tension in the string is T, as shown in Figure 2.

(a) Show that
$$v^2 = \frac{ga}{2} (5 + 4 \sin \theta)$$
.

(b) Find T in terms of
$$m$$
, g and θ .

(3)

The string becomes slack when $\theta = \alpha$.

(c) Find the value of
$$\alpha$$
.

(3)

The particle is projected again from A with the same velocity as before. When P is at the same level as O for the first time after leaving A, the string meets a small smooth peg B which has been fixed at a distance $\frac{1}{2}a$ from O. The particle now moves on an arc of a circle centre B. Given that the particle reaches the point C, which is $\frac{1}{2}a$ vertically above the point B, without the string going slack,

(d) find the tension in the string when P is at the point C.

(6)

- 7. A particle *P* of mass 2 kg is attached to one end of a light elastic string, of natural length 1 m and modulus of elasticity 98 N. The other end of the string is attached to a fixed point *A*. When *P* hangs freely below *A* in equilibrium, *P* is at the point *E*, 1.2 m below *A*. The particle is now pulled down to a point *B* which is 0.4 m vertically below *E* and released from rest.
 - (a) Prove that, while the string is taut, P moves with simple harmonic motion about E with period $\frac{2\pi}{7}$ s.

(5)

(b) Find the greatest magnitude of the acceleration of P while the string is taut.

(1)

(c) Find the speed of P when the string first becomes slack.

(3)

(d) Find, to 3 significant figures, the time taken, from release, for P to return to B for the first time.

(7)

TOTAL FOR PAPER: 75 MARKS

END