M2 REVISION QUESTIONS

1 Kinematics of a particle moving in a straight line or plane

Motion in a vertical plane with constant acceleration, eg under gravity.	
Simple cases of motion of a projectile.	
Velocity and acceleration when the displacement is a function of time.	*Notes:* *The setting up and solution of equations of the form* $\frac{\mathrm{d}x}{\mathrm{d}t}$ = *f(t) or* $\frac{\mathrm{d}v}{\mathrm{d}t}$ = *g(t) will be consistent with the level of calculus in C2.*
Differentiation and integration of a vector with respect to time.	*For example, given that* $\mathbf{r} = t^2\mathbf{i} + t^{3/2}\mathbf{j}$, *find* $\dot{\mathbf{r}}$ *and* $\ddot{\mathbf{r}}$ *at a given time.*

1.1 A particle of mass 0.8 kg is moving in a straight line on a rough horizontal plane. The speed of the particle is reduced from 15 m s^{-1} to 10 m s^{-1} as the particle moves 20 m. Assuming the only resistance to motion is the friction between the particle and the plane, find

(*a*) the work done by friction in reducing the speed of the particle from 15 m s^{-1} to 10 m s^{-1}, **(2)**

(*b*) the coefficient of friction between the particle and the plane. **(4)**

1.2 A particle P of mass 0.5 kg is moving under the action of a single force **F** newtons. At time t seconds, $\mathbf{F} = (1.5t^2 - 3)\mathbf{i} + 2t\mathbf{j}$. When $t = 2$, the velocity of P is $(-4\mathbf{i} + 5\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(*a*) Find the acceleration of P at time t seconds. **(2)**

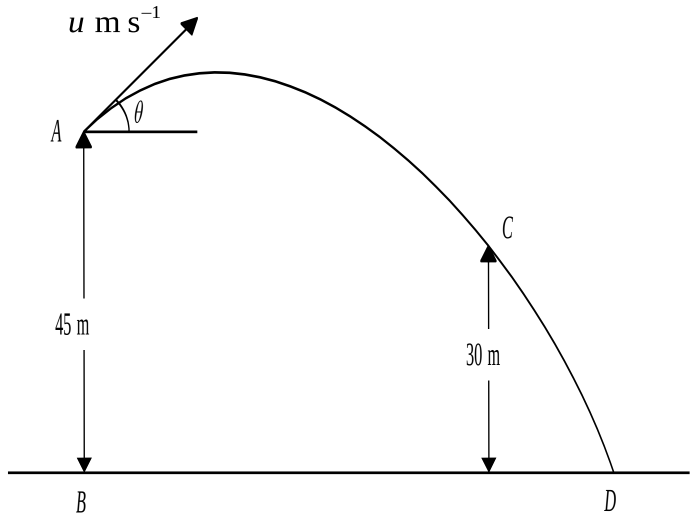
(*b*) Show that, when $t = 3$, the velocity of P is $(9\mathbf{i} + 15\mathbf{j})\,\mathrm{m\,s^{-1}}$. **(5)**

When $t = 3$, the particle P receives an impulse **Q** N s. Immediately after the impulse the velocity of P is $(-3\mathbf{i} + 20\mathbf{j})\,\mathrm{m\,s^{-1}}$. Find

(*c*) the magnitude of **Q**, **(3)**

(*d*) the angle between **Q** and **i**. **(3)**

1.3 **Figure 3**



A particle P is projected from a point A with speed u m s^{-1} at an angle of elevation θ, where cos $\theta = \frac{4}{5}$. The point B, on horizontal ground, is vertically below A and AB = 45 m. After projection, P moves freely under gravity passing through point C, 30 m above the ground, before striking the ground at the point D, as shown in Figure 3.

Given that P passes through C with speed 24.5 m s^{-1},

(*a*) using conservation of energy, or otherwise, show that u = 17.5, **(4)**

(*b*) find the size of the angle which the velocity of P makes with the horizontal as P passes through C, **(3)**

(*c*) find the distance BD. **(7)**

1.4 At time t seconds ($t \geq 0$), a particle P has position vector **p** metres, with respect to a fixed origin O, where

$$\mathbf{p} = (3t^2 - 6t + 4)\mathbf{i} + (3t^3 - 4t)\mathbf{j}.$$

Find

(*a*) the velocity of P at time t seconds, **(2)**

(*b*) the value of t when P is moving parallel to the vector **i**. **(3)**

When t = 1, the particle P receives an impulse of (2**i** – 6**j**) N s. Given that the mass of P is 0.5 kg,

(*c*) find the velocity of P immediately after the impulse. **(4)**

1.5

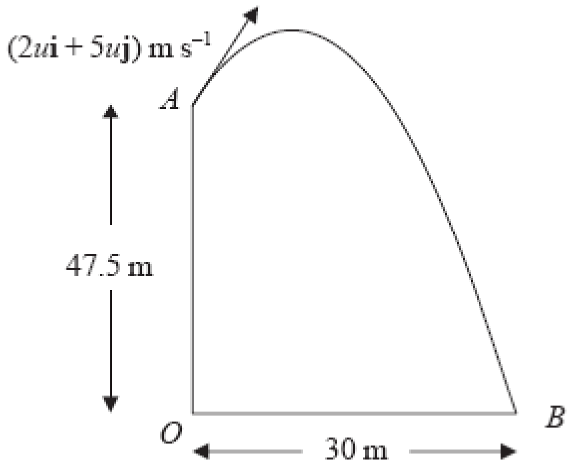


Figure 3

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.*]

A particle *P* is projected from the point *A* which has position vector 47.5**j** metres with respect to a fixed origin *O*. The velocity of projection of *P* is (2*u***i** + 5*u***j**) m s^{-1}. The particle moves freely under gravity passing through the point *B* with position vector 30**i** metres, as shown in Figure 3.

(*a*) Show that the time taken for *P* to move from *A* to *B* is 5 s. **(6)**

(*b*) Find the value of *u*. **(2)**

(*c*) Find the speed of *P* at *B*. **(5)**

1.6 A particle *P* moves along the *x*-axis in a straight line so that, at time *t* seconds, the velocity of *P* is *v* m s^{-1}, where

$$v = \begin{cases} 10t - t^2, & 0 \le t \le 6, \\ \dfrac{-432}{t^2}, & t > 6. \end{cases}$$

At *t* = 0, *P* is at the origin *O*. Find the displacement of *P* from *O* when

(*a*) *t* = 6, **(3)**

(*b*) *t* = 10. **(5)**

1.7

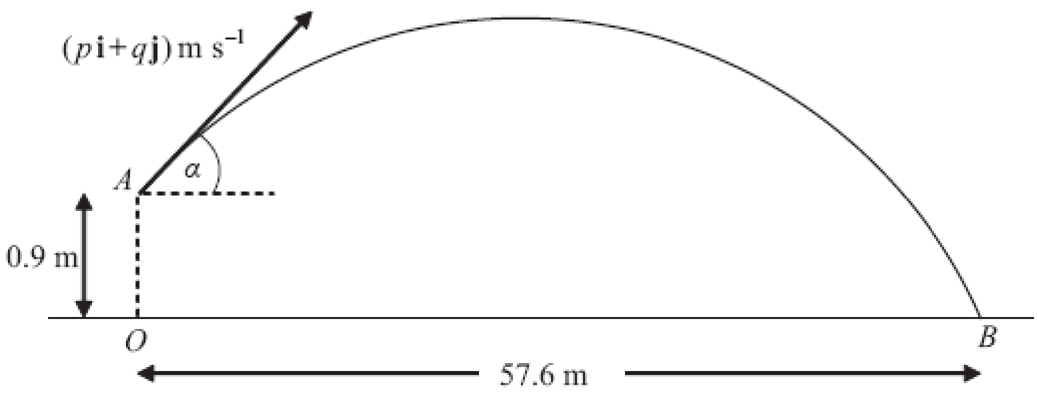


Figure 3

A cricket ball is hit from a point A with velocity of ($p\mathbf{i}$ + $q\mathbf{j}$) m s^{-1}, at an angle α above the horizontal. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and OB = 57.6 m, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

(*a*) find the value of p, **(2)**

(*b*) show that q = 14.4, **(3)**

(*c*) find the initial speed of the cricket ball, **(2)**

(*d*) find the exact value of tan α. **(1)**

(*e*) Find the length of time for which the cricket ball is at least 4 m above the ground. **(6)**

(*f*) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic. **(1)**

1.8 A particle P moves along the x-axis. At time t seconds the velocity of P is v m s^{-1} in the positive x-direction, where $v = 3t^2 - 4t + 3$. When t = 0, P is at the origin O. Find the distance of P from O when P is moving with minimum velocity. **(8)**

1.9 *[In this question* ***i*** *and* ***j*** *are unit vectors in a horizontal and upward vertical direction respectively.]*

A particle P is projected from a fixed point O on horizontal ground with velocity u($\mathbf{i}$ + $c\mathbf{j}$) m s^{-1}, where c and u are positive constants. The particle moves freely under gravity until it strikes the ground at A, where it immediately comes to rest. Relative to O, the position vector of a point on the path of P is ($x\mathbf{i}$ + $y\mathbf{j}$) m.

(*a*) Show that $y = cx - \dfrac{4.9x^2}{u^2}$. **(5)**

Given that u = 7, OA = R m and the maximum vertical height of P above the ground is H m,

(*b*) using the result in part (*a*), or otherwise, find, in terms of c,

(i) R

(ii) H. **(6)**

Given also that when P is at the point Q, the velocity of P is at right angles to its initial velocity,

(*c*) find, in terms of c, the value of x at Q. **(6)**

2 Centres of mass

Centre of mass of a discrete mass distribution in one and two dimensions.	
Centre of mass of uniform plane figures, and simple cases of composite plane figures.	*Notes:* *The use of an axis of symmetry will be acceptable where appropriate. Use of integration is not required. Figures may include the shapes referred to in the formulae book. Results given in the formulae book may be quoted without proof.*
Simple cases of equilibrium of a plane lamina.	*The lamina may* *(i) be suspended from a fixed point;* *(ii) free to rotate about a fixed horizontal axis;* *(iii) be put on an inclined plane.*

2.1

Figure 1

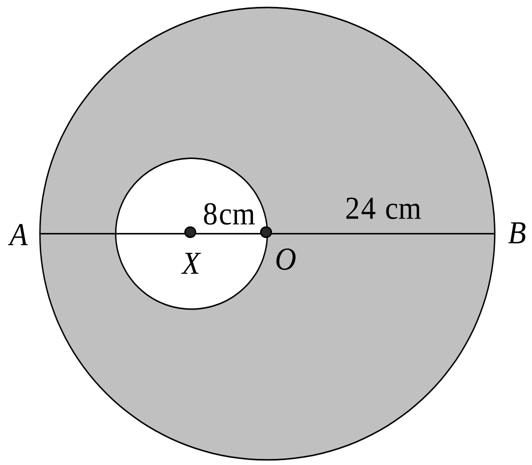


Figure 1 shows a template T made by removing a circular disc, of centre X and radius 8 cm, from a uniform circular lamina, of centre O and radius 24 cm. The point X lies on the diameter AOB of the lamina and AX = 16 cm. The centre of mass of T is at the point G.

(*a*) Find AG. **(6)**

The template T is free to rotate about a smooth fixed horizontal axis, perpendicular to the plane of T, which passes through the mid-point of OB. A small stud of mass $\frac{1}{4}m$ is fixed at B, and T and the stud are in equilibrium with AB horizontal. Modelling the stud as a particle,

(*b*) find the mass of T in terms of m. **(4)**

2.2

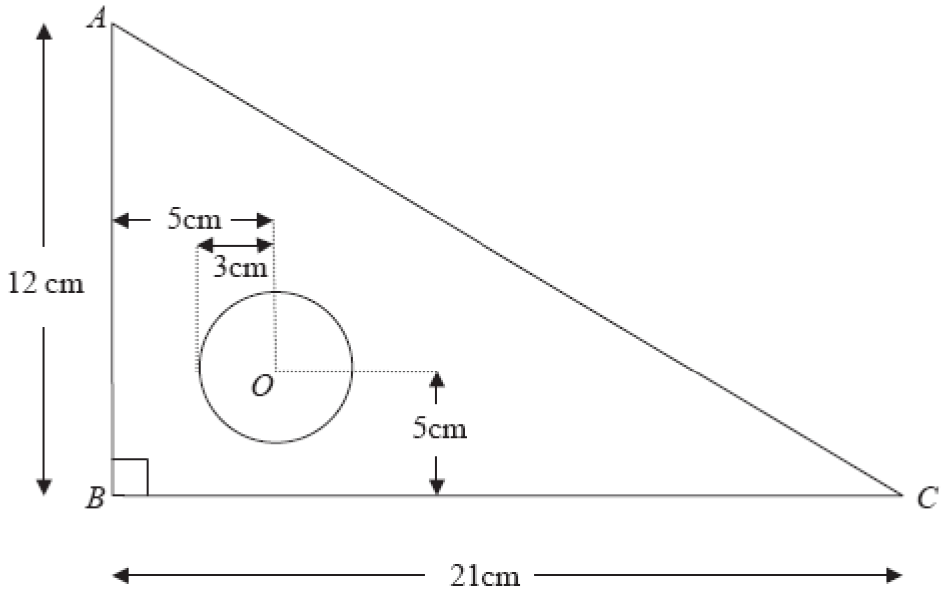


Figure 1

A set square S is made by removing a circle of centre O and radius 3 cm from a triangular piece of wood. The piece of wood is modelled as a uniform triangular lamina ABC, with $\angle ABC = 90°$, $AB = 12$ cm and $BC = 21$ cm. The point O is 5 cm from AB and 5 cm from BC, as shown in Figure 1.

(*a*) Find the distance of the centre of mass of S from

(i) AB,

(ii) BC. **(9)**

The set square is freely suspended from C and hangs in equilibrium.

(*b*) Find, to the nearest degree, the angle between CB and the vertical. **(3)**

2.3

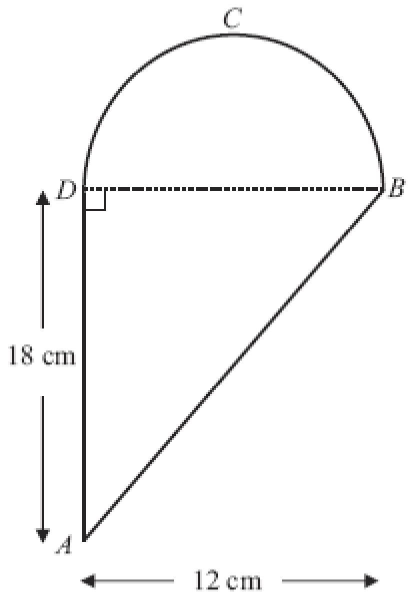


Figure 2

A uniform lamina *ABCD* is made by joining a uniform triangular lamina *ABD* to a uniform semi-circular lamina *DBC*, of the same material, along the edge *BD*, as shown in Figure 2. Triangle *ABD* is right-angled at *D* and *AD* = 18 cm. The semi-circle has diameter *BD* and *BD* = 12 cm.

(*a*) Show that, to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *AD* is 4.69 cm.

(4)

Given that the centre of mass of a uniform semicircular lamina, radius *r*, is at a distance $\frac{4r}{3\pi}$ from the centre of the bounding diameter,

(*b*) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *BD*.

(4)

The lamina is freely suspended from *B* and hangs in equilibrium.

(*c*) Find, to the nearest degree, the angle which *BD* makes with the vertical.

(4)

2.4 [*The centre of mass of a semi-circular lamina of radius r is* $\frac{4r}{3\pi}$ *from the centre*.]

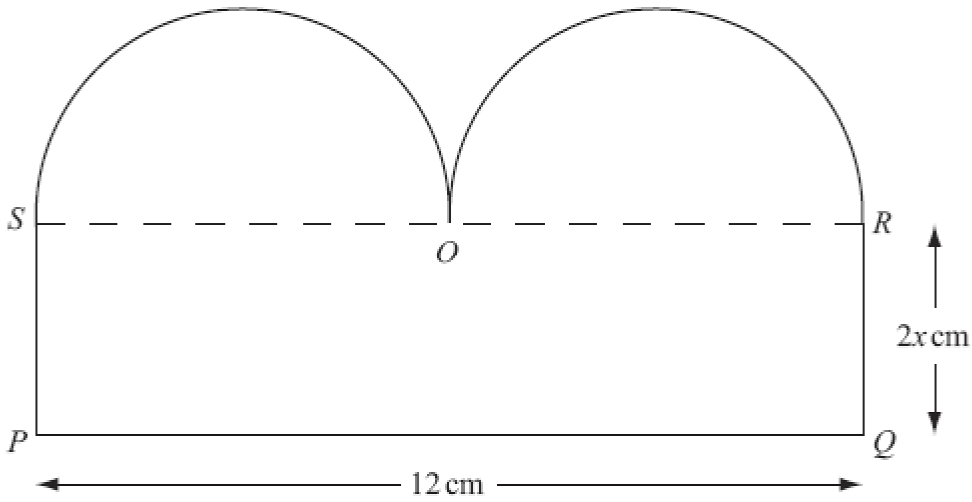


Figure 3

A template *T* consists of a uniform plane lamina *PQROS*, as shown in Figure 3. The lamina is bounded by two semicircles, with diameters *SO* and *OR*, and by the sides *SP*, *PQ* and *QR* of the rectangle *PQRS*. The point *O* is the mid-point of *SR*, *PQ* = 12 cm and *QR* = 2*x* cm.

(*a*) Show that the centre of mass of *T* is a distance $\frac{4|2x^2 - 3|}{8x + 3\pi}$ cm from *SR*.

(7)

The template *T* is freely suspended from the point *P* and hangs in equilibrium.

Given that *x* = 2 and that *θ* is the angle that *PQ* makes with the horizontal,

(*b*) show that tan *θ* = $\frac{48 + 9\pi}{22 + 6\pi}$.

(4)

3 Work and energy

Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.	*Notes:* *Problems involving motion under a constant resistance and/or up and down an inclined plane may be set.*

3.1 A car of mass 800 kg is moving at a constant speed of 15 m s^{-1} down a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{24}$. The resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 900 N.

(*a*) Find, in kW, the rate of working of the engine of the car. **(4)**

When the car is travelling down the road at 15 m s^{-1}, the engine is switched off. The car comes to rest in time *T* seconds after the engine is switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 900 N.

(*b*) Find the value of *T*. **(4)**

3.2 A parcel of mass 2.5 kg is moving in a straight line on a smooth horizontal floor. Initially the parcel is moving with speed 8 m s^{-1}. The parcel is brought to rest in a distance of 20 m by a constant horizontal force of magnitude *R* newtons. Modelling the parcel as a particle, find

(*a*) the kinetic energy lost by the parcel in coming to rest, **(2)**

(*b*) the value of *R*. **(3)**

3.3 A car of mass 1000 kg is moving at a constant speed of 16 m s^{-1} up a straight road inclined at an angle ϑ to the horizontal. The rate of working of the engine of the car is 20 kW and the resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 550 N.

(*a*) Show that $\sin\vartheta = \frac{1}{14}$. **(5)**

When the car is travelling up the road at 16 m s^{-1}, the engine is switched off. The car comes to rest, without braking, having moved a distance *y* metres from the point where the engine was switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 550 N.

(*b*) Find the value of *y*. **(4)**

3.4 A car of mass 1500 kg is moving up a straight road, which is inclined at an angle ϑ to the horizontal, where $\sin \vartheta = \frac{1}{14}$. The resistance to the motion of the car from non-gravitational forces is constant and is modelled as a single constant force of magnitude 650 N. The car's engine is working at a rate of 30 kW.

Find the acceleration of the car at the instant when its speed is 15 m s^{-1}. **(5)**

3.5. A block of mass 10 kg is pulled along a straight horizontal road by a constant horizontal force of magnitude 70 N in the direction of the road. The block moves in a straight line passing through two points *A* and *B* on the road, where *AB* = 50 m. The block is modelled as a particle and the road is modelled as a rough plane. The coefficient of friction between the block and the road is $\frac{4}{7}$.

(*a*) Calculate the work done against friction in moving the block from *A* to *B*. **(4)**

The block passes through *A* with a speed of 2 m s^{-1}.

(*b*) Find the speed of the block at *B*. **(4)**

3.6 A particle of mass 0.5 kg is projected vertically upwards from ground level with a speed of 20 m s^{-1}. It comes to instantaneous rest at a height of 10 m above the ground. As the particle moves it is subject to air resistance of constant magnitude *R* newtons. Using the work-energy principle, or otherwise, find the value of *R*.

(6)

3.7 A cyclist and her bicycle have a total mass of 70 kg. She cycles along a straight horizontal road with constant speed 3.5 m s^{-1}. She is working at a constant rate of 490 W.

(*a*) Find the magnitude of the resistance to motion.

(4)

The cyclist now cycles down a straight road which is inclined at an angle ϑ to the horizontal, where $\sin \vartheta = \frac{1}{14}$, at a constant speed *U* m s^{-1}. The magnitude of the non-gravitational resistance to motion is modelled as 40*U* Newtons. She is now working at a constant rate of 24 W.

(*b*) Find the value of *U*.

4 Collisions

Momentum as a vector. The impulse-momentum principle in vector form. Conservation of linear momentum.	
Direct impact of elastic particles. Newton's law of restitution. Loss of mechanical energy due to impact.	*Notes:* *Candidates will be expected to know and use the inequalities* $0 \leq e \leq 1$ *(where e is the coefficient of restitution).*
Successive impacts of up to three particles or two particles and a smooth plane surface.	*Collision with a plane surface will not involve oblique impact.*

4.1 A particle P of mass m is moving in a straight line on a smooth horizontal table. Another particle Q of mass km is at rest on the table. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. After the collision, the speed of P is v and the speed of Q is $3v$. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(*a*) Find, in terms of v only, the speed of P before the collision.

(3)

(*b*) Find the value of k.

(3)

After being struck by P, the particle Q collides directly with a particle R of mass $11m$ which is at rest on the table. After this second collision, Q and R have the same speed and are moving in opposite directions. Show that

(*c*) the coefficient of restitution between Q and R is $\frac{3}{4}$,

(4)

(*d*) there will be a further collision between P and Q.

(2)

4.2 A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P. The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(*a*) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$.

(5)

(*b*) Find the total kinetic energy lost in the collision.

(5)

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

(*c*) Calculate the range of values of e for which there will be a second collision between P and Q.

(7)

4.3 A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(*a*) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$. **(5)**

The speed of P immediately after the collision is $\frac{1}{2}u$.

(*b*) Show that $e = \frac{1}{4}$. **(4)**

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(*c*) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall. **(4)**

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(*d*) find, in terms of d, the distance of the point B from the wall. **(4)**

4.4 Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $2u$ and the speed of Q is u. The coefficient of restitution between the particles is e, where $e < 1$. Find, in terms of u and e,

(i) the speed of P immediately after the collision,

(ii) the speed of Q immediately after the collision. **(7)**

4.5

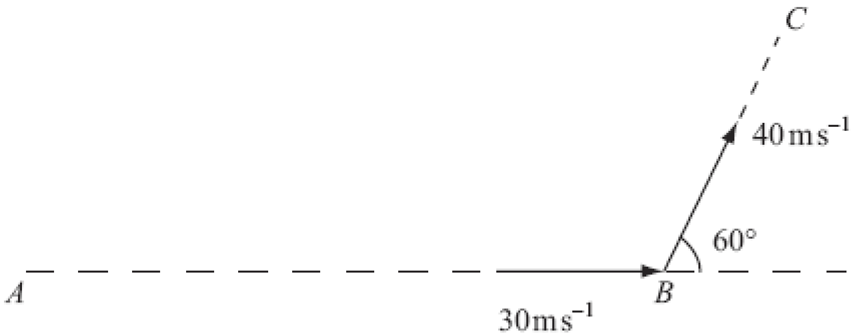


Figure 1

The points A, B and C lie in a horizontal plane. A batsman strikes a ball of mass 0.25 kg. Immediately before being struck, the ball is moving along the horizontal line AB with speed 30 m s^{-1}. Immediately after being struck, the ball moves along the horizontal line BC with speed 40 m s^{-1}. The line BC makes an angle of 60° with the original direction of motion AB, as shown in Figure 1.

Find, to 3 significant figures,

(i) the magnitude of the impulse given to the ball,

(ii) the size of the angle that the direction of this impulse makes with the original direction of motion AB.

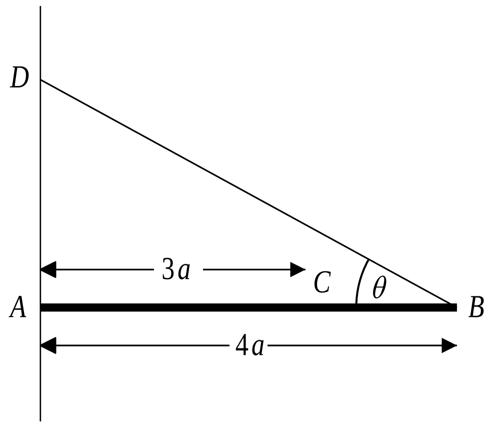
(8)

5 Statics of rigid bodies

Moment of a force.	
Equilibrium of rigid bodies.	*Notes:* *Problems involving parallel and non-parallel coplanar forces. Problems may include rods or ladders resting against smooth or rough vertical walls and on smooth or rough ground.*

5.1

Figure 2



A horizontal uniform rod *AB* has mass *m* and length 4*a*. The end *A* rests against a rough vertical wall. A particle of mass 2*m* is attached to the rod at the point *C*, where *AC* = 3*a*. One end of a light inextensible string *BD* is attached to the rod at *B* and the other end is attached to the wall at a point *D*, where *D* is vertically above *A*. The rod is in equilibrium in a vertical plane perpendicular to the wall. The string is inclined at an angle θ to the horizontal, where tan $\theta = \frac{3}{4}$, as shown in Figure 2.

(*a*) Find the tension in the string.

(5)

(*b*) Show that the horizontal component of the force exerted by the wall on the rod has magnitude $\frac{8}{3}$ *mg*.

(3)

The coefficient of friction between the wall and the rod is μ. Given that the rod is in limiting equilibrium,

(*c*) find the value of μ.

(4)

5.2

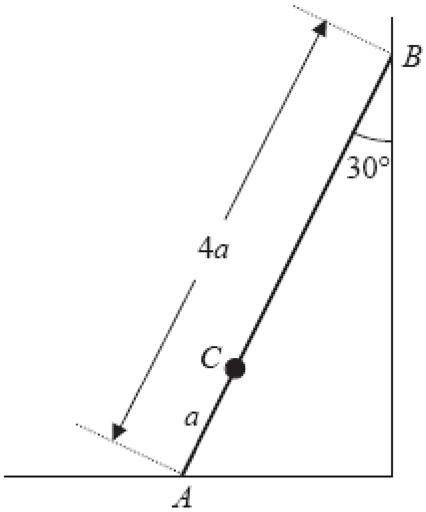


Figure 2

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground. **(10)**

5.3

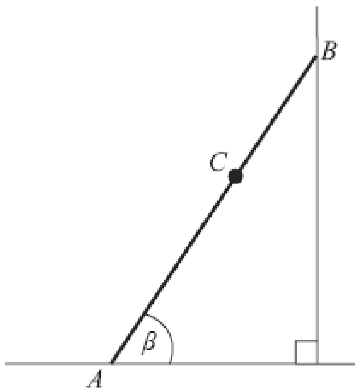


Figure 1

Figure 1 shows a ladder AB, of mass 25 kg and length 4 m, resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point C on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(*a*) Find the magnitude of the frictional force of the ground on the ladder. **(3)**

(*b*) Find, to the nearest degree, the value of β. **(6)**

(*c*) State how you have used the modelling assumption that Reece is a particle. **(1)**

5.4

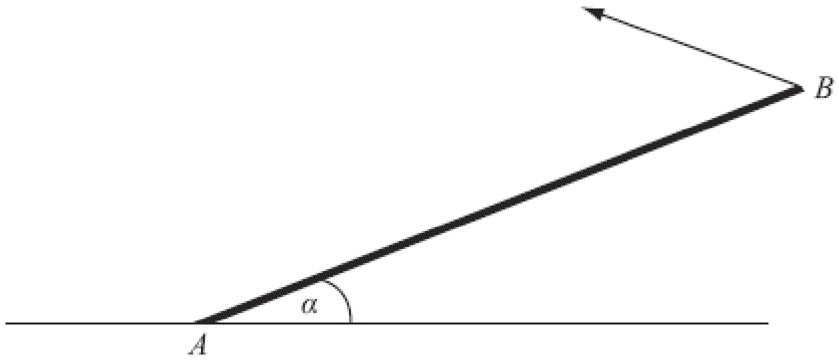


Figure 2

A uniform rod *AB*, of mass 20 kg and length 4 m, rests with one end *A* on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$, by a force acting at *B*, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5.

Find the magnitude of the normal reaction of the ground on the rod at *A*.

(7)

Answers

1.1) a) 50 J b) 0.32

1.2 a) $(3t^2-6)\boldsymbol{i}+4t\,\boldsymbol{j}$ b) Proof c) 6.5 d) $157°$

1.3 a) Proof b) $55°$ c) 60

1.4 a) $(6\mathrm{t}-6)\mathbf{i}+(9\mathrm{t}^2-4)\mathbf{j}$ b) $\frac{2}{3}$ c) 4**i** – 7**j**

1.5 a) Proof b) 3 c) 34.5

1.6 a) 36 b) 7.2

1.7 a) 19.2 b) Proof c) 24 d) $\frac{3}{4}$ e) 2.5 s

f) e.g. Variable '*g*', Air resistance, Speed of wind, Swing of ball, The ball is not a particle.

1.8 $\frac{38}{27}$

1.9 a) Proof b) i) 10c ii) 2.5 c^2 c) 5 $\left(c+\frac{1}{c}\right)$

2.1 a) 25 b) $\frac{3}{11}m$

2.2 a)i) 7.6 ii) 3.7 b) $15°$

2.3 a) Proof b) 3.06 c) $23°$

2.4 a) Proof b) Proof

3.1 a) 8.6 kW b) 21

3.2 a) 80 J b) 4

3.3 a) Proof b) 102

3.4 0.2

3.5 a) 2800 J b) 12

3.6 5.1

3.7 a) 140 N b) 1.6

4.1 a) 8v b) 3 c) Proof d) $\frac{9}{8}V > V\ hence\ further\ collision$

4.2 a) Proof b) $\frac{9}{20}mu^2$ c) $e > \frac{1}{4}$

4.3 a) Proof b) Proof c) Proof d) $\frac{1}{5}d$

4.4 i) u(1 - e) ii) u(1 + 2e)

4.5 i) 9.01 ii) $106°$

5.1 a) $\frac{10}{3}mg$ b) Proof c) $\frac{3}{8}$

5.2 $\frac{5}{16\sqrt{3}}$

5.3 a) 44g b) $56°$ c) So that Reece's weight acts directly at the point *C*

5.4 160 N