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Mechanics M2
Advanced/Advanced Subsidiary

Friday 16 June 2017 - Afternoon Paper Reference
Time: $\mathbf{1}$ hour $\mathbf{3 0}$ minutes
6678/01

You must have:
Total Marks
Mathematical Formulae and Statistical Tables (Pink)

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/ sketches/ graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided.
- there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.


## Information

- The total mark for this paper is 75 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. A particle $P$ of mass 0.5 kg is moving with velocity $4 \mathbf{j} \mathrm{~m} \mathrm{~s}^{-1}$ when it receives an impulse I Ns. Immediately after $P$ receives the impulse, the velocity of $P$ is $(2 \mathbf{i}+3 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$.

Find
(a) the magnitude of $\mathbf{I}$,
(b) the angle between $\mathbf{I}$ and $\mathbf{j}$.
2. A truck of mass 900 kg is towing a trailer of mass 150 kg up an inclined straight road with constant speed $15 \mathrm{~m} \mathrm{~s}^{-1}$. The trailer is attached to the truck by a light inextensible towbar which is parallel to the road. The road is inclined at an angle $\theta$ to the horizontal, where $\sin \theta=\frac{\mathbf{1}}{\boldsymbol{9}}$. The resistance to motion of the truck from non-gravitational forces has constant magnitude 200 N and the resistance to motion of the trailer from non-gravitational forces has constant magnitude 50 N .
(a) Find the rate at which the engine of the truck is working.

When the truck and trailer are moving up the road at $15 \mathrm{~m} \mathrm{~s}^{-1}$ the towbar breaks, and the trailer is no longer attached to the truck. The rate at which the engine of the truck is working is unchanged. The resistance to motion of the truck from non-gravitational forces and the resistance to motion of the trailer from non-gravitational forces are still forces of constant magnitudes 200 N and 50 N respectively.
(b) Find the acceleration of the truck at the instant after the towbar breaks.
(c) Use the work-energy principle to find out how much further up the road the trailer travels before coming to instantaneous rest.
(Total 12 marks)
3.


Figure 1


Figure 2

The uniform rectangular lamina $A B D E$, shown in Figure 1, has side $A B$ of length $2 a$ and side $B D$ of length $6 a$. The point $C$ divides $B D$ in the ratio 1:2 and the point $F$ divides $E A$ in the ratio 1: 2. The rectangular lamina is folded along $F C$ to produce the folded lamina $L$, shown in Figure 2.
(a) Show that the centre of mass of $L$ is $\frac{\mathbf{1 6}}{\mathbf{9}} a$ from $E F$.

The folded lamina, $L$, is freely suspended from $C$ and hangs in equilibrium.
(b) Find the size of the angle between $C F$ and the downward vertical.
4. At time $t=0$ a particle $P$ leaves the origin $O$ and moves along the $x$-axis. At time $t$ seconds, the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$ in the positive $x$ direction, where

$$
v=3 t^{2}-16 t+21
$$

The particle is instantaneously at rest when $t=t_{1}$ and when $t=t_{2}\left(t_{1}<t_{2}\right)$.
(a) Find the value of $t_{1}$ and the value of $t_{2}$.
(b) Find the magnitude of the acceleration of $P$ at the instant when $t=t_{1}$.
(c) Find the distance travelled by $P$ in the interval $t_{1} \leqslant t \leqslant t_{2}$.
(d) Show that $P$ does not return to $O$.
5.


Figure 3
A uniform rod $A B$, of mass 5 kg and length 8 m , has its end $B$ resting on rough horizontal ground. The rod is held in limiting equilibrium at an angle $\alpha$ to the horizontal, where $\tan \alpha=\frac{\mathbf{3}}{\mathbf{4}}$, by a rope attached to the rod at $C$. The distance $A C=1 \mathrm{~m}$. The rope is in the same vertical plane as the rod. The angle between the rope and the rod is $\beta$ and the tension in the rope is $T$ newtons, as shown in Figure 3. The coefficient of friction between the rod and the ground is $\frac{2}{3}$. The vertical component of the force exerted on the rod at $B$ by the ground is $R$ newtons.
(a) Find the value of $R$.
(b) Find the size of angle $\beta$.
6.


Figure 4
The points $A$ and $B$ lie 40 m apart on horizontal ground. At time $t=0$ the particles $P$ and $Q$ are projected in the vertical plane containing $A B$ and move freely under gravity. Particle $P$ is projected from $A$ with speed $30 \mathrm{~m} \mathrm{~s}^{-1}$ at $60^{\circ}$ to $A B$ and particle $Q$ is projected from $B$ with speed $q \mathrm{~m} \mathrm{~s}^{-1}$ at angle $\theta$ to $B A$, as shown in Figure 4.

At $t=2$ seconds, $P$ and $Q$ collide.
(a) Find
(i) the size of angle $\theta$,
(ii) the value of $q$.
(b) Find the speed of $P$ at the instant before it collides with $Q$.
(Total 11 marks)
7. Two particles $A$ and $B$, of masses $3 m$ and $4 m$ respectively, lie at rest on a smooth horizontal surface. Particle $B$ lies between $A$ and a smooth vertical wall which is perpendicular to the line joining $A$ and $B$. Particle $B$ is projected with speed $5 u$ in a direction perpendicular to the wall and collides with the wall. The coefficient of restitution between $B$ and the wall is $\frac{3}{5}$.
(a) Find the magnitude of the impulse received by $B$ in the collision with the wall.

After the collision with the wall, $B$ rebounds from the wall and collides directly with $A$. The coefficient of restitution between $A$ and $B$ is $e$.
(b) Show that, immediately after they collide, $A$ and $B$ are both moving in the same direction.

The kinetic energy of $B$ immediately after it collides with $A$ is one quarter of the kinetic energy of $B$ immediately before it collides with $A$.
(c) Find the value of $e$.
(Total 14 marks)

