

Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE in Mechanics 2
(6678_01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \checkmark will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

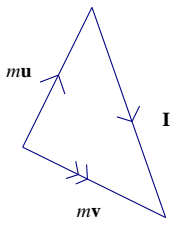
RHS, LHS Right hand side, left hand side.

Q	Scheme	Marks	Notes
1a	$t = 0, v = 11 \Rightarrow r = 11$	B1	
	$t = 2, v = 3 \Rightarrow 4p + 2q + 11 = 3,$	M1	Accept $4p + 2q + r = 3$
	$4p + 2q = -8$	A1	Any equivalent unsimplified form with 11 used
	Differentiate to find acceleration	M1	OR use symmetry, $t = 4, v = 11$
	$a = 2pt + q$	A1	$\Rightarrow 11 = 16p + 4q + 11, 4p + q = 0$
	$t = 2, a = 0 \Rightarrow 4p + q = 0$	DM1	2 nd eqn in p & q and solve for p & q Dependent on both previous m marks
	$\Rightarrow -q + 2q = -8, q = -8, p = 2$	A1	
	$(v = 2t^2 - 8t + 11)$		
	$t = 3, a = 4t - 8 = 4 \text{ (ms}^{-2}\text{)}$	A1	
		(8)	
1a alt	Min speed at $t = 2 \Rightarrow$ $v = (pt^2 + qt + r) = k(t - 2)^2 + c$	B1	
		M1	Completed square form.
	$v = k(t - 2)^2 + 3$	A1	Correct completed square form
	$t = 0, v = 11 \Rightarrow 4k + 3 = 11,$	M1	Solve for k
	$k = 2$	A1	$v = 2(t - 2)^2 + 3 (= 2t^2 - 8t + 11)$
	Differentiate to find acceleration	DM1	Dependent on both previous m marks
	$a = 4(t - 2)$	A1	
	$t = 3, a = 4 \text{ (m s}^{-2}\text{)}$	A1	
		(8)	
1b	Integrate: $\int 2(t - 2)^2 + 3 dt = \frac{2}{3}(t - 2)^3 + 3t (+C)$ or $\int 2t^2 - 8t + 11 dt = \frac{2}{3}t^3 - 4t^2 + 11t (+C)$	M1	follow their coefficients found in (a) Accept in p, q, r
	At most one error seen	A1ft	For their coefficients
	All correct	A1ft	For their coefficients provided $\neq 0$
	$\left[\frac{2}{3}(t - 2)^3 + 3t \right]_2^3 = \left(\frac{2}{3} + 9 \right) - (0 + 6)$ or $\left[\frac{2}{3}t^3 - 4t^2 + 11t \right]_2^3$ $= (18 - 36 + 33) - \left(\frac{16}{3} - 16 + 22 \right)$	DM1	Use of $t = 2, t = 3$ as limits on a definite integral (or subtract distances to cancel C). Dependent on having integrated. Allow with p, q, r

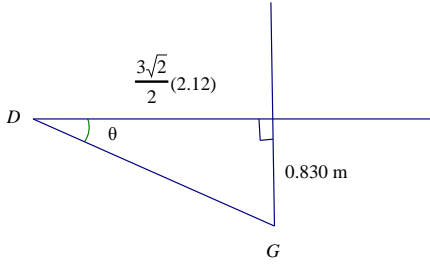
Q	Scheme	Marks	Notes
	$3\frac{2}{3}$ (m)	A1	Accept exact equivalent or 3.7 or better
		(5)	
		[13]	

Q	Scheme	Marks	Notes
2a		M1	Equation of motion up or down the road. Requires all 3 terms. Condone sign errors and trig confusion. Must be dimensionally correct.
	$F = mg \sin \theta + R$ ($F = R + 392$)	A1	Correct equation up the road
	$G + mg \sin \theta = R$ ($G = R - 392$)	A1	Correct equation down the road
	$F = \frac{3P}{12.5}$ or $G = \frac{P}{12.5}$ $\Rightarrow \frac{3P}{12.5} = 392 + R$ or $\frac{P}{12.5} = R - 392$	B1	Use of $F = \frac{P}{v}$ at least once
	$\frac{2P}{12.5} = 2 \times 392$, $2R = \frac{4P}{12.5}$	M1	Solve simultaneous equations for P or R , provided $F \neq G$ and P and $3P$ used correctly
	$P = 4900$ (500g), $R = 784$ (80g)	A1	CSO. Both values correct. Accept 2sf, 3sf or an exact multiple of g
		(6)	
2b	Must be using work-energy.		
	KE lost = PE gained + WD against R	M1	Equation needs all 3 terms and no extras. Condone sign errors.
	$\frac{1}{2} \times 800 \times 12.5^2$ $= 800 \times 9.8 \times \frac{d}{20} + (\text{their } R) \times d$	A1	At most 1 error. Allow with R (with trig. substituted) $(62500 = 392d + Rd)$
		A1ft	Correct equation in their R (with trig. substituted)
	$d = \frac{62500}{1176} = 53.1(\text{m})$	A1	CSO. Accept 53(m)
		(4)	
		[10]	

Q	Scheme	Marks	Notes
3.	Since this question is about the magnitude of the impulse, condone subtraction in the "wrong order" throughout.		
	$m\mathbf{v} - m\mathbf{u} = 0.6(2c\mathbf{i} - c\mathbf{j} - c\mathbf{i} - 2c\mathbf{j})$	M1	Impulse = change in momentum Marking the RHS only
	$= 0.6(c\mathbf{i} - 3c\mathbf{j})$	A1	
	Magnitude $= 0.6\sqrt{c^2 + 9c^2}$	DM1	Correct use of Pythagoras' theorem on $m\mathbf{v} - m\mathbf{u}$ or $\mathbf{v} - \mathbf{u}$ Marking the RHS only. Dependent on the previous M1
	$= 0.6\sqrt{10}c \quad (= 0.6\sqrt{10}c^2)$	A1	Accept $\sqrt{10}c$ for change in velocity
The next two marks are not available to a candidate who has equated a scalar to a vector.			
	$2\sqrt{10} = 0.6\sqrt{10}c$	DM1	Equate & solve for c Dependent on the previous M1
	$c = \frac{10}{3}$	A1	Accept 3.3 or better
		(6)	
alt	$m\mathbf{v} - m\mathbf{u} = 0.6(2c\mathbf{i} - c\mathbf{j} - c\mathbf{i} - 2c\mathbf{j})$	M1	change in momentum
	$= 0.6(c\mathbf{i} - 3c\mathbf{j})$	A1	
	Square of magnitude	DM1	
	$= 0.36(10c^2)$	A1	
The next two marks are not available to a candidate who has equated a scalar to a vector.			
	$40 = 0.36(c^2 + 9c^2) \quad ,$	DM1	Equate & solve for c
	$c = \frac{10}{3}$	A1	
		(6)	
alt	$\begin{pmatrix} 2\sqrt{10} \cos \theta \\ 2\sqrt{10} \sin \theta \end{pmatrix} = 0.6 \begin{pmatrix} 2c - c \\ -c - 2c \end{pmatrix}$	M1	Impulse momentum equation
	$= 0.6c \begin{pmatrix} 1 \\ -3 \end{pmatrix}$	A1	Correct equation
	$2\sqrt{10} \cos \theta = 0.6c$ $2\sqrt{10} \sin \theta = -3 \times 0.6c$	DM1	Compare coefficients and form equation for θ
	$\tan \theta = -3 \Rightarrow \cos \theta = (\pm) \frac{1}{\sqrt{10}}$	A1	$\cos \theta$ or $\sin \theta$ correct
	$2\sqrt{10} \cos \theta = 0.6c$	DM1	
	$\Rightarrow c = \frac{10}{3}$	A1	

alt		M1	Impulse momentum triangle Units used for the vectors must be dimensionally correct
	Sides of magnitude $\sqrt{5}c, \sqrt{5}c, \frac{10\sqrt{10}}{3}$ or $\frac{3\sqrt{5}c}{5}, \frac{3\sqrt{5}c}{5}, 2\sqrt{10}$	A1	
	$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} c \\ 2c \end{pmatrix} \cdot \begin{pmatrix} 2c \\ -c \end{pmatrix}$	DM1	Use of scalar product
	$= 2c^2 - 2c^2 = 0 \therefore \text{at } 90^\circ$	A1	...to show velocities perpendicular
	$(0.6 \times \sqrt{5}c)^2 + (0.6 \times \sqrt{5}c)^2 = (2\sqrt{10})^2$	DM1	Use of Pythagoras' theorem in a right angled triangle
	$\frac{18c^2}{5} = 40, \quad c = \frac{10}{3}$	A1	
		(6)	

Q	Scheme			Marks	
4a		Triangle	sector	B1	Mass ratios Distances Distances from AD are $-\frac{1}{3} \times \frac{3\sqrt{2}}{2}$ and $\frac{16\sqrt{2}}{3\pi} - \frac{3\sqrt{2}}{2}$ ($= 0.280$)
	mass	4.5	4π ($= 12.56..$)		
	c of m from O	$\frac{2}{3} \times \frac{3\sqrt{2}}{2}$ ($= 1.41...$)	$\frac{16\sqrt{2}}{3\pi}$ ($= 2.40....$)	B1	
	: $4\pi \times \frac{16\sqrt{2}}{3\pi} - 4.5\sqrt{2} \left(= \frac{101\sqrt{2}}{6} \right) = (4\pi - 4.5)d$			M1	Moments about an axis through O and parallel to DA . Terms must be dimensionally correct. Shapes combined correctly.
				A1	Correct unsimplified equation
	$d = \frac{101\sqrt{2}}{6(4\pi - 4.5)} = 2.951.....$				(distance from O)
	Distance from $DA = 2.951... - \frac{3\sqrt{2}}{2}$ $= 0.830$ (0.83) m			A1	Accept $\frac{101\sqrt{2}}{6(4\pi - 4.5)} - \frac{3\sqrt{2}}{2}$
				(5)	
4a alt		Triangle	sector	B1	Mass ratios Distances
	mass	4.5	4π ($= 12.56..$)		
	c of m from both axes OC, OB	1	$\frac{16\sqrt{2}}{3\pi} \times \frac{1}{\sqrt{2}} = \frac{16}{3\pi}$	B1	
	$4\pi \times \frac{16}{3\pi} - 4.51 = (4\pi - 4.5)\bar{x}$			M1	Moments about an axis through O . Terms must be dimensionally correct. Condone sign error(s)
	$\left(\bar{x} = \bar{y} = \frac{101}{6(4\pi - 4.5)} \right)$			A1	Correct unsimplified equation
	$d = \frac{101\sqrt{2}}{6(4\pi - 4.5)}$				Distance from O
	Distance from $DA = 2.951... - \frac{3\sqrt{2}}{2}$ $= 0.830$ (0.83) m			A1	
				(5)	

4b			
	$\tan \theta = \frac{\text{their } 0.830}{2.12}$ or $\tan \phi = \frac{2.12}{\text{their } 0.830}$	M1	Use of tan to find a relevant angle:
	21.4° or 68.6°	A1	
	Angle between DC and downward vertical = 135° - their θ	M1	Correct method for the required angle
	= 114°	A1	The Q asks for the angle to the nearest degree.
		(4)	
4balt	$GD^2 = OD^2 + OG^2 - 2OD \cdot OG \cos 45$ $(GD = 2.28) \quad \frac{\sin 45}{DG} = \frac{\sin \theta}{OG}$	M1	Complete method to find angle ODG
	$\Rightarrow \theta = 66.4^\circ$	A1	
		M1	Correct method for the required angle
	Required angle = 180 - 66.4 = 114°	A1	The Q asks for the angle to the nearest degree.
		(4)	
		[9]	

Q	Scheme	Marks	Notes
5a	M(A): $d \cos \theta \times 5g = 4P$	M1	Terms must be dimensionally correct. Condone trig confusion
		A1	
	Resolving horizontally: $P \sin \theta = F$	B1	
	Resolving vertically: $P \cos \theta + R = 5g$	M1	Requires all 3 terms. Condone trig confusion and sign errors
		A1	Correct equation
		DM1	Substitute for P to find R or F Dependent on both previous M marks
	$R = 5g - \frac{5gd \cos^2 \theta}{4}$	A1	One force correct. Accept equivalent forms e.g. $R = \frac{20g - 5gd + 20g \tan^2 \theta}{4(1 + \tan^2 \theta)}$
	$F = \frac{5gd \cos \theta \sin \theta}{4}$	A1	Both forces correct. Accept equivalent forms e.g. $F = \frac{5gd \tan \theta}{4 \sec^2 \theta}$
		(8)	
	5a alt	M(B): $5g \cos \theta \times (4 - d) + F \sin \theta \times 4 = R \cos \theta \times 4$	M1
		A1	At most one error
Resolve parallel to the rod: $5g \sin \theta = R \sin \theta + F \cos \theta$		M1	Requires all 3 terms. Condone trig confusion and sign errors
		B1	At most one error
		A1	Correct equation
$\Rightarrow R = 5g - \frac{F \cos \theta}{\sin \theta}$			
$5g \cos \theta \times (4 - d) + F \sin \theta \times 4$ $= 4 \cos \theta \left(5g - \frac{F \cos \theta}{\sin \theta} \right)$		DM1	Eliminate one variable to find F or R Dependent on both previous M marks
$4F \left(\sin \theta + \frac{\cos^2 \theta}{\sin \theta} \right)$ $= 20g \cos \theta - 20g \cos \theta + 5gd \cos \theta$			
$F = \frac{5gd \cos \theta \sin \theta}{4}$		A1	One force correct
$R = 5g - \frac{5gd \cos^2 \theta}{4}$		A1	Both forces correct
			See next page for part (b)

Q	Scheme	Marks	Notes
6a	Horizontal motion: $x = 3t$	B1	
	Vertical motion: $y = 4t - \frac{g}{2}t^2$	M1	Correct use of <i>suvat</i> . Condone sign error(s)
		A1	
	$\left(y = 4 \times \frac{x}{3} - \frac{g}{2} \times \frac{x^2}{9} \right), \lambda = - \left(\frac{4\lambda}{3} - \frac{g\lambda^2}{18} \right)$	M1	Use $y = -x$ and form an equation in one variable
	$\frac{7\lambda}{3} = \frac{g\lambda^2}{18}$	M1	solve for λ
	$\lambda = \frac{42}{g}$ or 4.3 (4.29)	A1 (6)	Not $\frac{30}{7}$
alta	Horizontal motion: $x = 3t$	B1	
	Vertical motion: $y = 4t - \frac{g}{2}t^2$	M1	Correct use of <i>suvat</i> . Condone sign error(s)
		A1	
	$\Rightarrow -3t = 4t - \frac{1}{2}gt^2, \left(t = \frac{14}{g} \right)$	M1	Use $y = -x$ and form an equation in one variable
	$\lambda = 3t$	M1	Solve for λ
	$\lambda = 4.3$ (4.29)	A1 (6)	
6b	At A: $v \rightarrow 3 \text{ (m s}^{-1}\text{)}$	B1	
	$v \uparrow 4 - g \times \frac{14}{g}$	M1	Complete method using <i>suvat</i> to find $v \uparrow$ with their t or λ
	$= -10 \text{ (m s}^{-1}\text{)}$	A1	Accept +10 with direction confirmed by diagram
	Speed = $\sqrt{(\text{their } 10)^2 + (3)^2}$	DM1	Dependent on the first M1 in (b)
	$= \sqrt{109} \text{ (m s}^{-1}\text{)}$	A1	(10.4) Allow for $v \uparrow = 10$
	$\tan^{-1}\left(\frac{\text{their } 10}{3}\right)$ or $\tan^{-1}\left(\frac{3}{\text{their } 10}\right)$	DM1	Use trig to find a relevant angle. Dependent on the first M1 in (b)
	Direction = 73.3° below the horizontal	A1	(1.28 radians) Accept direction $3\mathbf{i} - 10\mathbf{j}$ Do not accept a bearing
		(7)	
Alt 6b	Loss in GPE : $mg\lambda = 42m$	B1	
	Gain in KE : $\frac{1}{2}mv^2 - \frac{1}{2}m \times 25$	M1	Terms must be dimensionally correct. Condone sign error.
		A1	
	Solve for v : $42 = \frac{1}{2}v^2 - \frac{25}{2}$	M1	
	$v = \sqrt{109}$	A1	
	$v \cos \theta = 3$	M1	Use trig. to find a relevant angle
	$\theta = 73.3^\circ$ below the horizontal	A1 (7)	Accept correct angle marked correctly on a diagram.
		[13]	

Q	Scheme	Marks	Notes
7a			
	CLM: $6mu = 2mv + 3mw$	M1	Requires all 3 terms. Must be dimensionally correct. Condone sign error(s)
	$(6u = 2v + 3w)$	A1	This equation defines their directions
	Impact: $w - v = \frac{3}{4} \times 3u \left(= \frac{9}{4}u \right)$	M1	Must be used with e on the correct side
		A1	Penalise inconsistent directions here
	$6u = 2w - \frac{9}{2}u + 3w$	DM1	Solve simultaneous equations for v or w Dependent on the 2 previous M marks
	$w = \frac{21}{10}u = v_B$	A1	One correct
	$v = w - \frac{9}{4}u = \left(\frac{21}{10} - \frac{9}{4} \right)u = -\frac{3}{20}u, v_A = \frac{3}{20}u$	A1	Both correct
		(7)	
7b	Speed of B after hitting wall $= \frac{21}{10}ue$	M1(B1)	$e \times$ their w
	Impulse $= \frac{27}{4}mu = 3m \left(\frac{21}{10}u + \frac{21}{10}ue \right)$	M1	for their w . Must be trying to use the correct equation with $3m$.
	$\frac{9}{4} = \frac{21}{10}(1+e), e = \frac{1}{14}$	A1 (3)	
7b alt	Impulse $= \frac{27}{4}mu = 3m \left(\frac{21}{10}u + V \right), \left(V = \frac{3u}{20} \right)$	M1(B1)	Use impulse to find V . Must be trying to use the correct equation with $3m$.
	$\frac{21u}{10}e = \frac{3u}{20}$,	M1	$V = e \times$ their w .
	$e = \frac{1}{14}$	A1 (3)	
7c	Speed of B after second impact $=$ $\frac{1}{14} \times \frac{21}{10}u = \frac{3}{20}u$	B1ft	Compare two relevant speeds. (ft on their V or their $e \times$ their w)
	Same velocity (and A has a head start), so no collision.	B1 (2)	From correct work only
		[12]	

