Name: ________________________________________

TT5 remedial work

**Date:**

**Time:** 0 minute

**Total marks available:** 60

**Total marks achieved:** ______

**Peter Vermeer**

# Questions

Q1.

(a) Express 2cos$\theta$ – sin$\theta$ in the form $R\cos(\theta + a)$, where $R$ and $a$ are constants, $R > 0$ and $0 < a$ 90° Give the exact value of $R$ and give the value of $a$ to 2 decimal places.

**(3)**

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

**(5)**

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of $\theta$ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

**(2)**

**(Total for question = 10 marks)**

Q2.

The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{dx}{dt} = -\frac{5}{2}x, \qquad t \geqslant 0$$

where $x$ is the mass of the substance measured in grams and $t$ is the time measured in days.

Given that $x = 60$ when $t = 0$,

(a) solve the differential equation, giving $x$ in terms of $t$. You should show all steps in your working and give your answer in its simplest form.

**(4)**

(b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams.

Give your answer to the nearest minute.

**(3)**

**(Total for question = 7 marks)**

Q3.

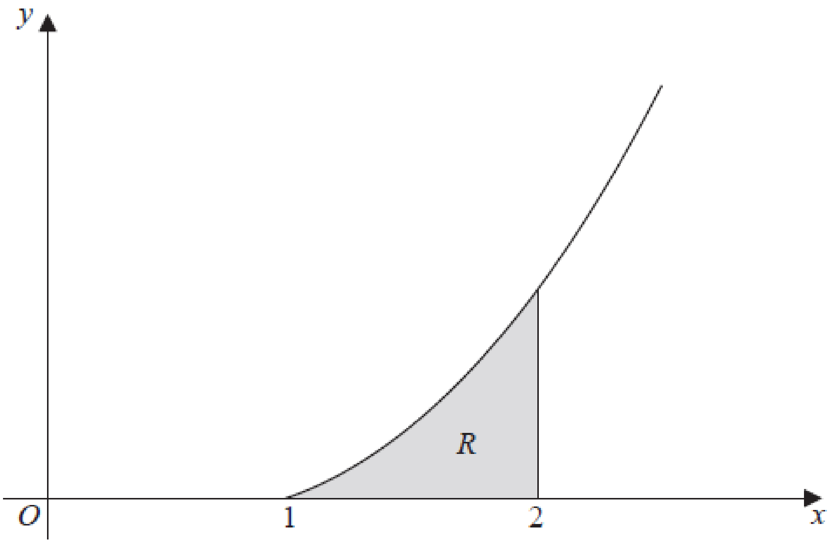


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x$, $x \geq 1$

The finite region *R*, shown shaded in Figure 1, is bounded by the curve, the *x*-axis and the line $x = 2$

The table below shows corresponding values of *x* and *y* for $y = x^2 \ln x$

| $x$ | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
|---|---|---|---|---|---|---|
| $y$ | 0 | 0.2625 |  | 1.2032 | 1.9044 | 2.7726 |

(a) Complete the table above, giving the missing value of *y* to 4 decimal places.

**(1)**

(b) Use the trapezium rule with all the values of *y* in the completed table to obtain an estimate for the area of *R*, giving your answer to 3 decimal places.

**(3)**

(c) Use integration to find the exact value for the area of *R*.

**(5)**

**(Total for question = 9 marks)**

Q4.

With respect to a fixed origin $O$, the line $l_1$ is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where $\mu$ is a scalar parameter.

The point $A$ lies on $l_1$ where $\mu = 1$

(a) Find the coordinates of $A$.

**(1)**

The point $P$ has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$.

The line $l_2$ passes through the point $P$ and is parallel to the line $l_1$

(b) Write down a vector equation for the line $l_2$

**(2)**

(c) Find the exact value of the distance $AP$.

Give your answer in the form $k\sqrt{2}$ , where $k$ is a constant to be determined.

**(2)**

The acute angle between $AP$ and $l_2$ is $\theta$.

(d) Find the value of $\cos\theta$

**(3)**

*A* point $E$ lies on the line $l_2$
Given that $AP = PE$,

(e) find the area of triangle $APE$,

**(2)**

(f) find the coordinates of the two possible positions of $E$.

**(5)**

**(Total for question = 15 marks)**

Q5.

A car of mass 800kg is moving on a straight road which is inclined at an angle $\theta$ to the horizontal, where sin $\theta$ = $\frac{1}{20}$. The resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude $R$ newtons. When the car is moving up the road at a constant speed of 12.5ms$^{-1}$, the engine of the car is working at a constant rate of 3$P$ watts. When the car is moving down the road at a constant speed of 12.5ms$^{-1}$, the engine of the car is working at a constant rate of $P$ watts.

(a) Find

(i) the value of $P$,

(ii) the value of $R$.

When the car is moving up the road at 12.5ms$^{-1}$ the engine is switched off and the car comes to rest, without braking, in a distance $d$ metres. The resistance to the motion of the car from non-gravitational forces is still modelled as a constant force of magnitude $R$ newtons.

(b) Use the work-energy principle to find the value of $d$.

**(4)**

**(Total for question = 4 marks)**

Q6.

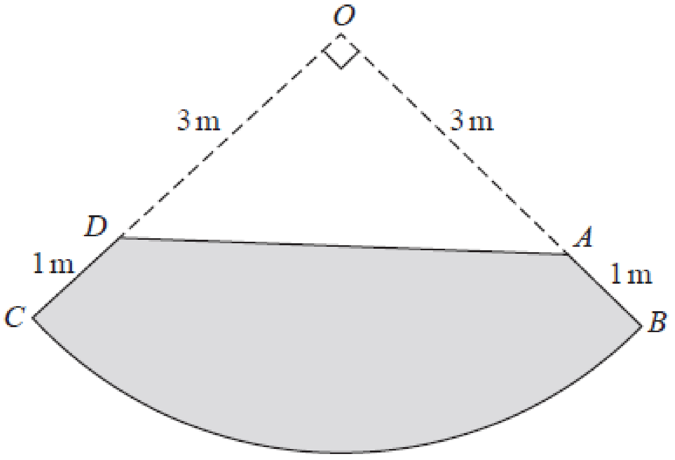


**Figure 1**

The uniform lamina *OBC* is one quarter of a circular disc with centre *O* and radius 4m. The points *A* and *D*, on *OB* and *OC* respectively, are 3 m from *O*. The uniform lamina *ABCD*, shown shaded in Figure 1, is formed by removing the triangle *OAD* from *OBC*.

Given that the centre of mass of one quarter of a uniform circular disc of radius $r$ is at a distance $r$ from the centre of the disc,

(a) find the distance of the centre of mass of the lamina *ABCD* from *AD*.

**(5)**

The lamina is freely suspended from *D* and hangs in equilibrium.

(b) Find, to the nearest degree, the angle between *DC* and the downward vertical.

**(4)**

**(Total for question = 9 marks)**

## Examiner's Report

Q1.

In part (a) the majority of students found $R$ and $\alpha$ correctly. The most frequent error was in stating $\tan \alpha = -\frac{1}{2}$ or occasionally $\tan \alpha = 2$. In just a few responses, a decimal value rather than a surd was given for $R$ losing the last mark.

A majority of students scored 3 or 4 marks out of 5 on part (b). A common error was the lack of accuracy in calculations, with answers of 32.9 (rather than 33.0) commonly seen. Many students were able to successfully find a second solution to the given equation.

Part (c) proved to be discriminating with only a small minority of students being successful. Having solved part (b) by going from $\cos(\theta + 26.57) = 0.5068$ to $\theta + 26.57 = 59.54$ to $\theta = 59.54 - 26.57$, it was hoped that for part (c) we would see students go from $\cos(\theta - 26.57) = 0.5068$ to $\theta = 59.54 + 26.57$. However, most correct answers came from a total restart and students therefore undertaking a great deal of work. One common error was to state the smallest answer in part (b) with little or no working, assuming that the equation in part (c) was the same as that in part (b). Another error was to somehow attempt to use the minimum or maximum values of the cosine function, sometimes using –1 in any way possible.

## Q2.

This question on the topic of differential equations discriminated well across students of all abilities.

In part (a), the majority of students separated the variables correctly to give $\int \frac{1}{x}\mathrm{d}x = \int -\frac{5}{2}\mathrm{d}t$ and integrated both sides to give a correct equation containing a constant of integration. A minority who separated their variables to obtain either $\int x\mathrm{d}x = \int -\frac{5}{2}\mathrm{d}t$ or $\int \frac{2x}{5}\mathrm{d}x = \int -1\mathrm{d}t$ made little progress with the rest of the question. Although the majority of students used $t = 0$, $x = 60$ to find the value of their constant of integration, some wrote their final answer as $\ln x = -\frac{5}{2}t + \ln 60$ without progressing to make $x$ the subject whilst others used incorrect algebraic manipulation to obtain a final answer of $x = e^{-\frac{5}{2}t} + 60$.

Those students who obtained either $x = 60e^{-\frac{5}{2}t}$ or $\ln x = -\frac{5}{2}t + \ln 60$ in part (a) were often able to complete part (b) successfully, with most converting their time in days to a time in minutes, although some made rounding errors. Although most students substituted $x = 20$ into their part (a) equation there were a minority who substituted $x = 40$ after misunderstanding the question posed in part (b).

## Q3.

The majority of students found part (a) and part (b) to be accessible and gained full marks. In part (a), it was extremely rare to see anything other than 0.6595 for the missing value, with occasionally a few students writing 0.6594. The application of the trapezium rule was usually correct and the final answer usually rounded to 3 decimal places. Errors seen included the use of an incorrect multiplier of $\frac{1}{5}$ or $\frac{1}{12}$ or a missing/extra term in the bracket. Some miscalculated their answer from a correctly written expression for the approximate area. It was rare for students to find the approximate area as a sum of separate trapezia.

In part (c) most students recognised that integration by parts was required and this was often completed correctly. Some students labelled $u$ and $\frac{dv}{dx}$ the wrong way round and a common error was to integrate $x^2$ to give $2x$. Following the first stage of integration by parts, some students attempted to integrate $\frac{x^3}{3} . \frac{1}{x}$ using 'by parts' a second time rather than simplifying this expression first to give $\frac{x^2}{3}$. Other students incorrectly obtained $\frac{x^3}{6}$ after integrating $\frac{x^2}{3}$. Whilst the majority applied the limits of 2 and 1 correctly and subtracted the correct way round to give a correct exact answer of $\frac{8}{3}\ln 2 - \frac{7}{9}$, some used a lower limit of 0 or made bracketing or sign errors whilst others gave a decimal answer to varying degrees of accuracy including some who possibly believed that an answer of 1.070614704 was exact.

Q4.

This question on the topic of vectors discriminated well across students of all abilities. Students generally scored well on part (a) to part (d) with parts part (e) and part (f) proving to be effective discriminators.

In part (a), most students substituted $\mu = 1$ into $l_1$ to find the correct coordinates of $A$.

In part (b), most students used the information given in the question to write down a correct equation for $l_2$ , but some students were penalised for writing an expression for $l_2$ rather than an equation for $l_2$.

In part (c) most students used correct algebra to find an expression for $\overrightarrow{AP}$ and used Pythagoras to calculate the length of this vector. A small number, however, stopped after finding (the vector) $\overrightarrow{AP}$.

In part (d), the majority of students applied the dot product formula between $\overrightarrow{AP}$ and the direction vector of $l_2$ and found $\cos\theta = \frac{4}{5}$. Some students, however, who applied the dot product formula between $\overrightarrow{PA}$ and the direction vector of $l_2$ found $\cos\theta = -\frac{4}{5}$, but only a few then argued that $\cos\theta = \frac{4}{5}$ because the angle $\theta$ is acute. Other students lost the final mark in part (d) by finding $\theta$ as 36.87° without making reference to $\cos\theta = \frac{4}{5}$. A minority of students applied the dot product formula between a pair of non-relevant vectors which sometimes included $\overrightarrow{OA}$, $\overrightarrow{OB}$ or $8\mathbf{i} + \mathbf{j} - 3\mathbf{k}$.

Many students struggled to make progress in part (e) and part (f). Successful solutions almost always followed a good diagram of the situation and students need to be encouraged to set out the given information in such a form before attempting to write up their solution.

In part (e), the area of a triangle *APE* was usually found using $x=\frac{1}{2}ab\sin C\cdot$ with students using their answers to part (c) and part (d). Some students incorrectly assumed triangle *APE* was right-angled and applied the formula $\frac{1}{2}(\text{base})(\text{height})$. Part (f) was often not attempted. Students who formed an appropriate equation in *λ* also usually knew how to use their values of *λ* to find the coordinates of the two possible positions of E. Some errors were made with the algebraic solution of their equation in *λ* and some arithmetical errors were made in calculating the possible coordinates of *E*.

### Q5.

The quality of the responses to this question was generally very good and showed that many students had a good understanding of Power, Energy and Work. (a) The majority of students wrote down correct equations for the motion of the car up and down the road. Most errors were due to sign confusion in the equation for the motion down the road - two separate diagrams were often the key to success here. Some students used the powers 3*P* and *P* as their driving forces, but most students were able to use $P = Fv$ correctly in their equations and go on to solve for *P* and *R*. A number of algebraic errors occurred in solving the simultaneous equations. Some students lost the final accuracy mark because they never expressed their final answers as exact multiples of g or to 2 or 3 significant figures.

(b) Most students tackled this part of the question successfully. The most common error in the work-energy equation was to account for work done against gravity in addition to the change in GPE, meaning that the equation contained an extra term. A few students omitted the work done against *R* or created a dimensionally incorrect expression by omitting the distance in their term for work done against *R*. The final accuracy mark was lost by some students for not rounding their final answer to 2 or 3 significant figures. A small number of students scored no marks, despite finding the correct distance, because they did not follow the instruction to "use the work-energy principle".

### Q6.

(a) Students who adopted a standard structured approach to this question with masses and clear calculations of distances of centres of mass from named lines were generally more successful than students who went direct to forming an attempted moments equation. There were a surprising number of errors in finding the areas of the triangle and quarter circle. The formula was given for the position of the centre of mass of the sector, but some students preferred to work from the formula given in the formula booklet. Some students found the geometry of the triangle challenging and there were many errors in finding the position of its centre of mass. Most students worked from an axis through *O* and adjusted their answer to find the required distance. Those students who used *AD* as their axis were more likely to make sign errors in their moments equation.

(b) Most students used their answer from part (a) correctly to find a relevant angle. Some students stopped at this point, but the majority went on to use a correct method to find the required angle. A clearly labelled diagram was very helpful here. Some students did not notice that the question asked for the answer to be given to the nearest degree.

# Mark Scheme

Q1.

| Question | Scheme | Marks |
|---|---|---|
| **(a)** | $R=\sqrt{5}$ | B1 |
| | $\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$ | M1A1 |
| | | **(3)** |
| **(b)** | $\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$ | |
| | $\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(awrt\ 0.507)$ | M1A1 |
| | $\theta+26.57°=59.54°$ | |
| | $\Rightarrow\theta=awrt\ 33.0°$ or $awrt\ 273.9°$ | A1 |
| | $\theta+26.6°=360°-$ their $'59.5°'$ | dM1 |
| | $\Rightarrow\theta=awrt\ 273.9°$ and $awrt\ 33.0°$ | A1 |
| | | **(5)** |
| **(c)** | $\theta-$ their $26.57°=$ their $59.54°\Rightarrow\theta=\ldots$ | M1 |
| | $\theta=\text{awrt}\ 86.1°$ | A1 |
| | | **(2)** |
| | | **(10 marks)** |

| | |
|---|---|
| (a) | |
| B1 | $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals |
| M1 | $\tan\alpha=\pm\frac{1}{2}$, $\tan\alpha=\pm\frac{2}{1}\Rightarrow\alpha=...$ <br> If their value of $R$ is used to find the value of $\alpha$ only accept $\cos\alpha=\pm\frac{2}{R}$ OR $\sin\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$ |
| A1 | $\alpha=$ awrt 26.57° |
| (b) | |
| M1 | Attempts to use part (a) $\Rightarrow\cos(\theta\pm\text{their }26.6°)=K$, $\lvert K\rvert\leqslant 1$ |
| A1 | $\cos(\theta\pm\text{their }26.6°)=\frac{17}{15\sqrt{5}}=(\text{awrt }0.507)$. Can be implied by $(\theta\pm\text{their }26.6°)=\text{awrt }59.5°/59.6°$ |
| A1 | One solution correct, $\theta=\textit{awrt }33.0°$ or $\theta=\textit{awrt }273.9°$ Do not accept 33 for 33.0. |
| dM1 | Obtains a second solution in the range. It is dependent upon having scored the previous M. <br> Usually for $\theta\pm\text{their }26.6°=360°-\text{their }59.5°\Rightarrow\theta=...$ |
| A1 | Both solutions $\theta=\textit{awrt }33.0°$ and $\textit{awrt }273.9°$. Do not accept 33 for 33.0. <br> Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\leqslant\theta<360°$ |
| (c) | |
| M1 | $\theta-\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$ <br> Alternatively $-\theta+\text{their }26.6°=-\text{their }59.5°\Rightarrow\theta=...$ <br> If the candidate has an incorrect sign for $\alpha$, for example they used $\cos(\theta-26.57°)$ in part (b) it would be scored for $\theta+\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$ |
| A1 | awrt 86.1° ONLY. Allow both marks following a correct (a) and (b) <br> They can restart the question to achieve this result. Do not award if 86.1 was their smallest answer in (b). This occurs when they have $\cos(\theta-26.57°)$ instead of $\cos(\theta+26.57°)$ in (b) |

Answers in radians: Withhold only one A mark, the first time a solution in radians appears
FYI (a) $\alpha=0.46$ (b) $\theta_1=\text{awrt }0.58$ and $\theta_2=\text{awrt }4.78$ (c) $\theta_3=\text{awrt }1.50$. Require 2 dp accuracy

Q2.

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| | $\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$ | | |
| (a)<br>**Way 1** | $\int \frac{1}{x}\,dx = \int -\frac{5}{2}\,dt$ | Separates variables as shown. $dx$ and $dt$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs. | B1 |
| | $\ln x = -\frac{5}{2}t + c$ | Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to $t$); $k, \alpha \neq 0$ | M1 |
| | | $\ln x = -\frac{5}{2}t + c$, including "$+c$" | A1 |
| | $\{t=0, x=60 \Rightarrow\}\ \ln 60 = c$<br>$\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$ | Finds their $c$ and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ **with no incorrect working seen** | A1 **cso** |
| | | | **[4]** |
| (a)<br>**Way 2** | $\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}dx$ | **Either** $\frac{dt}{dx} = -\frac{2}{5x}$ **or** $t = \int -\frac{2}{5x}dx$ | B1 |
| | $t = -\frac{2}{5}\ln x + c$ | Integrates both sides to give **either** $t = ...$ **or** $\pm\alpha\ln px; \alpha \neq 0, p > 0$ | M1 |
| | | $t = -\frac{2}{5}\ln x + c$, including "$+c$" | A1 |
| | $\{t = 0, x = 60 \Rightarrow\}\ c = \frac{2}{5}\ln 60 \Rightarrow t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60$<br>$\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$ | Finds their $c$ and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ **with no incorrect working seen** | A1 **cso** |
| | | | **[4]** |
| (a)<br>**Way 3** | $\int_{60}^{x} \frac{1}{x}dx = \int_{0}^{t} -\frac{5}{2}dt$ | Ignore limits | B1 |
| | $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ | Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to $t$); $k, \alpha \neq 0$ | M1 |
| | | $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ including the correct limits | A1 |
| | $\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$ | Correct algebra leading to a correct result | A1 **cso** |
| | | | **[4]** |

| | | | |
|---|---|---|---|
| (b) | $20 = 60e^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$ | Substitutes $x = 20$ into an equation in the form of **either** $x = \pm\lambda e^{\pm\mu t} \pm \beta$ **or** $x = \pm\lambda e^{\pm\mu t \pm \alpha\ln\delta x}$ **or** $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ **or** $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and $\beta$ can be 0 | M1 |
| | $t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$<br>$\{= 0.4394449...\text{ (days)}\}$<br>**Note: $t$ must be greater than 0** | **dependent on the previous M mark**<br>Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ($A \in$□ , $t > 0$) | dM1 |
| | $\Rightarrow t = 632.8006... = 633$(to the nearest minute) | awrt 633 **or** 10 hours and awrt 33 minutes | A1 cso |
| | Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working. | | |
| | | | 7 |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| | $\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$ | | |
| (a)<br>**Way 4** | $\int \frac{2}{5x}\,dx = -\int dt$ | Separates variables as shown. $dx$ and $dt$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs. | B1 |
| | $\frac{2}{5}\ln(5x) = -t + c$ | Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \rightarrow \pm kt$ (with respect to $t$); $k, \alpha \neq 0$; $p > 0$ | M1 |
| | | $\frac{2}{5}\ln(5x) = -t + c$, including "$+c$" | A1 |
| | $\{t = 0, x = 60 \Rightarrow\}$ $\frac{2}{5}\ln 300 = c$<br>$\frac{2}{5}\ln(5x) = -t + \frac{2}{5}\ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or<br>$\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$ | Finds their $c$ and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ **with no incorrect working seen** | A1 cso |
| | | | [4] |
| (a)<br>**Way 5** | $\left\{\frac{dt}{dx} = -\frac{2}{5x} \Rightarrow\right\}$ $t = \int_{60}^{x} -\frac{2}{5x}\,dx$ | Ignore limits | B1 |
| | $t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$ | Integrates both sides to give **either** $\pm k \rightarrow \pm kt$ (with respect to $t$) **or** $\pm\frac{\alpha}{x} \rightarrow \pm\alpha\ln x$; $k, \alpha \neq 0$ | M1 |
| | | $t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits | A1 |
| | $t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$<br>$\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$ | Correct algebra leading to a correct result | A1 cso |
| | | | [4] |

| | **Question Notes** | |
|---|---|---|
| (a) | **B1** | For the correct separation of variables. E.g. $\int \frac{1}{5x}\,dx = \int -\frac{1}{2}\,dt$ |
| | **Note** | B1 can be implied by seeing **either** $\ln x = -\frac{5}{2}t + c$ **or** $t = -\frac{2}{5}\ln x + c$ with or without $+c$ |
| | **Note** | B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ |
| | **Note** | Allow A1 for $x = 60\sqrt{e^{-5t}}$ or $x = \frac{60}{\sqrt{e^{5t}}}$ with no incorrect working seen |
| | **Note** | Give final A0 for $x = e^{-\frac{5}{2}t} + 60 \rightarrow x = 60e^{-\frac{5}{2}t}$ |
| | **Note** | Give final A0 for writing $x = e^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60e^{-\frac{5}{2}t}$) |
| | **Note** | Way 1 to Way 5 do not exhaust all the different methods that candidates can give. |
| | **Note** | Give B0M0A0A0 for writing down $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no evidence of working or integration seen. |
| (b) | **A1** | You can apply **cso** for the work only seen in part (b). |
| | **Note** | Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working. |
| | **Note** | Substitutes $x = 40$ into their equation from part (a) is M0dM0A0 |

Q3.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| (a) | $\begin{array}{c\|c\|c\|c\|c\|c\|c} x & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 \\ \hline y & 0 & 0.2625 & 0.659485... & 1.2032 & 1.9044 & 2.7726 \end{array}$ | $y = x^2 \ln x$ | |
| | $\{\text{At } x = 1.4\}\ y = 0.6595$ (4 dp) | 0.6595 | B1 **cao** |
| | | | **[1]** |
| (b) | $\frac{1}{2} \times (0.2) \times \left[0 + 2.7726 + 2\left(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044\right)\right]$ | Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{10}$ | B1 o.e. |
| | {**Note:** The "0" does not have to be included in [……]} | For structure of $[\ldots\ldots\ldots\ldots]$ | M1 |
| | $\left\{= \frac{1}{10}(10.8318)\right\} = 1.08318 = 1.083$ (3 dp) | anything that rounds to 1.083 | A1 |
| | | | **[3]** |
| (c)<br>**Way 1** | $\left\{I = \int x^2 \ln x \, dx\right\}$, $\left\{\begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^2 \Rightarrow v = \frac{1}{3}x^3 \end{array}\right\}$ | | |
| | $= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{dx\}$ | Either $x^2 \ln x \to \pm\lambda x^3 \ln x - \int \mu x^3 \left(\frac{1}{x}\right)\{dx\}$<br>or $\pm\lambda x^3 \ln x - \int \mu x^2 \{dx\}$, where $\lambda, \mu > 0$ | M1 |
| | | $x^2 \ln x \to \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{dx\}$, simplified or un-simplified | A1 |
| | $= \frac{x^3}{3}\ln x - \frac{x^3}{9}$ | $\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified | A1 |
| | Area $(R) = \left\{\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2\right\} = \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right)$ | **dependent on the previous M mark.** Applies limits of 2 and 1 and subtracts the correct way round | dM1 |
| | $= \frac{8}{3}\ln 2 - \frac{7}{9}$ | $\frac{8}{3}\ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24\ln 2 - 7)$ | A1 oe cso |
| | | | **[5]** |

| | | | |
|---|---|---|---|
| (c)<br>**Way 2** | $I = x^2(x\ln x - x) - \int 2x(x\ln x - x)dx$ | $\left\{\begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = \ln x \Rightarrow v = x\ln x - x \end{array}\right\}$ | |
| | So, $3I = x^2(x\ln x - x) + \int 2x^2\{dx\}$ | | |
| | and $I = \frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{dx\}$ | A full method of applying $u = x^2, v' = \ln x$ to give $\pm\lambda x^2(x\ln x - x) \pm \mu\int x^2\{dx\}$ | M1 |
| | | $\frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{dx\}$ simplified or un-simplified | A1 |
| | $= \frac{1}{3}x^2(x\ln x - x) + \frac{2}{9}x^3$ | $\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified | A1 |
| | | *Then award dM1A1 in the same way as above* | M1 A1 |
| | | | **[5]** |
| | | | **9** |

|  | **Question Notes** |  |
|---|---|---|
| (a) | **B1** | 0.6595 correct answer only. Look for this on the table or in the candidate's working. |
| (b) | **B1** | Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{2}\times\frac{1}{5}$ or $\frac{1}{10}$ or equivalent. |
|  | **M1** | For structure of trapezium rule $[\ \ldots\ldots\ldots\ldots\ ]$ |
|  | **Note** | No errors are allowed [eg. an omission of a $y$-ordinate or an extra $y$-ordinate or a repeated $y$ ordinate]. |
|  | **A1** | anything that rounds to 1.083 |
|  | **Note** | Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.070614704…) |
|  | **Note** | Full marks can be gained in part (b) for using an incorrect part (a) answer of 0.6594 |
|  | **Note** | Award B1M1A1 for $\frac{1}{10}(2.7726)+\frac{1}{5}(0.2625+\text{their } 0.6595+1.2032+1.9044)=\text{awrt } 1.083$ |
|  | <u>**Bracketing mistake:**</u> **Unless the final answer implies that the calculation has been done correctly**<br>Award B1M0A0 for $\frac{1}{2}(0.2)+2(0.2625+\text{their } 0.6595+1.2032+1.9044)+2.7726$ (answer of 10.9318)<br>Award B1M0A0 for $\frac{1}{2}(0.2)(2.7726)+2(0.2625+\text{their } 0.6595+1.2032+1.9044)$ (answer of 8.33646) |  |
|  | <u>**Alternative method: Adding individual trapezia**</u><br>Area $\approx 0.2\times\left[\frac{0+0.2625}{2}+\frac{0.2625+\text{"}0.6595\text{"}}{2}+\frac{\text{"}0.6595\text{"}+1.2032}{2}+\frac{1.2032+1.9044}{2}+\frac{1.9044+2.7726}{2}\right]=1.08318...$ |  |
|  | **B1** | 0.2 and a divisor of 2 on all terms inside brackets |
|  | **M1** | First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2 |
|  | **A1** | anything that rounds to 1.083 |
| (c) | **A1** | Exact answer needs to be a two term expression in the form $a\ln b+c$ |
|  | **Note** | Give A1 e.g. $\frac{8}{3}\ln 2-\frac{7}{9}$ or $\frac{1}{9}(24\ln 2-7)$ or $\frac{4}{3}\ln 4-\frac{7}{9}$ or $\frac{1}{3}\ln 256-\frac{7}{9}$ or $-\frac{7}{9}+\frac{8}{3}\ln 2$<br>or $\ln 2^{\frac{8}{3}}-\frac{7}{9}$ or equivalent. |
|  | **Note** | Give final A0 for a final answer of $\frac{8\ln 2-\ln 1}{3}-\frac{7}{9}$ or $\frac{8\ln 2}{3}-\frac{1}{3}\ln 1-\frac{7}{9}$ or $\frac{8\ln 2}{3}-\frac{8}{9}+\frac{1}{9}$<br>or $\frac{8}{3}\ln 2-\frac{7}{9}+c$ |
|  | **Note** | $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2$ followed by awrt 1.07 with no correct answer seen is dM1A0 |
|  | **Note** | Give dM0A0 for $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2 \rightarrow \left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\frac{1}{9}$ (adding rather than subtracting) |
|  | **Note** | Allow dM1A0 for $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2 \rightarrow \left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\left(0+\frac{1}{9}\right)$ |
|  | **SC** | A candidate who uses $u=\ln x$ and $\frac{dv}{dx}=x^2$, $\frac{du}{dx}=\frac{\alpha}{x}$, $v=\beta x^3$, writes down the correct "by parts" formula but makes only one error when applying it can be awarded Special Case 1st M1. |

Q4.

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| | $l_1:\mathbf{r}=\begin{pmatrix}8\\1\\-3\end{pmatrix}+\mu\begin{pmatrix}-5\\4\\3\end{pmatrix}$ So $\mathbf{d}_1=\begin{pmatrix}-5\\4\\3\end{pmatrix}$. $\overrightarrow{OA}$ occurs when $\mu=1$. $\overrightarrow{OP}=\begin{pmatrix}1\\5\\2\end{pmatrix}$ | | |
| (a) | $A(3,5,0)$ | $(3,5,0)$ | B1 |
| | | | **[1]** |
| (b) | $\{l_2:\}\ \mathbf{r}=\begin{pmatrix}1\\5\\2\end{pmatrix}+\lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ | $\mathbf{a}+\lambda\mathbf{d}$ or $\mathbf{a}+\mu\mathbf{d}$, $\mathbf{a}+t\mathbf{d}$, $\mathbf{a}\neq 0$, $\mathbf{d}\neq 0$ with either $\mathbf{a}=\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ or $\mathbf{d}=-5\mathbf{i}+4\mathbf{j}+3\mathbf{k}$, or a multiple of $-5\mathbf{i}+4\mathbf{j}+3\mathbf{k}$ | M1 |
| | | Correct vector equation using $\mathbf{r}=$ or $l=$ or $l_2=$ | A1 |
| | $\mathbf{d}_2$ is the direction vector of $l_2$ | Do not allow $l_2:$ or $l_2\rightarrow$ or $l_1=$ for the A1 mark. | **[2]** |
| (c) | $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}1\\5\\2\end{pmatrix}-\begin{pmatrix}3\\5\\0\end{pmatrix}=\begin{pmatrix}-2\\0\\2\end{pmatrix}$ | | |
| | $AP=\sqrt{(-2)^2+(0)^2+(2)^2}=\sqrt{8}=2\sqrt{2}$ | Full method for finding $AP$ | M1 |
| | | $2\sqrt{2}$ | A1 |
| | | | **[2]** |
| (d) | So $\overrightarrow{AP}=\begin{pmatrix}-2\\0\\2\end{pmatrix}$ and $\mathbf{d}_2=\begin{pmatrix}-5\\4\\3\end{pmatrix}\Rightarrow\begin{pmatrix}-2\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-5\\4\\3\end{pmatrix}$ | Realisation that the dot product is required between $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | M1 |
| | $\{\cos\theta=\}\ \dfrac{\overrightarrow{AP}\bullet\mathbf{d}_2}{\lvert\overrightarrow{AP}\rvert\lvert\mathbf{d}_2\rvert}=\dfrac{\pm\left(\begin{pmatrix}-2\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-5\\4\\3\end{pmatrix}\right)}{\sqrt{(-2)^2+(0)^2+(2)^2}.\sqrt{(-5)^2+(4)^2+(3)^2}}$ | **dependent on the previous M mark.** Applies dot product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | dM1 |
| | $\{\cos\theta\}=\dfrac{\pm(10+0+6)}{\sqrt{8}.\sqrt{50}}=\dfrac{4}{5}$ | $\{\cos\theta\}=\dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$ | A1 cso |
| | | | **[3]** |
| (e) | $\{\text{Area }APE=\}\ \dfrac{1}{2}(\text{their }2\sqrt{2})^2\sin\theta$ | $\dfrac{1}{2}(\text{their }2\sqrt{2})^2\sin\theta$ or $\dfrac{1}{2}(\text{their }2\sqrt{2})^2\sin(\text{their }\theta)$ | M1 |
| | $=2.4$ | 2.4 or $\dfrac{12}{5}$ or $\dfrac{24}{10}$ or awrt 2.40 | A1 |
| | | | **[2]** |
| (f) | $\overrightarrow{PE}=(-5\lambda)\mathbf{i}+(4\lambda)\mathbf{j}+(3\lambda)\mathbf{k}$ and $PE=\text{their }2\sqrt{2}$ from part (c) | | |
| | $\{PE^2=\}\ (-5\lambda)^2+(4\lambda)^2+(3\lambda)^2=(\text{their }2\sqrt{2})^2$ | This mark can be implied. | M1 |
| | $\left\{\Rightarrow 50\lambda^2=8\Rightarrow\lambda^2=\dfrac{4}{25}\Rightarrow\right\}\ \lambda=\pm\dfrac{2}{5}$ | Either $\lambda=\dfrac{2}{5}$ or $\lambda=-\dfrac{2}{5}$ | A1 |
| | $l_2:\mathbf{r}=\begin{pmatrix}1\\5\\2\end{pmatrix}\pm\dfrac{2}{5}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ | **dependent on the previous M mark** Substitutes at least one of their values of $\lambda$ into $l_2$. | dM1 |
| | $\{\overrightarrow{OE}\}=\begin{pmatrix}3\\\frac{17}{5}\\\frac{4}{5}\end{pmatrix}$ or $\begin{pmatrix}3\\3.4\\0.8\end{pmatrix}$, $\{\overrightarrow{OE}\}=\begin{pmatrix}-1\\\frac{33}{5}\\\frac{16}{5}\end{pmatrix}$ or $\begin{pmatrix}-1\\6.6\\3.2\end{pmatrix}$ | At least one set of coordinates are correct. | A1 |
| | | Both sets of coordinates are correct. | A1 |
| | | | **[5]** |
| | | | **15** |

| | Question Notes | | |
|---|---|---|---|
| (a) | **B1** | Allow $A(3,5,0)$ or $3\mathbf{i}+5\mathbf{j}$ or $3\mathbf{i}+5\mathbf{j}+0\mathbf{k}$ or $\begin{pmatrix}3\\5\\0\end{pmatrix}$ or benefit of the doubt $\begin{matrix}3\\5\\0\end{matrix}$ | |
| (b) | **A1** | Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ **or** Line 2 $=$ <br> i.e. Writing $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\mathbf{d}$, where $\mathbf{d}$ is a multiple of $\begin{pmatrix}-5\\4\\3\end{pmatrix}$. | |
| | **Note** | Allow the use of parameters $\mu$ or $t$ instead of $\lambda$. | |
| (c) | **M1** | Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find $AP$ | |
| | **Note** | Allow M1A1 for $\begin{pmatrix}2\\0\\2\end{pmatrix}$ leading to $AP = \sqrt{(2)^2+(0)^2+(2)^2} = \sqrt{8} = 2\sqrt{2}$. | |
| (d) | **Note** | For both the M1 and dM1 marks $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) must be the vector used in part (c) or the difference $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ from part (a). | |
| | **Note** | Applying the dot product formula correctly without $\cos\theta$ as the subject is fine for M1dM1 | |
| | **Note** | *Evaluating* the dot product (i.e. $(-2)(-5)+(0)(4)+(2)(3)$) is not required for M1 and dM1 marks. | |
| | **Note** | In part (d) allow one slip in writing $\overrightarrow{AP}$ and $\mathbf{d}_2$ | |
| | **Note** | $\cos\theta = \frac{-10+0-6}{\sqrt{8}.\sqrt{50}} = -\frac{4}{5}$ followed by $\cos\theta = \frac{4}{5}$ is fine for A1 cso | |
| | **Note** | Give M1dM1A1 for $\{\cos\theta =\} = \frac{\begin{pmatrix}-2\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-10\\8\\6\end{pmatrix}}{\sqrt{8}.10\sqrt{2}} = \frac{20+12}{40} = \frac{4}{5}$ | |
| | **Note** | Allow final A1 (ignore subsequent working) for $\cos\theta = 0.8$ followed by $36.869...^{\circ}$ | |
| | *Alternative Method: Vector Cross Product* | | |
| | **Only apply this scheme if it is clear that a candidate is applying a vector cross product method.** | | |
| | $\overrightarrow{AP}\times\mathbf{d}_2 = \underline{\begin{pmatrix}-2\\0\\2\end{pmatrix}\times\begin{pmatrix}-5\\4\\3\end{pmatrix}} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -2 & 0 & 2\\ -5 & 4 & 3\end{vmatrix} = -8\mathbf{i}-4\mathbf{j}-8\mathbf{k}\right\}$ | Realisation that the vector cross product is required between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | M1 |
| | $\sin\theta = \frac{\sqrt{(-8)^2+(-4)^2+(-8)^2}}{\sqrt{(-2)^2+(0)^2+(2)^2}.\sqrt{(-5)^2+(4)^2+(3)^2}}$ | Applies the vector product formula between their $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | dM1 |
| | $\sin\theta = \frac{12}{\sqrt{8}.\sqrt{50}} = \frac{3}{5} \Rightarrow \underline{\cos\theta = \frac{4}{5}}$ | $\cos\theta = \frac{4}{5}$ or 0.8 or $\frac{8}{10}$ or $\frac{16}{20}$ | A1 |
| (e) | **Note** | Allow M1;A1 for $\frac{1}{2}(2\sqrt{2})^2\sin(36.869...^{\circ})$ or $\frac{1}{2}(2\sqrt{2})^2\sin(180^{\circ}-36.869...^{\circ})$; = awrt 2.40 | |
| | **Note** | Candidates must use their $\theta$ from part (d) or apply a correct method of finding their $\sin\theta = \frac{3}{5}$ from their $\cos\theta = \frac{4}{5}$ | |

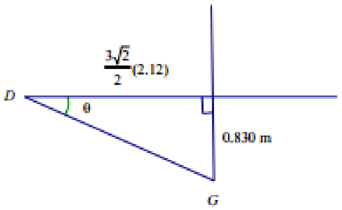
| | Question Notes Continued | | |
|---|---|---|---|
| (f) | **Note** | Allow the first M1A1 for deducing $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$ from no incorrect working | |
| | **SC** | Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working | |
| | **Note** | Give 1st M1 for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$ | |
| | **Note** | Give 1st M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent | |
| | **Note** | Give 1st M1 for $\lambda = \frac{\text{their } AP = "2\sqrt{2}"}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1st A1 for $\lambda = \frac{2\sqrt{2}}{5\sqrt{2}}$ | |
| | **Note** | So $\left\{\hat{\mathbf{d}}_1 = \frac{1}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow\right\}$ "vector" $= \frac{2\sqrt{2}}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ is M1A1 | |
| | **Note** | The 2nd dM1 in part (f) can be implied for at least 2 (out of 6) correct $x, y, z$ ordinates from their values of $\lambda$. | |
| | **Note** | Giving their "coordinates" as a column vector or position vector is fine for the final A1A1. | |
| | **CAREFUL** | Putting $l_2$ equal to $A$ gives<br>$\begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix} = \begin{pmatrix}3\\5\\0\end{pmatrix} \rightarrow \begin{pmatrix}\lambda = \frac{2}{5}\\ \lambda = 0\\ \lambda = -\frac{2}{3}\end{pmatrix}$ | Give M0 dM0 for finding and using $\lambda = \frac{2}{5}$ from this incorrect method. |
| | **CAREFUL** | Putting $\lambda\mathbf{d}_2 = \overrightarrow{AP}$ gives<br>$\lambda\begin{pmatrix}-5\\4\\3\end{pmatrix} = \begin{pmatrix}2\\0\\-2\end{pmatrix} \rightarrow \begin{pmatrix}\lambda = -\frac{2}{5}\\ \lambda = 0\\ \lambda = -\frac{2}{3}\end{pmatrix}$ | Give M0 dM0 for finding and using $\lambda = -\frac{2}{5}$ from this incorrect method. |
| | **General** | You can follow through the part (c) answer of their $AP = 2\sqrt{2}$<br>for (d) M1dM1, (e) M1, (f) M1dM1 | |
| | **General** | You can follow through their $\mathbf{d}_2$ in part (b) for (d) M1dM1, (f) M1dM1. | |

Q5.

| Q | Scheme | Marks | Notes |
|---|---|---|---|
| **a** | | M1 | Equation of motion up or down the road. Requires all 3 terms. Condone sign errors and trig confusion. Must be dimensionally correct. |
| | $F = mg\sin\theta + R \quad (F = R + 392)$ | A1 | Correct equation up the road |
| | $G + mg\sin\theta = R \quad (G = R - 392)$ | A1 | Correct equation down the road |
| | $F = \frac{3P}{12.5}$ or $G = \frac{P}{12.5}$<br>$\Rightarrow \frac{3P}{12.5} = 392 + R$ or $\frac{P}{12.5} = R - 392$ | B1 | Use of $F = \frac{P}{v}$ at least once |
| | $\frac{2P}{12.5} = 2 \times 392$ , $2R = \frac{4P}{12.5}$ | M1 | Solve simultaneous equations for $P$ or $R$, provided $F \neq G$ and $P$ and $3P$ used correctly |
| | $P = 4900\,(500\text{g}), \quad R = 784\,(80\text{g})$ | A1 | CSO. Both values correct. Accept 2sf, 3sf or an exact multiple of g |
| | | (6) | |
| **b** | **Must be using work-energy.** | | |
| | KE lost = PE gained + WD against R | M1 | Equation needs all 3 terms and no extras. Condone sign errors. |
| | $\frac{1}{2} \times 800 \times 12.5^2$<br>$= 800 \times 9.8 \times \frac{d}{20} + (\text{their } R) \times d$ | A1 | At most 1 error. Allow with $R$ (with trig. substituted)<br>$(62500 = 392d + Rd)$ |
| | | A1ft | Correct equation in their $R$(with trig. substituted) |
| | $d = \frac{62500}{1176} = 53.1\text{(m)}$ | A1 | CSO. Accept 53(m) |
| | | (4) | |
| | | [10] | |
| | | | |

Q6.

<table>
<tr><th>Q</th><th>Scheme</th><th>Marks</th><th></th></tr>
<tr><td><b>a</b></td><td><table><tr><th></th><th>Triangle</th><th>sector</th></tr><tr><td>mass</td><td>4.5</td><td>$4\pi$<br>$(=12.56..)$</td></tr><tr><td>c of m from $O$</td><td>$\frac{2}{3}\times\frac{3\sqrt{2}}{2}$<br>$(=1.41...)$</td><td>$\frac{16\sqrt{2}}{3\pi}(=2.40....)$</td></tr></table></td><td>B1<br>B1</td><td>Mass ratios<br>Distances<br>Distances from $AD$ are $-\frac{1}{3}\times\frac{3\sqrt{2}}{2}$<br>and $\frac{16\sqrt{2}}{3\pi}-\frac{3\sqrt{2}}{2}(=0.280)$</td></tr>
<tr><td></td><td>:<br>$4\pi\times\frac{16\sqrt{2}}{3\pi}-4.5\sqrt{2}\left(=\frac{101\sqrt{2}}{6}\right)=(4\pi-4.5)d$</td><td>M1</td><td>Moments about an axis through $O$ and parallel to $DA$.<br>Terms must be dimensionally correct.<br>Shapes combined correctly.</td></tr>
<tr><td></td><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>$d=\frac{101\sqrt{2}}{6(4\pi-4.5)}=2.951.....$</td><td></td><td>(distance from $O$)</td></tr>
<tr><td></td><td>Distance from $DA$ $=2.951...-\frac{3\sqrt{2}}{2}$<br>$=0.830(0.83)$ m</td><td>A1</td><td>Accept $\frac{101\sqrt{2}}{6(4\pi-4.5)}-\frac{3\sqrt{2}}{2}$</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td><b>a</b><br><b>alt</b></td><td><table><tr><th></th><th>Triangle</th><th>sector</th></tr><tr><td>mass</td><td>4.5</td><td>$4\pi(=12.56..)$</td></tr><tr><td>c of m from both axes $OC,OB$</td><td>1</td><td>$\frac{16\sqrt{2}}{3\pi}\times\frac{1}{\sqrt{2}}=\frac{16}{3\pi}$</td></tr></table></td><td>B1<br>B1</td><td>Mass ratios<br>Distances</td></tr>
<tr><td></td><td>$4\pi\times\frac{16}{3\pi}-4.51=(4\pi-4.5)\bar{x}$</td><td>M1</td><td>Moments about an axis through $O$.<br>Terms must be dimensionally correct.  Condone sign error(s)</td></tr>
<tr><td></td><td>$\left(\bar{x}=\bar{y}=\frac{101}{6(4\pi-4.5)}\right)$</td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>$d=\frac{101\sqrt{2}}{6(4\pi-4.5)}$</td><td></td><td>Distance from $O$</td></tr>
<tr><td></td><td>Distance from $DA$ $=2.951...-\frac{3\sqrt{2}}{2}$<br>$=0.830(0.83)$ m</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

| | | | |
|---|---|---|---|
| **b** |  | | |
| | $\tan\theta = \frac{\text{their } 0.830}{2.12}$ or $\tan\phi = \frac{2.12}{\text{their } 0.830}$ | M1 | Use of tan to find a relevant angle: |
| | 21.4° or 68.6° | A1 | |
| | Angle between DC and downward vertical<br>= 135° - their θ | M1 | Correct method for the required angle |
| | = 114° | A1 | The Q asks for the angle to the nearest degree. |
| | | (4) | |
| **balt** | $GD^2 = OD^2 + OG^2 - 2OD.OC\cos 45$<br>$(GD = 2.28)$ $\frac{\sin 45}{DG} = \frac{\sin\theta}{OG}$ | M1 | Complete method to find angle *ODG* |
| | $\Rightarrow \theta = 66.4°$ | A1 | |
| | | M1 | Correct method for the required angle |
| | Required angle $= 180 - 66.4 = 114°$ | A1 | The Q asks for the angle to the nearest degree. |
| | | (4) | |
| | | | |
| | | [9] | |