Paper Reference(s)

6666A/01

Pearson Edexcel International Advanced Level

Core Mathematics C4

Advanced

Monday 27 January 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. (*a*) Find the binomial expansion of

$$\frac{1}{(4+3x)^3}, \qquad |x| < \frac{4}{3}$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a simplified fraction.

(6)

In the binomial expansion of

$$\frac{1}{(4-9x)^3}, \qquad |x| < \frac{4}{9}$$

the coefficient of x^2 is A.

(*b*) Using your answer to part (*a*), or otherwise, find the value of A.
Give your answer as a simplified fraction.

(2)

---

2. (i) Find

$$\int x\cos\left(\frac{x}{2}\right)\mathrm{d}x$$

(3)

(ii) (*a*) Express $\dfrac{1}{x^2(1-3x)}$ in partial fractions.

(4)

(*b*) Hence find, for $0 < x < \dfrac{1}{3}$

$$\int \frac{1}{x^2(1-3x)}\mathrm{d}x$$

(3)

---

3. The number of bacteria, N, present in a liquid culture at time t hours after the start of a scientific study is modelled by the equation

$$N = 5000(1.04)^t, \qquad t \geq 0$$

where N is a continuous function of t.

(a) Find the number of bacteria present at the start of the scientific study.
(1)

(b) Find the percentage increase in the number of bacteria present from $t = 0$ to $t = 2$.
(2)

Given that $N = 15\,000$ when $t = T$,

(c) find the value of $\frac{\mathrm{d}N}{\mathrm{d}t}$ when $t = T$, giving your answer to 3 significant figures.
(4)

4.

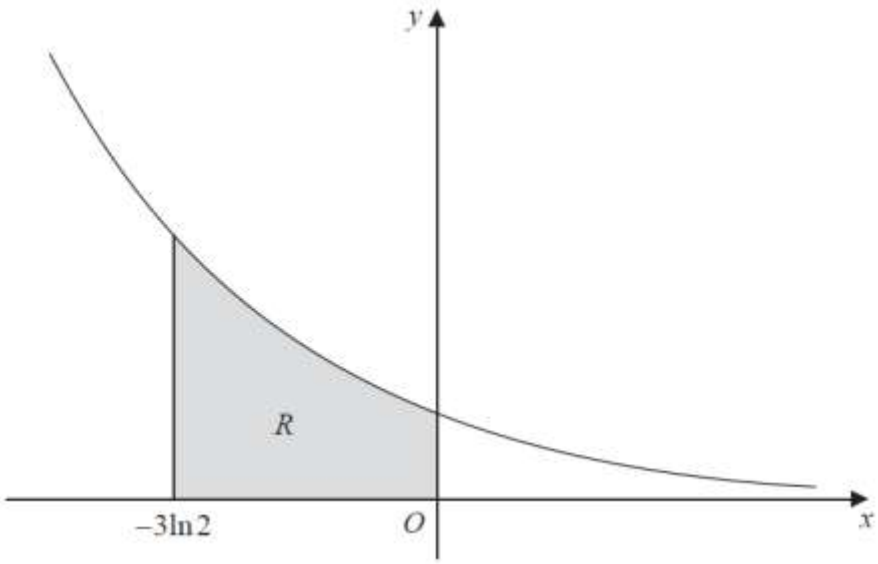

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line $x = -3\ln 2$ and the y-axis.

The table below shows corresponding values of x and y for $y = \dfrac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}$.

x	$-3\ln 2$	$-2\ln 2$	$-\ln 2$	0
y	2.1333		1.0079	0.6667

(*a*) Complete the table above by giving the missing value of y to 4 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(3)

(*c*) (i) Using the substitution $u = 1 + 3e^{-x}$, or otherwise, find

$$\int \frac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}\,dx$$

(5)

(ii) Hence find the value of the area of R.

(2)

5. Given that $y = 2$ at $x = \frac{\pi}{8}$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3y^2}{2\sin^2 2x}$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

---

6. Oil is leaking from a storage container onto a flat section of concrete at a rate of 0.48 $\mathrm{cm}^3\ \mathrm{s}^{-1}$. The leaking oil spreads to form a pool with an increasing circular cross-section. The pool has a constant uniform thickness of 3 mm.

Find the rate at which the radius r of the pool of oil is increasing at the instant when $r = 5$ cm. Give your answer, in $\mathrm{cm\ s}^{-1}$, to 3 significant figures.

(5)

---

7. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \qquad 0 \leq t \leq \pi$$

where t is a parameter.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$.

The line l is a normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.

(6)

---

8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$$

where λ and μ are scalar parameters.

(*a*) Find, to the nearest 0.1°, the acute angle between l_1 and l_2.

(3)

The point A has position vector $\begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$.

(*b*) Show that A lies on l_1.

(1)

The lines l_1 and l_2 intersect at the point X.

(*c*) Write down the coordinates of X.

(1)

(*d*) Find the exact value of the distance AX.

(2)

The distinct points B_1 and B_2 both lie on the line l_2.

Given that $AX = XB_1 = XB_2$,

(*e*) find the area of the triangle AB_1B_2 giving your answer to 3 significant figures.

(3)

Given that the x coordinate of B_1 is positive,

(*f*) find the exact coordinates of B_1 and the exact coordinates of B_2.

(5)

TOTAL FOR PAPER: 75 MARKS

END