Paper Reference(s)

6665A/01

Pearson Edexcel International Advanced Level

Core Mathematics C3

Advanced

Monday 27 January 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1.

$$\text{f}(x) = \sec x + 3x - 2, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(*a*) Show that there is a root of f(x) = 0 in the interval [0.2, 0.4].

(2)

(*b*) Show that the equation f(x) = 0 can be written in the form

$$x = \frac{2}{3} - \frac{1}{3\cos x}$$

(1)

The solution of f(x) = 0 is α, where α = 0.3 to 1 decimal place.

(*c*) Starting with $x_0 = 0.3$, use the iterative formula

$$x_{n+1} = \frac{2}{3} - \frac{1}{3\cos x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) State the value of α correct to 3 decimal places.

(1)

2.

$$\text{f}(x) = \frac{15}{3x+4} - \frac{2x}{x-1} + \frac{14}{(3x+4)(x-1)}, \qquad x > 1$$

(*a*) Express f(x) as a single fraction in its simplest form.

(4)

(*b*) Hence, or otherwise, find $\text{f}'(x)$, giving your answer as a single fraction in its simplest form.

(3)

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3. (*a*) By writing cosec x as $\dfrac{1}{\sin x}$, show that

$$\frac{\mathrm{d}(\operatorname{cosec} x)}{\mathrm{d}x} = -\operatorname{cosec} x \cot x$$

(3)

Given that $y = \mathrm{e}^{3x} \operatorname{cosec} 2x, \quad 0 < x < \dfrac{\pi}{2}$,

(*b*) find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

The curve with equation $y = \mathrm{e}^{3x} \operatorname{cosec} 2x, \quad 0 < x < \dfrac{\pi}{2}$, has a single turning point.

(*c*) Show that the x-coordinate of this turning point is at $x = \dfrac{1}{2}\arctan k$ where the value of the constant k should be found.

(2)

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4. A pot of coffee is delivered to a meeting room at 11am. At a time t minutes after 11am the temperature, θ°C, of the coffee in the pot is given by the equation

$$\theta = A + 60\mathrm{e}^{-kt}$$

where A and k are positive constants.

Given also that the temperature of the coffee at 11am is 85°C and that 15 minutes later it is 58°C,

(*a*) find the value of A.

(1)

(*b*) Show that $k = \dfrac{1}{15}\ln\left(\dfrac{20}{11}\right)$.

(3)

(*c*) Find, to the nearest minute, the time at which the temperature of the coffee reaches 50°C.

(4)

5.

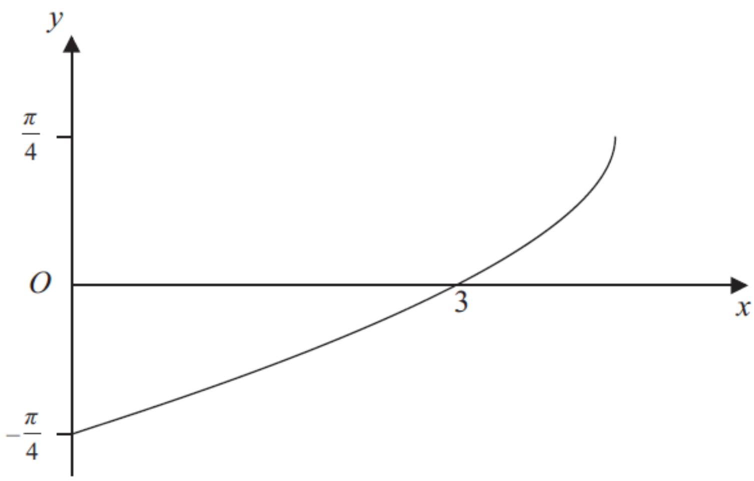

Figure 1

The curve shown in Figure 1 has equation

$$x = 3\sin y + 3\cos y, \qquad -\frac{\pi}{4} < y < \frac{\pi}{4}$$

(*a*) Express the equation of the curve in the form

$$x = R\sin(y + \alpha), \text{ where } R \text{ and } \alpha \text{ are constants, } R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

(3)

(*b*) Find the coordinates of the point on the curve where the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ is $\frac{1}{2}$.

Give your answers to 3 decimal places.

(6)

6. Given that a and b are constants and that $0 < a < b$,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x + a|$,

(ii) $y = |2x + a| - b$.

Show on each sketch the coordinates of each point at which the graph crosses or meets the axes.

(6)

(b) Solve, for x, the equation

$$|2x + a| - b = \frac{1}{3}x$$

giving any answers in terms of a and b.

(4)

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7. (i) (a) Prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$

(You may use the double angle formulae and the identity
$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$)

(4)

(b) Hence solve the equation

$$2\cos 3\theta + \cos 2\theta + 1 = 0$$

giving answers in the interval $0 < \theta < \pi$.
Solutions based entirely on graphical or numerical methods are not acceptable.

(6)

(ii) Given that $\theta = \arcsin x$ and that $0 < \theta < \frac{\pi}{2}$, show that

$$\cot\theta = \frac{\sqrt{(1 - x^2)}}{x}, \qquad 0 < x < 1$$

(3)

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8. The function f is defined by

$$\mathrm{f} : x \to 3 - 2\mathrm{e}^{-x}, \qquad x \in \mathbb{R}$$

(*a*) Find the inverse function, $\mathrm{f}^{-1}(x)$, and give its domain.

(5)

(*b*) Solve the equation $\mathrm{f}^{-1}(x) = \ln x$.

(4)

The equation $\mathrm{f}(t) = k\mathrm{e}^{t}$, where *k* is a positive constant, has exactly one real solution.

(*c*) Find the value of *k*.

(4)

TOTAL FOR PAPER: 75 MARKS

END