Paper Reference(s)

# WMA02/01

# Pearson Edexcel International Advanced Level

## Core Mathematics C34

## Advanced

## Monday 27 January 2014 – Morning

## Time: 2 hours 30 minutes

| Materials required for examination | Items included with question papers |
|---|---|
| Mathematical Formulae (Blue) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

**1.** $$\mathrm{f}(x) = \frac{2x}{x^2+3}, \qquad x \in \mathbb{R}$$

Find the set of values of $x$ for which $\mathrm{f}'(x) > 0$.

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

**(6)**

---

**2.** Solve, for $0 \leq x \leq 270°$, the equation

$$\frac{\tan 2x + \tan 50°}{1 - \tan 2x \tan 50°} = 2$$

Give your answers in degrees to 2 decimal places.

**(6)**

---

**3.** Given that

$$4x^3 + 2x^2 + 17x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$$

(*a*) find the values of the constants $A$, $B$, $C$ and $D$.

**(4)**

(*b*) Hence find

$$\int_1^4 \frac{4x^3 + 2x^2 + 17x + 8}{x^2+4}\,\mathrm{d}x$$

giving your answer in the form $p + \ln q$, where $p$ and $q$ are integers.

**(6)**

---

**4.**

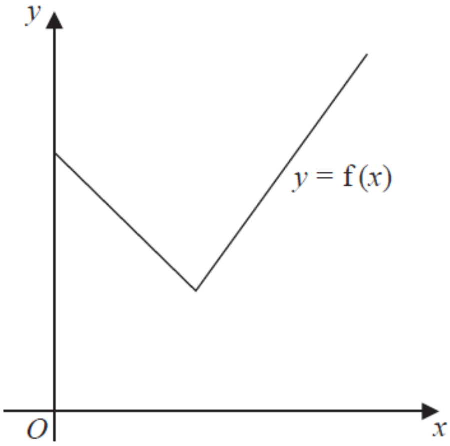


**Figure 1**

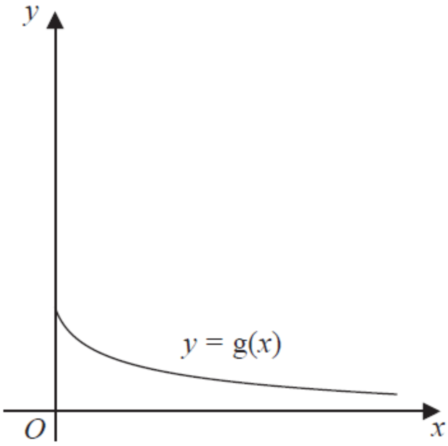


**Figure 2**

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$f(x) = 2|3 - x| + 5, \qquad x \geq 0$$

Figure 2 shows a sketch of part of the graph $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x+9}{2x+3}, \qquad x \geq 0$$

(*a*) Find the value of fg(1).

**(2)**

(*b*) State the range of g.

**(2)**

(*c*) Find $\mathrm{g}^{-1}(x)$ and state its domain.

**(4)**

Given that the equation $\mathrm{f}(x) = k$, where $k$ is a constant, has exactly two roots,

(*d*) state the range of possible values of $k$.

**(3)**

**5.** (*a*) Prove, by using logarithms, that

$$\frac{\mathrm{d}}{\mathrm{d}x}(2^x) = 2^x \ln 2$$

**(3)**

The curve $C$ has the equation

$$2x + 3y^2 + 3x^2y + 12 = 4 \times 2^x$$

The point $P$, with coordinates (2, 0), lies on $C$.

(*b*) Find an equation of the tangent to $C$ at $P$.

**(6)**

---

**6.** Given that the binomial expansion, in ascending powers of $x$, of

$$\frac{6}{\sqrt{(9 + Ax^2)}}, \qquad |x| < \frac{3}{\sqrt{|A|}}$$

is $$B - \frac{2}{3}x^2 + Cx^4 + \ldots$$

(*a*) find the values of the constants $A$, $B$ and $C$.

**(7)**

(*b*) Hence find the coefficient of $x^6$.

**(2)**

---

**7.**

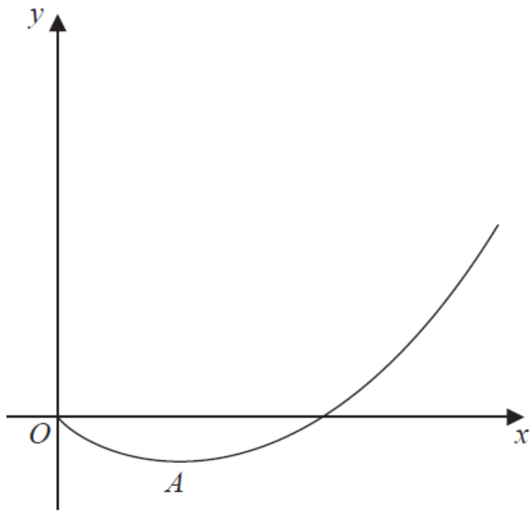


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x(1 + x)\ln x, \qquad x > 0$$

The curve has a minimum turning point at $A$.

(*a*) Find $\mathrm{f}'(x)$.

**(3)**

(*b*) Hence show that the $x$ coordinate of $A$ is the solution of the equation

$$x = \mathrm{e}^{-\frac{1+x}{1+2x}}$$

**(3)**

(*c*) Use the iteration formula

$$x_{n+1} = \mathrm{e}^{-\frac{1+x_n}{1+2x_n}}, \qquad x_0 = 0.46$$

to find the values of $x_1$, $x_2$ and $x_3$ to 4 decimal places.

**(3)**

(*d*) Use your answer to part (*c*) to estimate the coordinates of $A$ to 2 decimal places.

**(2)**

**8.** (*a*) Prove that

$$2\operatorname{cosec} 2A - \cot A \equiv \tan A, \qquad A \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

**(4)**

(*b*) Hence solve for $0 \leq \theta \leq \frac{\pi}{2}$

(i) $2\operatorname{cosec} 4\theta - \cot 2\theta = \sqrt{3}$,

(ii) $\tan \theta + \cot \theta = 5$.

Give your answers to 3 significant figures.

**(6)**

---

**9.** (*a*) Use the substitution $u = 4 - \sqrt{x}$ to find

$$\int \frac{\mathrm{d}x}{4 - \sqrt{x}}$$

**(6)**

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{4 - \sqrt{h}}{20}$$

where $h$ is the height in metres and $t$ is the time measured in years after the tree is planted.

(*b*) Find the range in values for $h$ for which the height of a tree in this species is increasing.

**(2)**

(*c*) Given that one of these trees is 1 metre high when it is planted, calculate the time it would take to reach a height of 10 metres. Write your answer to 3 significant figures.

**(7)**

---

**10.** With respect to a fixed origin $O$, the lines $l_1$ and $l_2$ are given by the equations

$$l_1 : \mathbf{r} = (\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{j} + 12\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where $\lambda$ and $\mu$ are scalar parameters.

(*a*) Show that $l_1$ and $l_2$ meet and find the position vector of their point of intersection.

**(6)**

(*b*) Show that $l_1$ and $l_2$ are perpendicular to each other.

**(2)**

The point $A$, with position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$, lies on $l_1$.

The point $B$ is the image of $A$ after reflection in the line $l_2$.

(*c*) Find the position vector of $B$.

**(3)**

---

**11.** The curve $C$ has parametric equations

$$x = 10\cos 2t, \quad y = 6\sin t, \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

The point $A$ with coordinates (5, 3) lies on $C$.

(*a*) Find the value of $t$ at the point $A$.

**(1)**

(*b*) Show that an equation of the normal to $C$ at $A$ is

$$3y = 10x - 41$$

**(6)**

The normal to $C$ at $A$ cuts $C$ again at the point $B$.

(*c*) Find the exact coordinates of $B$.

**(8)**

---

**12.**

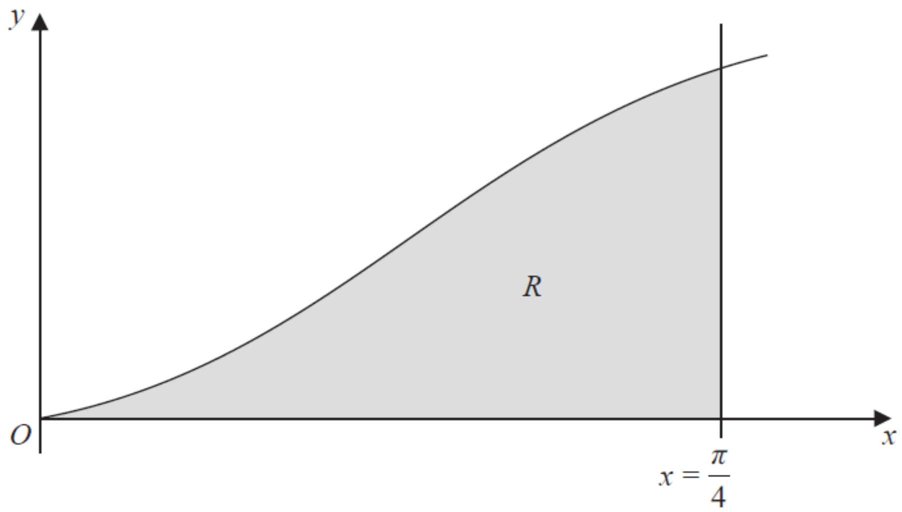


**Figure 4**

Figure 4 shows a sketch of part of the curve with equation

$$y = x(\sin x + \cos x), \qquad 0 \leq x \leq \frac{\pi}{4}$$

The finite region $R$, shown shaded in Figure 4, is bounded by the curve, the $x$-axis and the line $x = \frac{\pi}{4}$. This shaded region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution, with volume $V$.

(*a*) Assuming the formula for volume of revolution show that $V = \int_0^{\frac{\pi}{4}} \pi x^2 (1 + \sin 2x) \mathrm{d}x$ .

**(3)**

(*b*) Hence using calculus find the exact value of $V$.

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

**(9)**

**TOTAL FOR PAPER: 125 MARKS**

**END**