HOW TO USE THE VECTOR (OR CROSS) PRODUCT TO FIND A VECTOR PERPENDICULAR TO $\underline{a}$ and $\underline{b}$

If $\underline{a}$ is $\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\underline{b}$ is $\begin{pmatrix}4\\5\\6\end{pmatrix}$

what is $\underline{a} \times \underline{b}$?

By definition $\underline{a} \times \underline{b} = |\underline{a}||\underline{b}| \sin\theta \, \hat{\underline{n}}$

θ is the angle between the vectors
$\hat{\underline{n}}$ is a unit vector perpendicular to $\underline{a}$ and $\underline{b}$

We set up a determinant (a way of solving linear equations easily)

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - \underline{j}\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + \underline{k}\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

always -ve here

$$= \underline{i}\,(2\times6 - 5\times3) - \underline{j}\,(1\times6 - 4\times3) + \underline{k}\,(1\times5 - 4\times$$

$$= -3\underline{i} + 6\underline{j} - 3\underline{k}$$

This vector is $\perp$ to $\underline{a}$ and $\underline{b}$

Now consider $\underline{b} \times \underline{a}$

$$\underline{b} \times \underline{a} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= i \begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix} - \underline{j} \begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} + \underline{k} \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix}$$

$$= 3\underline{i} - 2\underline{j} + 3\underline{k}$$

acts in opposite direction to $\underline{a} \times \underline{b}$

* we will be looking at this in more detail later on in the course

TRY THIS EXAMPLE

$$\underline{a} = \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix} \text{ and } \underline{b} = \begin{pmatrix} 4 \\ -8 \\ 5 \end{pmatrix}$$

find $\underline{a} \times \underline{b}$

Set up the determinant

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ & & \\ & & \end{vmatrix}$$

$$= \underline{i} \begin{vmatrix} & \\ & \end{vmatrix} - \underline{j} \begin{vmatrix} & \\ & \end{vmatrix} + \underline{k} \begin{vmatrix} & \\ & \end{vmatrix}$$

$$=$$

Compare your answer with Example 16 P181

same vector but in the opposite direction

USE WHICHEVER METHOD YOU PREFER