Unit FP2 – Further Pure Mathematics

The examination

The examination will consist of one 1½ hour paper. It will contain about eight questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, –, ×, ÷, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, ln x, e^x, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Prerequisites

A knowledge of the specifications for C1, C2, C3, C4 and FP1 and their associated formulae is assumed and may be tested.

	SPECIFICATION	NOTES
	1. Coordinate Systems	
S*	Cartesian and parametric equations for the parabola, ellipse, hyperbola and rectangular hyperbola.	Candidates should be familiar with the equations: $y^2 = 4ax; x = at^2, y = 2at.$ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $x = a\cos t, y = b\sin t.$ $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$; $x = a\sec t, y = b\tan t$ and $x = a\cosh t, y = b\sinh t.$ $xy = c^2$; $x = ct, y = \frac{c}{t}$.
S*	The focus-directrix properties of the parabola, ellipse and hyperbola, including the eccentricity.	For example, candidates should know that, for the ellipse, $b^2 = a^2(1 - e^2)$, the foci are $(ae, 0)$ and $(-ae, 0)$ and the equations of the directrices are $x = +\frac{a}{e}$ and $x = -\frac{a}{e}$.
S*	Tangents and normals to these curves.	The condition for $y = mx + c$ to be a tangent to these curves is expected to be known.
S*	Simple loci problems.	

S*	Intrinsic coordinates (s, ψ).	Candidates should be familiar with the equations $\frac{dy}{dx} = \tan\psi$, $\frac{dx}{ds} = \cos\psi$, $\frac{dy}{ds} = \sin\psi$. In appropriate cases, candidates should be able to obtain intrinsic equations of curves given cartesian or parametric equations.
S*	Radius of curvature.	For curves with cartesian, parametric or intrinsic equations.
	2. Hyperbolic functions	
N	Definition of the six hyperbolic functions in terms of exponentials. Graphs and properties of the hyperbolic functions.	For example, $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\text{sech}\, x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$. Candidates should be able to derive and use simple identities such as $\cosh^2 x - \sinh^2 x \equiv 1$ and $\cosh^2 x + \sinh^2 x \equiv \cosh 2x$ and to solve equations such as $a \cosh x + b \sinh x = c$.
N	Inverse hyperbolic functions, their graphs, properties and logarithmic equivalents.	For example, $\text{arsinh}\, x = \ln[x + \sqrt{(1 + x^2)}]$ and candidates may be required to prove this and similar results.
	3. Differentiation	
S*	Differentiation of hyperbolic functions and expressions involving them.	For example, $\tanh 3x$, $x \sinh^2 x$, $\frac{\cosh 2x}{\sqrt{x+1}}$.
S*	Differentiation of inverse functions, including trigonometric and hyperbolic functions.	For example, $\arcsin x + x\sqrt{(1 - x^2)}$, $\frac{1}{2}(\text{artanh}\, x^2)$.
	4. Integration	
S*	Integration of hyperbolic functions and expressions involving them.	
S*	Integration of inverse trigonometric and hyperbolic functions.	For example, $\int \text{arsinh}\, x\, dx$, $\int \arctan x\, dx$.
S*	Integration using hyperbolic and trigonometric substitutions.	To include the integrals of $1/(a^2 + x^2)$, $1/\sqrt{(a^2 - x^2)}$, $1/\sqrt{(a^2 + x^2)}$, $1/\sqrt{(x^2 - a^2)}$.
S*	Use of substitution for integrals involving quadratic surds.	In more complicated cases, substitutions will be given.

S *	The derivation and use of simple reduction formulae.	Candidates should be able to derive formulae such as $nI_n = (n-1)I_{n-2}$, $n \geq 2$, for $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, \mathrm{d}x$, $I_{n+2} = \dfrac{2\sin(n+1)x}{n+1} + I_n$ for $I_n = \int \dfrac{\sin nx}{\sin x} \mathrm{d}x$, $n > 0$.
S *	The calculation of arc length and the area of a surface of revolution.	The equation of the curve may be given in cartesian or parametric form. Equations in polar or intrinsic form will not be set.