FP2 PRACTICE PAPER 6 Mark Schemes

1.	(a) $\frac{1}{r(r+2)} \equiv \frac{A}{r} + \frac{B}{r+2} \equiv \frac{A(r+2)+Br}{r(r+2)}$ and attempt to find A and B	M1
	$\equiv \frac{1}{2r} - \frac{1}{2(r+2)}$	A1 (2)
	(b) $\sum \frac{4}{r(r+2)} \equiv 2\left[\frac{1}{r} - \frac{1}{r+2}\right]$	
	$\sum_1^n \left[\frac{1}{r} - \frac{1}{r+2}\right] = \{1 - \frac{1}{3}\} + \left\{\frac{1}{2} - \frac{1}{4}\right\} + \left\{\frac{1}{3} - \frac{1}{5}\right\} + \ldots\ldots$ $+ \left\{\frac{1}{n-1} - \frac{1}{n+1}\right\} + \left\{\frac{1}{n} - \frac{1}{n+2}\right\}$	M1A1
	[If A and B incorrect, allow A1√ here only, providing still differences]	
	$= \frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}$	A1
	Forming single fraction: $\frac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{2(n+1)(n+2)}$	M1
	Deriving given answer $\frac{n(3n+5)}{(n+1)(n+2)}$, cso	A1 (5)
	(c) Using S(100) − S(49) = $\frac{100 \times 305}{101 \times 102} - \frac{49 \times 152}{50 \times 51}$	M1A1
	[= 2.96059… − 2.92078…]	
	= 0.0398 (4 d.p.)	A1 (3) [10]
	[Allow S(100) − S(50), (⇒ 0.0383) for M1]	

2.	(a) $\frac{dy}{dx} = x\frac{dv}{dx} + v, \quad \frac{d^2y}{dx^2} = x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}$	M1A1
	[M1 for diff. product, A1 both correct]	
	$\therefore x^2\left(x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}\right) - 2x\left(x\frac{dv}{dx} + v\right) + (2 + 9x^2)vx = x^5$	M1
	$x^3\frac{d^2v}{dx^2} + 2x^2\frac{dv}{dx} - 2x^2\frac{dv}{dx} - 2vx + 2vx + 9vx^3 = x^5$	A1
	$[x^3\frac{d^2v}{dx^2} + + 9vx^3 = x^5]$	
	Given result: $\frac{d^2v}{dx^2} + 9v = x^2$ cso	A1 (5)
	(b) CF: $v = A\sin 3x + B\cos 3x$ (may just write it down)	M1A1
	Appropriate form for P1: $v = \lambda x^2 + \mu$ (or $ax^2 + bx + c$)	M1
	Complete method to find λ and μ	M1
	$v = A\sin 3x + B\cos 3x + \frac{1}{9}x^2 - \frac{2}{81}$ [f.t. only on wrong CF]	M1A1√ (6)
	(c)$\therefore$ $y = Ax\sin 3x + Bx\cos 3x + \frac{1}{9}x^3 - \frac{2}{81}x$	B1√ (1) [12]
	[f.t. for $y = x$ (candidate's CF + PI), providing two arbitary constants]	

3.

(a) For C: Using polar/ cartesian relationships to form Cartesian equation — M1

so $x^2 + y^2 = 6x$ — A1

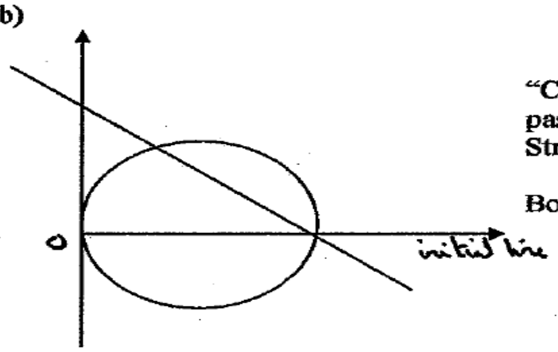
[Equation in any form: e.g. $(x-3)^2 + y^2 = 9$ from sketch.

or $\sqrt{x^2+y^2} = \dfrac{6x}{\sqrt{x^2+y^2}}$]

For D: $r\cos\left(\dfrac{\pi}{3} - \theta\right) = 3$ and attempt to expand — M1

$\dfrac{x}{2} + \dfrac{\sqrt{3}\,y}{2} = 3$ (any form) — M1A1 (5)

(b)



"Circle", symmetric in initial line passing through pole — B1

Straight line — B1

Both passing through (6, 0) — B1 (3)

(c) Polars: Meet where $6\cos\theta\cos(\frac{\pi}{3} - \theta) = 3$ — M1

$\sqrt{3}\sin\theta\cos\theta = \sin^2\theta$ — M1

$\sin\theta = 0$ or $\tan\theta = \sqrt{3}$ $[\theta = 0$ or $\frac{\pi}{3}]$ — M1

Points are $(6, 0)$ and $(3, \frac{\pi}{3})$ — B1,A1 (5) [13]

4.

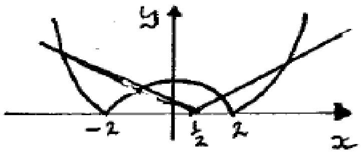
$\dfrac{dy}{dx} + \dfrac{2}{1+x}y = \dfrac{1}{x(x+1)}$ Attempt $y' + Py = Q$ form — M1

I.F. $= e^{\int \frac{2}{1+x}dx} = e^{2\ln(1+x)}$, $= (1+x)^2$ — M1, A1

$\therefore$ $y(1+x)^2 = \int\left(\dfrac{x+1}{x}\right)dx$ OR $\dfrac{d}{dx}\left(y(1+x)^2\right) = \dfrac{x+1}{x}$ — M1 (√ I.F.)

i.e. $(y(1+x)^2 =)\ x + \ln x + (C)$ — M1 A1

$y = \dfrac{x + \ln x + C}{(1+x)^2}$ — A1 c.a.o. (7)

5.(a)	 W Shape - Symmetric about y-axis	B1	
	V Shape. Vertex on positive x-axis	B1	
	$-2,\ 2$	B1	
	$\frac{1}{2}$	B1	(4)
(b)	$x^2 - 4 = 2x - 1$	M1	
	$x^2 - 2x - 3 = 0 \quad \Rightarrow \quad x = \underline{3\ ,\ -1}$	A1	
	$x^2 - 4 = -(2x-1)$	M1	
	$x^2 + 2x - 5 = 0\ , \quad \Rightarrow \quad x = \dfrac{-2 \pm \sqrt{4 + 20}}{2}$ Correct 3 term Quadratic = 0	A1,	
	$x = \underline{-1 \pm \sqrt{6}}$	A1	(5)
(c)	$x < -1 - \sqrt{6}\ ; \quad -1 < x < \sqrt{6} - 1\ ; \quad x > 3$ (√surds) Accept 3 s.f.	B1√; B1√; B1	(3) (12)

6.	(a) $z^n = e^{in\theta} = (\cos n\theta + i \sin n\theta), \quad z^{-n} = e^{-in\theta} = (\cos n\theta - i \sin n\theta)$	M1	
	Completion (needs to be convincing) $z^n - \dfrac{1}{z^n} = 2i \sin n\theta$ (*)AG	A1	(2)
	(b) $\left(z - \dfrac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \dfrac{10}{z} + \dfrac{5}{z^3} - \dfrac{1}{z^5}$	M1A1	
	$= \left(z^5 - \dfrac{1}{z^5}\right) - 5\left(z^3 - \dfrac{1}{z^3}\right) + 10\left(z - \dfrac{1}{z}\right)$		
	$(2i\sin\theta)^5 = 32i \sin^5\theta = 2i \sin 5\theta - 10i \sin 3\theta + 20i \sin\theta$	M1A1	
	$\Rightarrow \sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10 \sin\theta)$ (*) AG	A1	(5)
	(c) Finding $\sin^5\theta = ¼ \sin\theta$	M1	
	$\theta = 0, \pi$ (both)	B1	
	$(\sin^4\theta = ¼) \quad \Rightarrow \sin\theta = \pm \frac{1}{\sqrt{2}}$	M1	
	$\theta = \frac{\pi}{4}, \frac{3\pi}{4};\quad \frac{5\pi}{4}, \frac{7\pi}{4}$	A1;A1	(5)
			[12]

7.	(a)	$\left(x^2+1\right)\dfrac{d^3y}{dx^3}+2x\dfrac{d^2y}{dx^2}=4y\dfrac{dy}{dx}+(1-2x)\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}$	M1 A1
		$\left(x^2+1\right)\dfrac{d^3y}{dx^3}=\left(1-4x\right)\dfrac{d^2y}{dx^2}+\left(4y-2\right)\dfrac{dy}{dx}$ (*)	A1 (3)
	(b)	$\left(\dfrac{d^2y}{dx^2}\right)_0=3$	B1
		$\left(\dfrac{d^3y}{dx^3}\right)_0=5$ Follow through: $\dfrac{d^3y}{dx^3}=\dfrac{d^2y}{dx^2}+2$	B1ft
		$y=1+x+\dfrac{3}{2}x^2+\dfrac{5}{6}x^3....$	M1 A1 (4)
	(c)	$x=-0.5,\quad y\approx 1-0.5+0.375-0.104166\ldots$ $=0.77$ (2 d.p.) [awrt 0.77]	B1 (1)
			(8)