FP2 PRACTICE PAPER 6

1. (*a*) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

(2)

(*b*) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{4}{r(r+2)} = \frac{n(3n+5)}{(n+1)(n+2)}.$$

(5)

(*c*) Find the value of $\displaystyle\sum_{r=50}^{100} \frac{4}{r(r+2)}$, to 4 decimal places.

(3)

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2. (*a*) Show that the transformation $y = xv$ transforms the equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (2 + 9x^2)y = x^5, \qquad \text{I}$$

into the equation

$$\frac{d^2v}{dx^2} + 9v = x^2. \qquad \text{II}$$

(5)

(*b*) Solve the differential equation II to find v as a function of x.

(6)

(*c*) Hence state the general solution of the differential equation I.

(1)

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3. The curve C has polar equation $r = 6\cos\theta$, $-\frac{\pi}{2} \le \theta < \frac{\pi}{2}$,

and the line D has polar equation $r = 3\sec\left(\frac{\pi}{3} - \theta\right)$, $-\frac{\pi}{6} \le \theta < \frac{5\pi}{6}$.

(*a*) Find a cartesian equation of C and a cartesian equation of D.

(5)

(*b*) Sketch on the same diagram the graphs of C and D, indicating where each cuts the initial line.

(3)

The graphs of C and D intersect at the points P and Q.

(*c*) Find the polar coordinates of P and Q.

(5)

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4. Find the general solution of the differential equation

$$(x + 1)\frac{dy}{dx} + 2y = \frac{1}{x}, \qquad x > 0.$$

giving your answer in the form $y = f(x)$.

(7)

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5. (*a*) On the same diagram, sketch the graphs of $y = |x^2 - 4|$ and $y = |2x - 1|$, showing the coordinates of the points where the graphs meet the axes.

(4)

(*b*) Solve $|x^2 - 4| = |2x - 1|$, giving your answers in surd form where appropriate.

(5)

(*c*) Hence, or otherwise, find the set of values of x for which of $|x^2 - 4| > |2x - 1|$.

(3)

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6. (*a*) Given that $z = e^{i\theta}$, show that

$$z^n - \frac{1}{z^n} = 2i \sin n\theta,$$

where n is a positive integer.

(2)

(*b*) Show that

$$\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta).$$

(5)

(*c*) Hence solve, in the interval $0 \leq \theta < 2\pi$,

$$\sin 5\theta - 5 \sin 3\theta + 6 \sin \theta = 0.$$

(5)

7.

$$(x^2 + 1)\frac{d^2y}{dx^2} = 2y^2 + (1 - 2x)\frac{dy}{dx}. \qquad \text{(I)}$$

(*a*) By differentiating equation (I) with respect to x, show that

$$(x^2 + 1)\frac{d^3y}{dx^3} = (1 - 4x)\frac{d^2y}{dx^2} + (4y - 2)\frac{dy}{dx}.$$

(3)

Given that $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$,

(*b*) find the series solution for y, in ascending powers of x, up to and including the term in x^3.

(4)

(*c*) Use your series to estimate the value of y at $x = -0.5$, giving your answer to two decimal places.

(1)

TOTAL MARKS: 75