# FP2 PRACTICE PAPER 5 Mark Schemes

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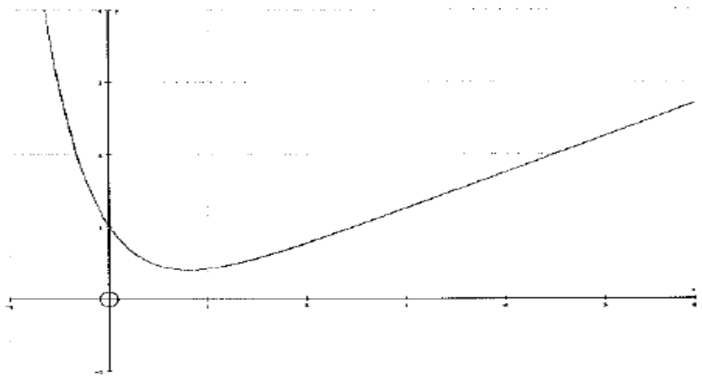
| 1. | (a) | Integrating Factor $= e^{2x}$ | Min point and passing through (0,1) | B1 |
| | | $\frac{d}{dx}(ye^{2x}) = xe^{2x}$ | | M1 |
| | | $ye^{2x} = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$ | | M1 |
| | | $= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$ | shape | A1 |
| | | $\therefore y = \frac{1}{2}x - \frac{1}{4} + ce^{-2x}$ | | A1 |
| | | | | (5) |
| | (b) | $1 = c - \frac{1}{4} \rightarrow c = \frac{5}{4}$ | | M1 |
| | | $\therefore y = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}$ and $\frac{dy}{dx} = \frac{1}{2} - \frac{5}{2}e^{-2x}$ | | M1 |
| | | When $y' = 0$, $e^{-2x} = \frac{1}{5}$ $\therefore 2x = \ln 5$ | | M1 |
| | | $x = \frac{1}{2}\ln 5$, $y = \frac{1}{4}\ln 5$ at minimum point. | | A1 |
| | | | | (4) |
| | (c) |  | | B1 |
| | | | | B1 |
| | | | | (2) |

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| 2. | (a) | Auxiliary equation: $m^2+2m+2=0 \to m=-1\pm i$ | M1 |
| | | Complementary Function is $y=e^{-t}(A\cos t+B\sin t)$ | M1A1 |
| | | Particular Integral is $y=\lambda e^{-t}$, *with* $y'=-\lambda e^{-t}$, *and* $y''=\lambda e^{-t}$ | M1 |
| | | $\therefore(\lambda-2\lambda+2\lambda)e^{-t}=2e^{-t}\to\lambda=2$ | A1 |
| | | $\therefore y=e^{-t}(A\cos t+B\sin t+2)$ | B1 **(6)** |
| | (b) | Puts $1=A+2$ and solves to obtain $A=-1$ | M1, |
| | | $y'=e^{-t}(-A\sin t+B\cos t)-e^{-t}(A\cos t+B\sin t+2)$ | M1 A1ft |
| | | Puts $1=B-A-2$ and uses value for $A$ to obtain $B$ | M1 |
| | | B=2 | A1cso |
| | | $\therefore y=e^{-t}(2\sin t-\cos t+2)$ | A1cso **(6)** |

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| **3.** | **(a)** | $3a(1-\cos\theta) = a(1+\cos\theta)$ | **M1** |
| | | $2a = 4a\cos\theta \rightarrow \cos\theta = \frac{1}{2} \quad \therefore \theta = \frac{\pi}{3} \text{ or } -\frac{\pi}{3}$ | **M1** |
| | | $r = \frac{3a}{2}$ | |
| | | [Co-ordinates of points are $(\frac{3a}{2}, \frac{\pi}{3})$ and $(\frac{3a}{2}, -\frac{\pi}{3})$ ] | **A1 A1** |
| | | | **(4)** |
| | **(b)** | $AB = 2r\sin\theta = \dfrac{3a\sqrt{3}}{2}$ | **M1A1** |
| | | | **(2)** |
| | **(c)** | Area $= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} r^2 d\theta$ | |
| | | $= \frac{1}{2}\int [a^2(1+\cos\theta)^2 - 9a^2(1-\cos\theta)^2] d\theta$ | **M1 M1** |
| | | $= \frac{a^2}{2}\int [1 + 2\cos\theta + \cos^2\theta - 9(1 - 2\cos\theta + \cos^2\theta)] d\theta$ | **A1** |
| | | $= \frac{a^2}{2}\int [-8 + 20\cos\theta - 8\cos^2\theta] d\theta$ | |
| | | $= \text{k}[-8\theta + 20\sin\theta \ldots$ | **B1** |
| | | $\ldots\ldots -2\sin 2\theta - 4\theta]$ | **B1** |
| | | Uses limits $\frac{\pi}{3}$ and $-\frac{\pi}{3}$ correctly or uses twice smaller area and uses limits $\frac{\pi}{3}$ and 0 correctly.(Need not see 0 substituted) | **M1** |
| | | $= a^2[-4\pi + 10\sqrt{3} - \sqrt{3}]$ or $= a^2[-4\pi + 9\sqrt{3}]$ or $3.022a^2$ | **A1** |
| | | | **(7)** |
| | **(d)** | $3a\dfrac{\sqrt{3}}{2} = 4.5 \rightarrow a = \sqrt{3}$ | **B1** |
| | | $\therefore$ Area $= 3[9\sqrt{3} - 4\pi], \ = 9.07\,\text{cm}^2$ | **M1, A1** |
| | | | **(3)** |

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**4.** (*a*) $\arg z = \frac{\pi}{4} \Rightarrow z = \lambda + \lambda\text{i}$ (or putting *x* and *y* equal at some stage) B1

$w = \frac{(\lambda+1)+\lambda\text{i}}{\lambda+(\lambda+1)\text{i}}$, and attempt modulus of numerator or denominator. M1

(Could still be in terms of *x* and *y*)

$|(\lambda+1)+\lambda\text{i}| = |\lambda+(\lambda+1)\text{i}| = \sqrt{(\lambda+1)^2+\lambda^2}$, $\therefore |w| = 1\ (*)$ A1, A1cso (4)

(*b*) $w = \frac{z+1}{z+\text{i}} \Rightarrow zw + w\text{i} = z+1 \Rightarrow z = \frac{1-w\text{i}}{w-1}$ M1

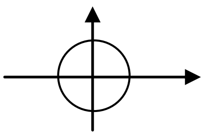
$|z| = 1 \Rightarrow |1-w\text{i}| = |w-1|$ M1 A1

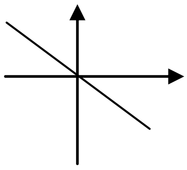
For $w = a + \text{i}b$, $|(1+b)-a\text{i}| = |(a-1)+\text{i}b|$ M1

$\sqrt{(1+b)^2+a^2} = \sqrt{(a-1)^2+b^2}$ M1

$b = -a$ Image is (line) $y = -x$ A1 (6)

(c)





B1 B1 (2)

(d) $z = \text{i}$ marked (*P*) on *z*-plane sketch. B1

$z = \text{i} \Rightarrow w = \frac{1+\text{i}}{2\text{i}} = \frac{\text{i}-1}{-2} = \frac{1}{2} - \frac{1}{2}\text{i}$ marked (*Q*) on *w*-plane sketch. B1 (2)

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| **5.** | Working from RHS:<br>(a) Combining $\frac{1}{r} - \frac{1}{r+1}$ $\left[ \frac{1}{r(r+1)} \right]$ | M1 |
| | Forming single fraction : $\frac{r(r-1)(r+1) + (r+1) - r}{r(r+1)}$ | M1 |
| | $= \frac{r(r^2-1)+1}{r(r+1)} = \frac{r^3 - r + 1}{r(r+1)}$ **AG** | A1cso (3) |
| | Note: For A1, must be intermediate step, as shown | |
| | Working from LHS:<br>(a) $\frac{r(r^2-1)+1}{r(r+1)} = \frac{r(r+1)(r-1)+1}{r(r+1)} = r - 1 + \frac{1}{r(r+1)}$ M1 | |
| | Splitting $\frac{1}{r(r+1)}$ into partial fractions M1 | |
| | Showing $= \frac{r(r^2-1)+1}{r(r+1)} = r - 1 + \frac{1}{r} - \frac{1}{r+1}$ no incorrect working seen A1 | |
| | Notes:<br>In first method, second M needs all necessary terms, allowing for sign errors<br>In second method first M is for division:<br>Second method mark is for method shown (allow "cover up" rule stated)<br>If long division, allow reasonable attempt which has remainder constant or linear function of r.<br>Setting $\frac{r(r^2-1)+1}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$ is M0<br>If 3 or 4 constants used in a correct initial statement,<br>M1 for finding 2 constants; M1 for complete method to find remaining constant(s) | |

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| (b) $\sum_1^n r - \sum_1^n 1 + \sum_1^n \left(\frac{1}{r} - \frac{1}{r+1}\right)$ | |
| $= \frac{n(n+1)}{2}$, $(-)\ n$, $+ \ldots\ldots\ldots\ldots\ \ldots\ldots +$ | B1, B1 |
| $\left[(1-\frac{1}{2}) + (\frac{1}{2}-\frac{1}{3}) + \ldots\ldots\ldots + (\frac{1}{n} - \frac{1}{n+1})\right] =$ | M1 |
| Simplification of method of differences: $1 - \frac{1}{n+1}$ | A1 |
| $\{ = \frac{n(n-1)}{2} + [1 - \frac{1}{(n+1)}]\}$ | |
| Attempt single fraction: $= \frac{n(n+1)(n-1)+2n}{2(n+1)}$ (dep. prev. M1) | depM1 |
| $= \frac{n(n^2+1)}{2(n+1)}$ *or* $\frac{n^3+n}{2(n+1)}$ | A1<br>**(6)** |
| | **[9]** |
| *Alternative*: Using Difference method on whole expression: | |
| $[0 + 1 - \frac{1}{2}] + [1 + \frac{1}{2} - \frac{1}{3}] + [2 + \frac{1}{3} - \frac{1}{4}] \ldots\ldots\ [n-1+\frac{1}{n} - \frac{1}{n+1}]$ | M1 |
| $= (1+2+3\ \ldots\ldots\ldots + n-1)$, $+ [(1 - \frac{1}{n+1})]$ any form | B1, + [A1] |
| $= \frac{n(n-1)}{2}$, $\{+ \frac{n}{n+1}\}$ | B1, |
| $= \frac{n(n+1)(n-1)+2n}{2(n+1)}$ [Attempt single fraction] | depM1 |
| $= \frac{n(n^2+1)}{2(n+1)}$ *or* $\frac{n^3+n}{2(n+1)}$ | A1 |
| Notes:<br>First M mark is for use of method of differences and attempt at some simplification<br>First A mark is for simplified result of this method (no more than 2 terms)<br>Second M mark for attempt at forming single fraction, dependent on first M mark<br>In alternative first B1 need not be added but need to see 1 2 …….. (n – 1) | |

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| **6.** | (a) | $(\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta \quad \therefore$ true for $n = 1$ | B1 |
| | | Assume true for $n = k$, $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ | |
| | | $(\cos\theta + i\sin\theta)^{k+1} = (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$ | M1 |
| | | $= \cos k\theta\cos\theta - \sin k\theta\sin\theta + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$ | M1 |
| | | (Can be achieved either from the line above or the line below) | |
| | | $= \cos(k+1)\theta + i\sin(k+1)\theta$ | A1 |
| | | Requires full justification of $(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$ | |
| | | ($\therefore$ true for $n = k + 1$ if true for $n = k$) $\quad \therefore$ true for $n \in \mathbb{Z}^+$ by induction | A1cso (5) |
| | (b) | $\cos 5\theta = \text{Re}\left[(\cos\theta + i\sin\theta)^5\right]$ | |
| | | $= \cos^5\theta + 10\cos^3\theta\, i^2\sin^2\theta + 5\cos\theta\, i^4\sin^4\theta$ | M1 A1 |
| | | $= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ | M1 |
| | | $= \cos^5\theta - 10\cos^3\theta(1 - \cos^2\theta) + 5\cos\theta(1 - \cos^2\theta)^2$ | M1 |
| | | $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*) | A1cso (5) |
| | (c) | $\dfrac{\cos 5\theta}{\cos\theta} = 0 \Rightarrow \cos 5\theta = 0$ | M1 |
| | | $5\theta = \dfrac{\pi}{2} \qquad \theta = \dfrac{\pi}{10}$ | A1 |
| | | $x = 2\cos\theta, \qquad x = 2\cos\dfrac{\pi}{10}$ is a root (*) | A1 (3) |
| | | | **(13)** |