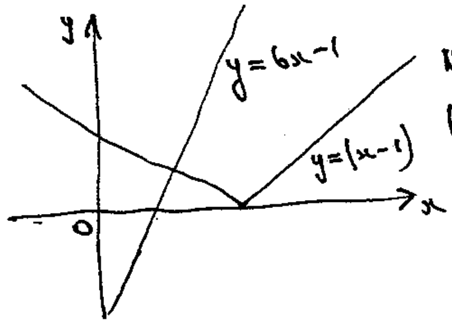
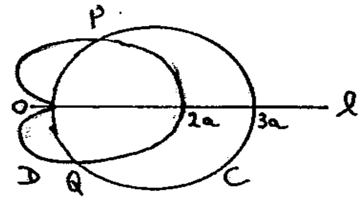
FP2 PRACTICE PAPER 1 Mark Schemes

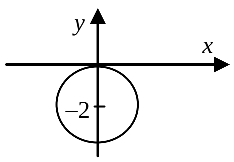
1.	$x \geqslant 1$ and $x - 1 > 6x - 1$	
	$x < 0$ No values	
	OR	
		
	No critical value from $y = x - 1$, $y = 6x - 1$	M1 A1
	$\left.\begin{array}{l} y = 1 - x \\ y = 6x - 1 \end{array}\right\} \rightarrow x = \frac{2}{7}$ as critical value	M1 A1
	Solution set $x < \frac{2}{7}$ [Correct final statement needed for A1 here]	A1 cso (5)

Question number	Scheme	Marks
2. (a)	$\frac{dv}{dt} - \frac{1}{t}v = 1 \rightarrow \text{I.F.} = e^{-\int \frac{1}{t} dt} = e^{-\ln t} = \frac{1}{t}$	M1 A1 A1
	$\frac{d}{dt}\left(\frac{v}{t}\right) = \frac{1}{t} \rightarrow \frac{v}{t} = \ln t + C$	M1 A1
	$v = t(\ln t + C)$ ⊛	A1 (6)
(b)	$v = 3$ at $t = 2$ so $C = \frac{3}{2} - \ln 2$ or $\cdot 807$	M1 A1
	At $t = 4$, $\frac{v}{4} = \ln 4 + \frac{3}{2} - \ln 2$	M1
	$v = 8.77$	A1 (4)

3.	(a)	$y = \frac{1}{2}x^2e^x$, $y' = \frac{1}{2}x^2e^x + xe^x$	B1
		$y'' = \frac{1}{2}x^2e^x + 2xe^x + e^x$	B1
		$y'' - 2y' + y = \frac{1}{2}x^2e^x + 2xe^x + e^x - x^2e^x - 2xe^x + \frac{1}{2}x^2e^x$	M1
		$= e^x$	A1 (4)
		OR $ye^{-x} = \frac{1}{2}x^2$, $y'e^{-x} - ye^{-x} = x$ M1, B1	
		$y''e^{-x} - 2y'e^{-x} + ye^{-x} = 1 \Rightarrow y'' - 2y' + y = e^x$ B1 A1	
	(b)	Auxiliary equation $m^2 - 2m + 1 = 0$, $\Rightarrow m = 1$ repeated	M1, A1
		Complementary function $e^x(A + Bx)$	A1
		General solution $y = e^x(A + Bx) + \frac{1}{2}x^2e^x$	A1 f.t.
		$x = 0$, $y = 1 \Rightarrow A = 1$ (cso)	B1
		$y' = e^x(A + Bx) + Be^x + xe^x + \frac{1}{2}x^2e^x$	M1
		$y' = 2$ at $x = 0 \Rightarrow 2 = A + B \Rightarrow B = 1$	M1 A1
		Specific solution $y = e^x\left(1 + x + \frac{1}{2}x^2\right)$	A1 cso (9)

Question number	Scheme	Marks
4. (a)		
	Circle	B1
	Diameter $O \rightarrow 3a$ on initial line	B1
	Cardioid cusp at O	B1
	symmetry on initial line and $\underline{2a}$	B1 (4)
(b)	$3a\cos\theta = a(1+\cos\theta) \longrightarrow \cos\theta = \frac{1}{2}$	M1
	$\theta = \pm\frac{\pi}{3}$ $\quad r = \frac{3a}{2}$ at P and Q	A1 A1 (3)
(c)	Area $A_1 = \frac{1}{2}\int a^2(1+\cos\theta)^2\,d\theta$	M1
	$= \frac{1}{2}a^2\int\left[1+2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right]d\theta$	M1 A1
	$= \frac{1}{2}a^2\left[\frac{3\theta}{2}+2\sin\theta+\frac{1}{4}\sin 2\theta\right]$	(A1, A1, A0)
	Evaluating A1 using limits 0 and $\frac{\pi}{3}$ to get $A_1 = \frac{\pi a^2}{4} + \frac{9\sqrt{3}a^2}{16}$	M1 A1 cao (7)
(d)	Area required $= \frac{9}{4}\pi a^2 - 2A_1 - 2A_2$	M1, B1
	$= \frac{9\pi a^2}{4} - \frac{\pi a^2}{2} - \frac{9\sqrt{3}a^2}{8} - \frac{3\pi a^2}{4} + \frac{9a^2\sqrt{3}}{8}$	M1
	$= \pi a^2$	A1 (4)

5. (*a*)	$\frac{dy}{dx} + y\left(\frac{\sin x}{\cos x}\right) = \cos^2 x$	M1
	Int. factor $e^{\int \tan x\,dx} = e^{-\ln(\cos x)} = \sec x$	M1, A1
	Integrate: $y \sec x = \int \cos x\,dx$	M1 , A1
	$y \sec x = \sin x + C$	A1
	$(y = \sin x \cos x + C\cos x)$	**(6)**
(*b*)	When $y = 0$, $\cos x(\sin x + C) = 0$, $\cos x = 0$	M1
	2 solutions for this $(x = {}^{\pi}/_{2}, {}^{3\pi}/_{2})$	A1 **(2)**
(*c*)	$y = 0$ at $x = 0$: $C = 0$: $y = \sin x \cos x$	M1
	$(y = ½\sin 2x)$	
	Shape	A1
		A1 **(3)**
	Scales	
		(11 marks)

6. (*a*)(i)	$\lvert x + (y - 2)\mathrm{i}\rvert = 2\lvert x + (y + \mathrm{i})\rvert$	M1
	$\therefore x^2 + (y - 2)^2 = 4(x^2 + (y + 1)^2)$	
(ii)	so $3x^2 + 3y^2 + 12y = 0$ any correct from; 3 terms; isw	A1 **(2)**
	 Sketch circle	B1
	Centre (0,–2)	B1
	$r = 2$ or touches axis	B1 **(3)**
(*b*)	$w = 3(z - 7 + 11\mathrm{i})$	B1
	$= 3z - 21 + 33\mathrm{i}$	B1 **(2)**
		(7 marks)

[P6 June 2002 Qn 3]

7.	$zw =$	
	$12\left(\cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3}\right) + 12\mathrm{i}\left(\sin\frac{\pi}{4}\cos\frac{2\pi}{3} + \cos\frac{\pi}{4}\sin\frac{2\pi}{3}\right)$	B1
	$= 12\left[\cos\frac{11\pi}{12} + \mathrm{i}\sin\frac{11\pi}{12}\right]$	M1 A1
		(3 marks)

[P4 January 2003 Qn 1]

8. (*a*)	$\dfrac{1}{r+1} - \dfrac{1}{r+3}$	B1 B1 (2)
(*b*)	$\displaystyle\sum_{1}^{n} \dfrac{1}{r+1} - \dfrac{1}{r+3} = \dfrac{1}{2} - \cancel{\dfrac{1}{4}}$	
	$+ \dfrac{1}{3} - \dfrac{1}{5}$	
	$+ \cancel{\dfrac{1}{4}} - \cancel{\dfrac{1}{6}}$	M1
	$\vdots$	
	$+ \cancel{\dfrac{1}{n}} - \dfrac{1}{n+2}$	
	$+ \cancel{\dfrac{1}{n+1}} - \dfrac{1}{n+3}$	
	$= \left(\dfrac{1}{2} + \dfrac{1}{3}\right) + \left(-\dfrac{1}{n+2} - \dfrac{1}{n+3}\right)$	A1 A1
	$= \dfrac{5}{6} - \left(\dfrac{5n^2 + 25n + \cancel{30} - 12n - \cancel{30}}{6(n+2)(n+3)}\right)$	M1
	$= \dfrac{n(5n+13)}{6(n+2)(n+3)}$ *	A1 cso (5)
		(7 marks)