BHASVIC MαTHS

A2 Doubles summer assignment *4*

Section: *FP1*

(14 questions this week as it is half term)

<u>Past</u>

1. Show that

$$\int_0^1 x^2 e^x \, dx = e - 2$$

Show that the use of the trapezium rule with five strips (six ordinates) gives an estimate that is about 3.8% too high.

Explain why approximate evaluation of this integral using the trapezium rule will always result in an overestimate, however many strips are used.

2. If

$$I_n = \int_0^1 t^n e^{-t} \, dt$$

Where n is an integer, show that

$$I_0 = 1 - e^{-1}$$

By integrating by parts, show that $I_n = nI_{n-1} - e^{-1}$ for $n \geq 1$. Hence evaluate I_3, leaving your answer in terms of e^{-1}

3. Find the area of the region between the negative x-axis and the graph $y = x\sqrt{x+1}$

a) Using integration by parts

b) Using substitution

4. Solve the following equations

a) $\sin(\theta + 40) = 0.7, \;\; 0 \leq \theta \leq 360$

b) $3\cos^2\theta + 5\sin\theta - 1 = 0\,, \;\; 0 \leq \theta \leq 36-$

c) $2\cos\left(\theta - \frac{\pi}{6}\right) = 1, \;\; \pi \leq \theta \leq pi$

5. Prove the following identities

$$\frac{1-\cos 2\theta}{1+\cos 2\theta} \equiv \tan^2 \theta$$

$$\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot \theta$$

$$\tan 4\theta = \frac{4t(1-t^2)}{1-6t^2+t^4}, \qquad where\ t = tan\theta$$

<u>Present</u>

6. Points A and B have position vectors $OA = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$ and $OB = \begin{pmatrix} -1 \\ 7 \\ 2 \end{pmatrix}$. Find the acute angle between AB and OA.

7. Four points are given with coordinates $A(2,-1,3), B(1,1,2), C(6,-1,2)$ and $D(7,-3,3)$. Find the angle between AC and BD.

8. Four points have coordinates $A(2,4,1), B(k,4,2k), C(k+4,2k+4,2k+2)$ and $D(6,2k+4,3)$

a) Show that $ABCD$ is a parallelogram

b) When $k=1$ find the angles of the parallelogram

c) Find the value of k for which $ABCD$ is a rectangle

9. Find in vector form, the equation of the planes which contain the point with position vector a and are perpendicular to the vector n.

a) $a = 3i + 5j - 2k, n = i + j + k$

b) $a = -3i + 2j + k, n = 1 + j + k$

c) $a = 3i + 5j - 2k, n = -i - j - k$

d) $a = 2i + 7j - k, n = 2i + 2j + 2k$

10. Find, to 1 decimal place, the smaller angle between the planes

a) $r \cdot \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} = 4$ and $r \cdot \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix} = 2$

b) $r \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 4$ and $r \cdot \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix} = 2$

c) $x + y - 4z = 4$ and $5x - 2y + 3z = 13$

11. The plane Π_1 has equation $-x + 3y - 2z - 13 = 0$. Find the Cartesian and vector equations of the plane Π_2 that is parallel to Π_1 and passes through the point $(3,0,-4)$

12. Use calculus to find the shortest distance between the point $(1,5,-7)$ and the line with equation

$$r = \begin{pmatrix} -1 \\ 9 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$$

13. Use calculus to find the shortest distance between the point $(-4,0,2)$ and the line with equation

$$\frac{x+2}{-1} = \frac{y+2}{2} = \frac{z-1}{1}$$

14. Use calculus to find the shortest distance between each pair of lines

a) $x = 3, \frac{y+1}{6} = \frac{z}{2}$ and $r = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$

b) $r = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $x + 5 = 2 - y = z + 1$