BHASVIC MαTHS

A2 Doubles assignment *summer 1*

Section: *Mech and FP1*

<u>Past</u>

1. Integrate

$$\int \frac{x^2}{x^2-1}\,dx$$

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} (\sin^2 x + 3\cos^2 x)\,dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos 2x}{\sin^2 2x}\,dx$$

2. Solve the following in $0 \le x \le 360$

a) $\frac{\tan 47 - \tan\theta}{1+\tan 47\tan\theta} = 1.5$

b) $\sin 2\theta + \sin\theta - \tan\theta = 0$

c) $2 + \cos\theta\sin\theta = 8\sin^2\theta$

3. Prove the following identities

a) $\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

b) $\frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1} \equiv \tan\theta$

4.

$OABC$ is a parallelogram and the point M is the midpoint of AB.

The point N lies on the diagonal AC so that $AN : NC = 1 : 2$.

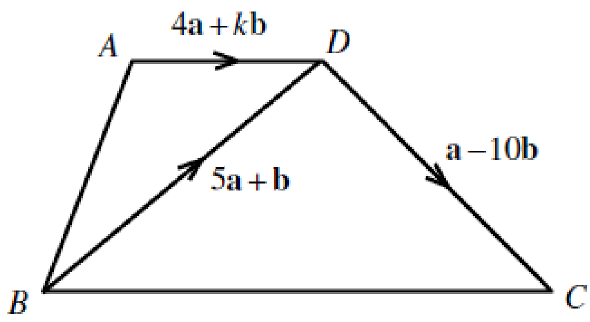
Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{AC}$, $\overrightarrow{AN}$, $\overrightarrow{ON}$ and $\overrightarrow{NM}$.

b) Deduce, showing your reasoning, that O, N and M are collinear.

5.

The figure below shows a trapezium $OBCA$ where AD is parallel to BC.



The following information is given for this trapezium.

$\overline{BD} = 5\mathbf{a} + \mathbf{b}$, $\overline{DC} = \mathbf{a} - 10\mathbf{b}$ and $\overline{AD} = 4\mathbf{a} + k\mathbf{b}$, where k is an integer.

a) Find the value of k.

b) Find a simplified expression for $\overline{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

6.

OAB is a triangle with the point P being the midpoint of OB and the point Q being the midpoint of AB.

The point R is such so that $\overrightarrow{AR} = \frac{2}{3}\overrightarrow{AP}$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{AP}$, $\overrightarrow{AQ}$ and $\overrightarrow{AR}$.

b) By finding simplified expressions, in terms $\mathbf{a}$ and $\mathbf{b}$, for two more suitable vectors, show that the points O, R and Q are collinear.

7.

Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = 2\mathbf{a}$ and $\overrightarrow{OD} = 2\mathbf{a} + \mathbf{b}$.

If $\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OD}$ prove that the point E lies on the straight line AB.

8.

$OABC$ is a square.

The point M is the midpoint of AB and the point N is the midpoint of MC.

The point D is such so that $\overrightarrow{AD} = \frac{3}{2}\overrightarrow{AB}$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{BD}$, $\overrightarrow{MC}$, $\overrightarrow{MN}$, $\overrightarrow{ON}$ and $\overrightarrow{ND}$.

b) Deduce, showing your reasoning, that O, N and D are collinear.

8.

With respect to a fixed origin O, the point A has position vector $8\mathbf{i}-6\mathbf{j}+5\mathbf{k}$ and the point B has position vector $t\mathbf{i}+t\mathbf{j}+2t\mathbf{k}$.

a) Show clearly that

$$|AB|^2 = 6t^2 - 24t + 125.$$

Let $f(t) = 6t^2 - 24t + 125$.

b) Find the value of t for which $f(t)$ takes a minimum value.

c) Hence determine the closest distance between A and B.

10.

Find the value of λ and μ, given that the vectors $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

a) $7\lambda\mathbf{a}+5\lambda\mathbf{b}+3\mu\mathbf{a}-\mu\mathbf{b}=5\mathbf{a}+2\mathbf{b}$

b) $2\lambda\mathbf{a}+3\lambda\mathbf{b}+3\mu\mathbf{a}-5\mu\mathbf{b}=-5\mathbf{a}+21\mathbf{b}$

c) $2\lambda\mathbf{a}+3\mu\mathbf{b}=7\mu\mathbf{a}+11\lambda\mathbf{b}+57\mathbf{a}+6\mathbf{b}$

d) $\lambda\mathbf{a}+3\lambda\mathbf{b}+\mu\mathbf{b}=2\mu\mathbf{a}+5\mathbf{a}+8\mathbf{b}$

11.

a)

Given the point $A(2,1,-3)$ and the vector $\overline{AB}=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$ determine the coordinates of the point B.

b)

Given the point $A(6,-4,1)$ and the vector $\overline{BA}=\mathbf{i}-\mathbf{j}+3\mathbf{k}$ determine the coordinates of the point B.

c)

Given the point $A(2,3,5)$ and the vectors $\overline{BA}=3\mathbf{i}-2\mathbf{j}$ and $\overline{BC}=2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, find the coordinates of the point C.

12.

$$A(2t,t,2) \quad \text{and} \quad B(t,4,1).$$

a) If A and B are variable points, where t is the time in seconds, show that

$$|\overline{AB}| = \sqrt{2t^2 - 8t + 17}.$$

b) Hence find the shortest distance between A and B.