EXAM STYLE PRACTICE SOLUTIONS

SECTION A STATISTICS.

1. Let X be the random variable that represent computer that last longer than 500 hours.

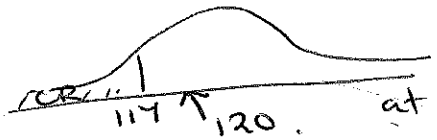
$X \sim B(40, 0.52)$

a) $P(X \geq 22) = 1 - P(X \leq 21)$
$= 0.4133.$

b) n large p close to 0.5

c) $X \sim N(np, np(1-p))$ $n = 250$ $p = 0.52$
$\sigma = 7.899$
$\sim N(130, 62.4)$
$P(X \leq 120) = P(Y \leq 120.5)$ continuity correction
$= 0.115$

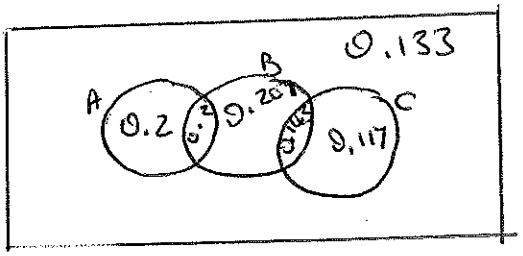
d) $P(X < x) = 0.05$



$x = 117.$

so 120 not in critical region so no evidence to reject engineer's claim.

2. a)



b) independent if
$P(A \cap B) = P(A) \times P(B)$
$0.2 = 0.4 \times 0.55$
$\neq 0.22$
so not independent

c) $P(A|B') = \frac{0.2}{0.45}$ since $P(B') = 0.45$

d) $P(A \cap B) = 0.2$
$P(A \cap B)' = 0.8$ so $P(C|(A \cap B)') = \frac{0.26}{0.8}$
$= \frac{13}{40}$

3. a) continuous - on a continuous scale

b) MENU 7 THEN (set up frequency ON) AC then OPTN 2

		cf.
13.5	2	2
16.5	6	8
19	11	19
21	7	26
24	5	31
	31	

Mean $\frac{\Sigma fx}{n} = 19.4°C$

S.d $\sqrt{\frac{Sxx}{n}} = 2.81°C$

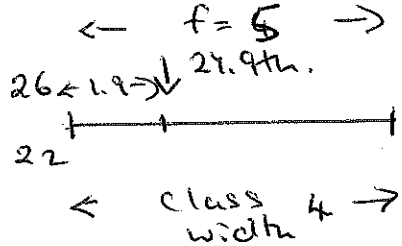
c) Continuous data, unequal class widths

d) $P_{90} = \frac{90}{100} \times 31 = 27.9$th.

(falls in class $22 \leq t < 26$)

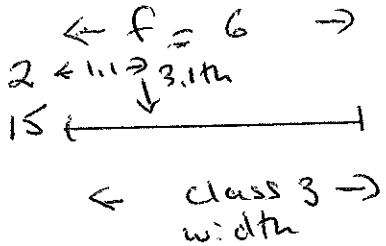
value $= 22 + \frac{1.9}{5} \times 4$

$= 23.52.$



$P_{10} = \frac{10}{100} \times 31 = 3.1$th.

(falls in class $15 \leq t < 18$)



P_{10}:

value $= 15 + \frac{1.1}{6} \times 3$

$= 15.55$

10th to 90th interpercetile range

is $23.52 - 15.55$

$= 7.97°C$

3 e) $r = 0.612$. sample 8 days.

$H_0: \rho = 0$

$H_\rho: \rho > 0$.

from tables critical value is 0.6215

since $r < 0.6215$ no evidence that the population correlation coefficient is greater than zero.

4 a) yes since r is close to 1 (use calculator)

b) $q = kt^n$

$\log q = \log k + n \log t$.

$y = \log k + nx$

$y = 0.0761 + 2.1317x$.

$\log k = 0.0761$

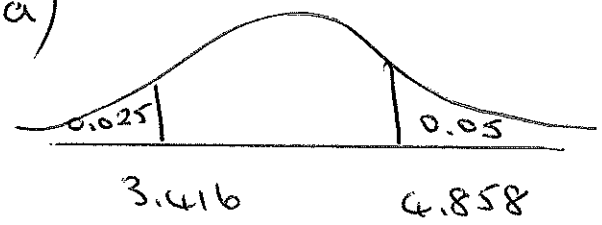
$k = 10^{0.0761}$

$k = 1.19$

$n = 2.1317$

c) extrapolation long way from range of data - not sensible.

5. a)



$$P\left(W < \frac{3.416-\mu}{\sigma}\right) = 0.025$$

negative sign as mean 0 lies at right

$$\frac{3.416-\mu}{\sigma} = 1.9599.$$

$$-(3.416-\mu) = 1.960\sigma \quad -①$$

$$P\left(W < \frac{4.858-\mu}{\sigma}\right) = 0.95$$

$$\frac{4.858-\mu}{\sigma} = 1.645$$

$$4.858-\mu = 1.645\sigma \quad -②$$

Solve ① and ②

gives $\mu = 4.2$.

$\sigma = 0.4$

calculator

lower bound

upp. bound

$\mu = 4.2$

$\sigma = 0.4$

$P(3.5 < W < 4.6) = 0.8012.$

80.1%.

b) $n = 15 \quad p = 0.801$

$X \sim B(15, 0.801)$

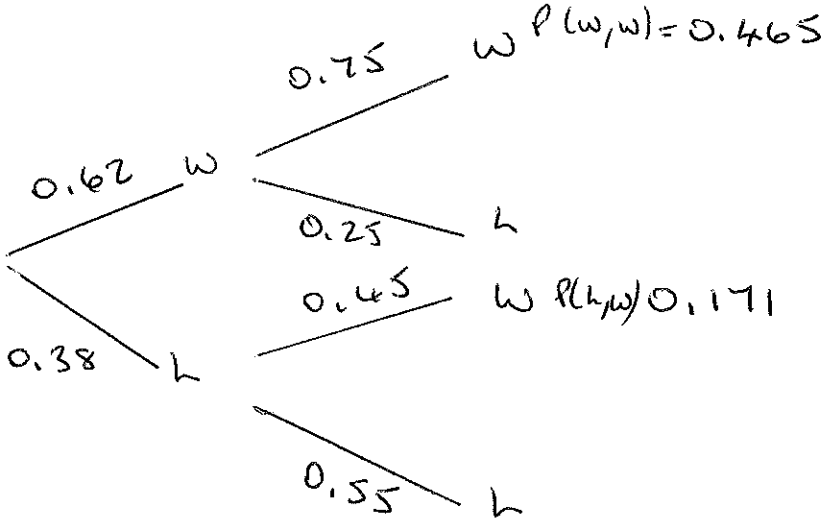
$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.05977$

$= 0.9402$

5c) IN CLASS TO DISCUSS.

6.



$$P(WW \mid W_{second}) = \frac{0.465}{0.465 + 0.171}$$

$$= 0.731 \quad (3\text{ s.f.})$$

EXAM STYLE PRACTICE SOLUTIONS

SECTION B MECHANICS

7. $\underline{v} = (2-6t^2)\underline{i} - t\underline{j}$

$\underline{s} = (2t-2t^3)\underline{i} - \frac{t^2}{2}\underline{j} + a\underline{i} + b\underline{j}$

when $t=1$, $\underline{s} = 5\underline{i}$:

$5\underline{i} = -\frac{1}{2}\underline{j} + a\underline{i} + b\underline{j}$

$5\underline{i} + \frac{1}{2}\underline{j} = a\underline{i} + b\underline{j}$

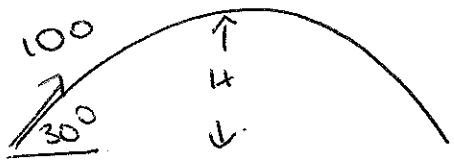
$\underline{s} = (2t - 2t^3 + 5)\underline{i} + \left(\frac{1}{2} - \frac{t^2}{2}\right)\underline{j}$

when $t=5$

$\underline{s} = -235\underline{i} + 12\underline{j}$

Distance $|\underline{s}| = 235$ m (3sf).

8.



$\uparrow$: $[s = ut + \frac{1}{2}at^2]$

$0 = 100\sin 30° t - \frac{1}{2}gt^2$

$0 = t(50 - \frac{1}{2}gt)$

$t = \frac{100}{g}$

$t = 10.2$s

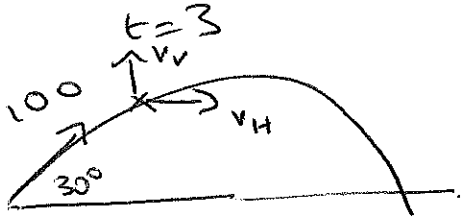
$\uparrow$ ~~$[v = u + at]$~~ $[v^2 = u^2 + 2as]$

~~$0 = 100\sin 30° -$~~ $0 = (100\sin 30°)^2 - 2gH$.

$H = \frac{50^2}{2g}$

$H = 128$m

8 c)



$\uparrow$: $[v = u + at]$

$v_V = 100\sin 30° - 3g$

$= 50 - 3g$

$\rightarrow$ $v_H = 100\cos 30°$

$= 50\sqrt{3}$

$V = \sqrt{(50-3g)^2 + (50\sqrt{3})^2}$

$= 89.0\ ms^{-1}$

9. a) $\underline{u} = 2\underline{i}$

$\underline{a} = 0.2\underline{i} - 0.8\underline{j}$

$t = 10$

$\underline{s}$? $(\underline{r})$

$[\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2]$

$\underline{r} = 2\underline{i} \times 10 + \frac{1}{2}(0.2\underline{i} - 0.8\underline{j})100$

$= 20\underline{i} + (0.1\underline{i} - 0.4\underline{j})100$

$= 20\underline{i} + 10\underline{i} - 40\underline{j}$

$= 30\underline{i} - 40\underline{j}$

b) $|\underline{r}| = \sqrt{30^2 + 40^2}$

$= 50m$

+ve i comp
-ve j comp.

c) $\underline{r} = 30\underline{i} - 40\underline{j} + 5t\underline{i}$

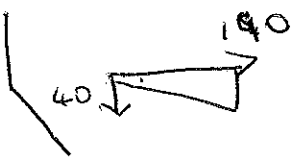
$30 + 5t = 40.$

$t = 2.$

after 12 seconds it is southeast of A

d) $\underline{r} = (30 + 5 \times 2)\underline{i} - 40\underline{j} + 30 \times 5\underline{i}$

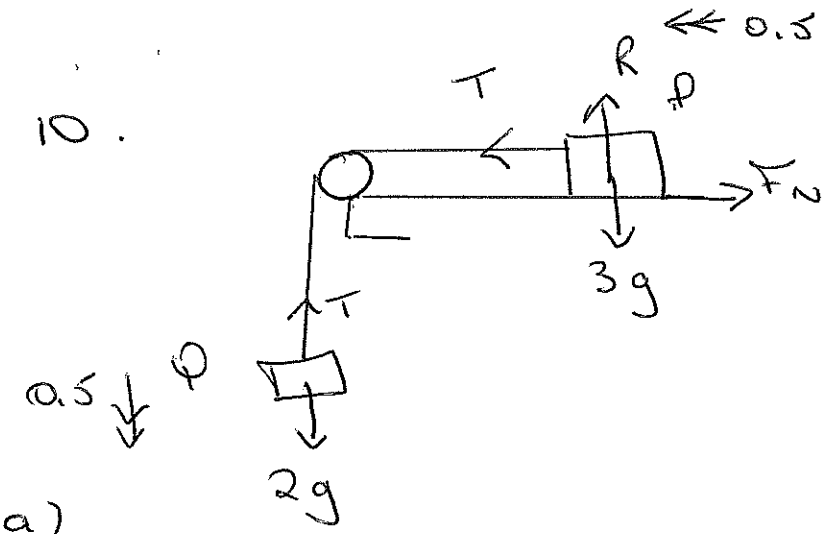
$= 190\underline{i} - 40\underline{j}$



Bearing $90° + \arctan\frac{40}{190}$

$= 102°$

10.



a)

For Q: [F = ma]

$2g - T = 2 \times 0.5$

$T = 18.6\,N$

b) $\uparrow$: $R = 3g$

For P: [F = ma]

$18.6 - \mu 3g = 3 \times 0.5$

$\frac{18.6 - 3 \times 0.5}{3g} = \mu$

$\mu = 0.58$

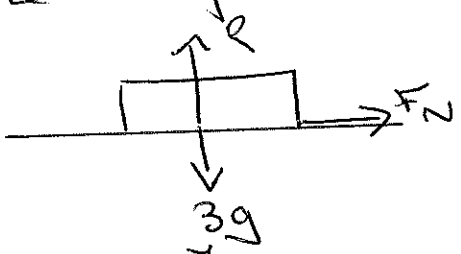
To find the velocity when the string breaks.

[v = u + at]

$v = 0 + 0.5 \times 2.$

$= 1\,ms^{-1}$

After the sting breaks



[F = ma]

$0 - 0.58 \times 3g = 3a.$

$a = -5.68\,ms^{-2}$

Continued →

10 continued

$u = 1$
$v = 0$
$a = -5.68$
t ?

$[v = u + at]$

$0 = 1 - 5.68t$

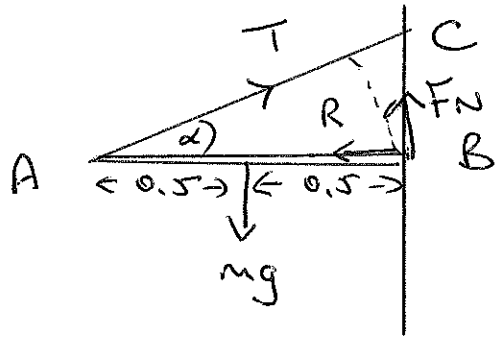
$t = \frac{1}{5.68}$

$t = 0.176$ S.

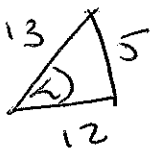
time to come to rest.

d) tension and acceleration equal in both strings

11.



$\tan\alpha = \frac{5}{12}$



$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

a)

$\curvearrowright$: $mg \times 0.5 = T \times 1 \times \sin\alpha$.

$mg\,0.5 = T \times \frac{5}{13}$

$\frac{13mg \times 0.5}{5} = T$

$T = \frac{13mg}{10}$ N

b) $\rightarrow$: $R = T\cos\alpha$

$= \frac{13mg}{10} \times \frac{12}{13}$

$= \frac{6mg}{5}$

$\uparrow$: $T\sin\alpha + F_N = mg$

$\frac{13mg}{10} \times \frac{5}{13} + F_N = mg$

continued →

11 b) continued

$$F_N = mg - \frac{mg}{2}$$

$$= \frac{mg}{2}$$

$$\mu = \frac{F_N}{R}$$

$$= \frac{mg/2}{6mg/5}$$

$$= \frac{mg}{2} \times \frac{5}{6mg}$$

$$= \frac{5}{12}$$